Heavy Quark Masses from Heavy Meson Masses

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Project in Collaboration with
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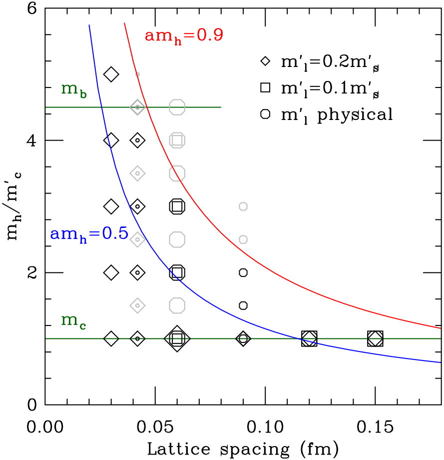
Motivation and Goal

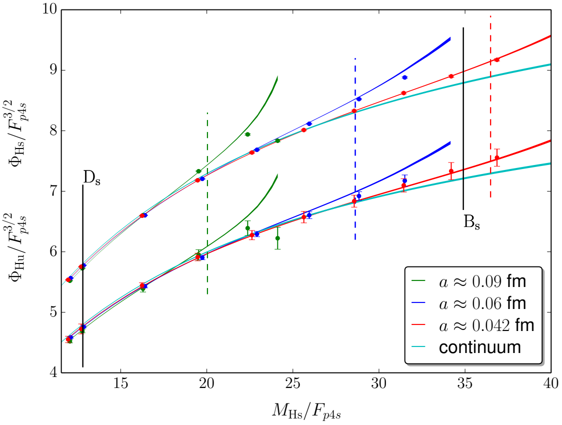
- Heavy-light meson systems can be used to test the standard model and look for signs of new physics:
  - Decay constants $\to$ CKM matrix elements
  - m_b and μ_π^2 $\to$ inclusive determination of V_{ub} and V_{cb}
  - m_b and m_c $\to$ Higgs boson branching ratios
- Lattice QCD allows us to calculate the decay constants and masses of heavy-light systems for various choices of heavy quark mass
- HQET allows us to describe the dependence of these quantities on heavy quark mass
- We take advantage of EFTs to analyze lattice data to extract decay constants and heavy quark masses

2+1+1 HISQ Ensembles Generated by MILC Collaboration



- Lattice spacings from 0.15fm to 0.03fm
- Various light sea masses
- am_c': simulated sea charm mass
- am_h: valence heavy mass
- Discretization effects: $\mathcal{O}(\alpha_s(am_h)^2)$ and $\mathcal{O}((am_h)^4)$
- Use only data with $am_h < 0.9$ for each ensemble

MILC/Fermilab Decay Constant Project



See arXiv:1407.3772 and arXiv:1511.02294 for details

- Decay constants for various combinations of light and heavy quark masses
- Dependence on heavy quark described by HQET
- Dependence on light quark mass and lattice artifacts described by SChPT for heavy-light mesons
- Yields decay constant of B and D systems ($F_{p4s} \approx 154$MeV)

Meson Mass in HQET

- Meson mass of a heavy-light system in terms of heavy quark mass

$$M_H = m_h + \bar{\Lambda} + \frac{\mu_\pi^2}{2m_h} - \frac{\mu_G^2(m_h)}{2m_h} + \cdots$$

- Challenges:
  - How apply it to lattice data?
    How to map the bare mass in lattice units to some renormalized mass?
  - What quark mass?
    Renormalon problem in the relation between pole and $\overline{\text{MS}}$ masses.

Renormalon in Pole Mass

- Consider the recurrence relation

$$R_n = 2n\beta_0 R_{n-1} + 2(n-1)\beta_1 R_{n-2} + \cdots + 2\beta_{n-1} R_0 \,, \quad n > 0$$

- Has one solution as

$$\begin{aligned} R_1 &= 2\beta_0 R_0 \\ R_2 &= (2\beta_0)^2 (2 + \frac{\beta_1}{2\beta_0^2}) R_0 \\ &\vdots \\ \Rightarrow \quad R_n &= N \; (2\beta_0)^n \frac{\Gamma(n+1+b)}{\Gamma(1+b)} \left(1 + \frac{s_1}{n+b} + \cdots \right) \end{aligned}$$

where $b = \beta_1/(2\beta_0^2)$, $s_1 = b^2 - \beta_2/(4\beta_0^3)$.

- R_n appears as the first renormalon in the relation between the pole mass and $\overline{\text{MS}}$ mass.

Renormalon-Subtracted Mass

- Consider

$$m^{\text{pole}} = \overline{m}\left(1+\sum_{n=0} r_n \alpha_s^{n+1}(\overline{m})\right)$$

where for large n

$$r_n \sim R_n = N_m\,(2\beta_0)^n \frac{\Gamma(n+1+b)}{\Gamma(1+b)}\left(1+\frac{s_1}{n+b}+\cdots\right)$$

- The overall constant N_m can be calculated perturbatively, *e.g.*, $N_m = 0.563(26)$ [Ayala et al hep-ph/1407.2128]

- Use renormalon-subtracted scheme to subtract the renormalon part [A. Pineda hep-ph/0105008]

$$\begin{aligned} m^{\text{RS}} &\equiv m^{\text{pole}} - \nu_f \sum_{n=0} R_n \alpha_s^{n+1}(\nu_f) \\ &= \overline{m}\left(1+\sum_{n=0} r_n \alpha_s^{n+1}(\overline{m})\right) - \nu_f \sum_{n=0} R_n \alpha_s^{n+1}(\nu_f) \\ &= \overline{m}\left(1+\sum_{n=0} r_n^{\text{RS}}(\overline{m},\nu_f,\mu)\alpha_s^{n+1}(\mu)\right) \end{aligned}$$

Quark Masses on Lattice

- The key relation

$$\frac{am_2}{am_1} \to \frac{m_2^{\overline{\mathsf{MS}}}(\mu)}{m_1^{\overline{\mathsf{MS}}}(\mu)} \quad \text{as} \quad a \to 0$$

- Following [Mason,et al hep-lat/0510053]

$$m^{\text{pole}} = \frac{am}{a}\left[1 + \alpha_{\mathsf{Lat}}\left(-\frac{2}{\pi}\log(am) + A_{10}\right) + \mathcal{O}(\alpha_{\mathsf{L}}^2)\right]$$

where $A_{10} = \text{const.} + \mathcal{O}\big((am)^2\big)$; lattice artifacts

- Use

$$\alpha_{\mathsf{Lat}}^{-1} = \alpha_{\overline{\mathsf{MS}}}^{-1}(\mu) - 2\beta_0 \ln\big(a\mu/\pi\big) + \text{const.} + \mathcal{O}(\alpha_{\overline{\mathsf{MS}}})$$

- For the ratios

$$\frac{m_2^{\text{pole}}}{m_1^{\text{pole}}} = \frac{am_2}{am_1}\left(1 - \frac{2}{\pi}\alpha_{\overline{\mathsf{MS}}}(\mu)\left[\log(\frac{am_2}{am_1}) + A_{10}\big((am_2)^2\big) - A_{10}\big((am_1)^2\big)\right] + \mathcal{O}(\alpha^2)\right),$$

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- Use

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- For the ratios

$$\frac{m_2^{\text{pole}}}{m_1^{\text{pole}}} = \frac{am_2}{am_1}\left(1 - \frac{2}{\pi}\alpha_{\overline{\mathsf{MS}}}(\mu)\Big[\log\big(\frac{am_2}{am_1}\big) + A_{10}\big((am_2)^2\big) - A_{10}\big((am_1)^2\big)\Big] + \mathcal{O}(\alpha^2)\right),$$

- Consider a fit parameter to capture this lattice artifact as

$$A_{10}\big((am_2)^2\big) - A_{10}\big((am_1)^2\big) \quad \to \quad K\left[(am_2)^2 - (am_1)^2\right]$$

Procedure to Map Lattice Mass to Pole Mass

- Take the continuum limit relation of the $\overline{\text{MS}}$ and pole mass at three-loop level

$$\frac{m_h^{\rm pole}}{m_c^{\rm pole}} = \frac{m_h(\mu)}{m_c(\mu)}\left[1 - \frac{2\alpha_s(\mu)}{\pi}\log(\frac{m_h(\mu)}{m_c(\mu)}) + \cdots\right],$$

- Add the lattice artifact at $\mathcal{O}\big(\alpha\ (am)^2\big)$ to the above expression
- At each ensemble calculate the tuned charm mass am_c
- Finally map

$$\frac{am_h}{am_c} \ \rightarrow \ \frac{m_h(\mu)}{m_c(\mu)} \ \rightarrow \ \frac{m_h^{\rm pole}}{m_c^{\rm pole}} \rightarrow \ \frac{m_h^{\rm RS}}{m_c^{\rm RS}}$$

- Define a fit parameter $m_c^{\rm RS}$, then $m_h^{\rm RS} = \left(\frac{m_h^{\rm RS}}{m_c^{\rm RS}}\right) m_c^{\rm RS}$

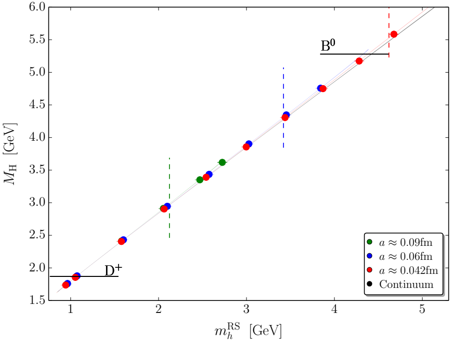
The Fit Function

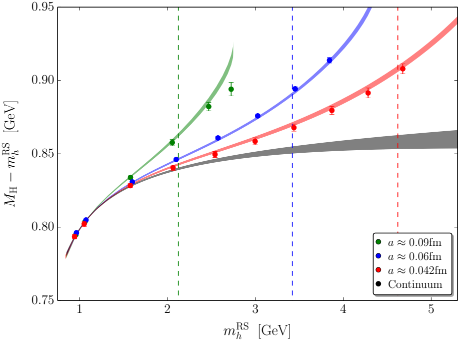
- The fit basic function in terms of RS mass

$$M_H = m_h^{\mathrm{RS}} + \bar{\Lambda} + \frac{\mu_\pi^2}{2m_h^{\mathrm{RS}}} - \frac{\mu_G^2(m_h^{\mathrm{RS}})}{2m_h^{\mathrm{RS}}} + \frac{\lambda_3}{(m_h^{\mathrm{RS}})^2}$$

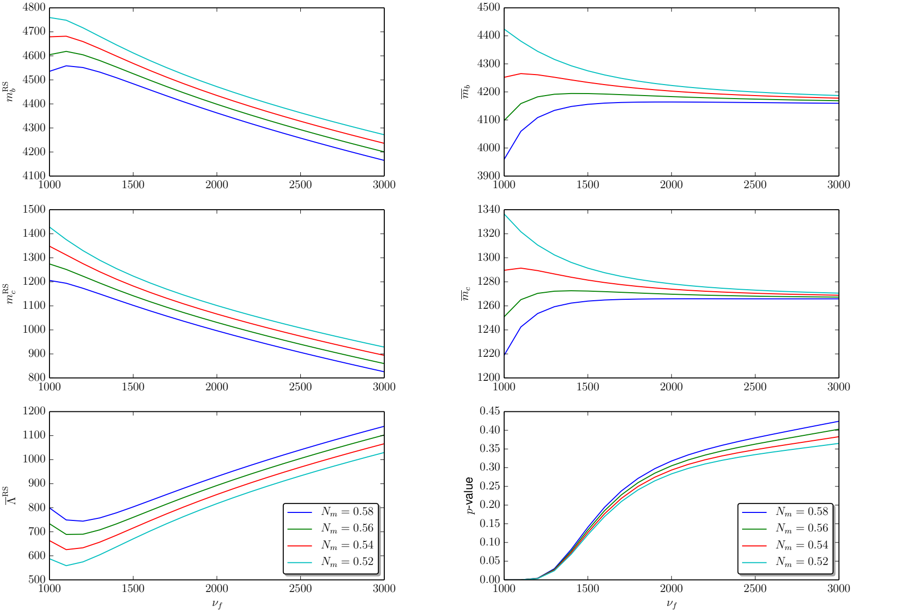
- Add more terms if required
- Recall previously introduced fit parameters:
  - K (to capture lattice artifacts)
  - m_c^{RS}

- Fit to heavy-light masses from HISQ ensembles of MILC collaboration
- A fit with 16 data points and 6 fit parameters
- Data points at three lattice spacings with $am'_c \leq am_h < 0.9$
- Scheme: RS, $N_m = 0.54$, $\nu_f = 2$GeV and $\mu = 2.5$GeV





Fit to Lattice Data: Dependence on N_m and $\nu_f = \mu$



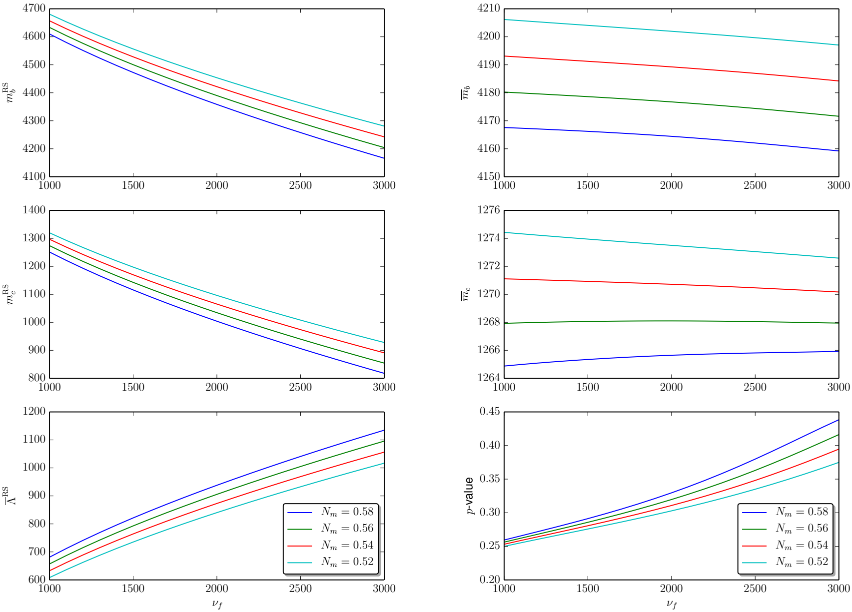
Conclusion

- HQET description of heavy-light meson masses is used to analyze the lattice data
- Various sources of systematic error will be taken into account for the full error analysis:
    - Statistical errors are tiny: a challenge to good fits
    - Stability under higher orders in a^2 and $1/m_h$
    - Improve determination of N_m and its uncertainty (in progress)
    - Choose well motivated ranges for ν_f and μ
    - Error of α_s

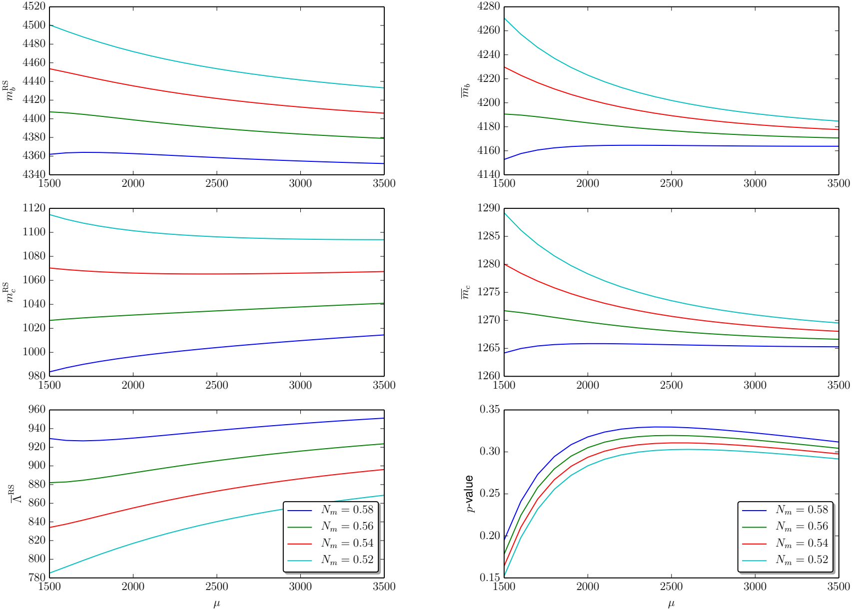
Thank you for your attention!

Back-up Slides

Fit to Lattice Data (Dependence on N_m and ν_f)



Fit to Lattice Data (Dependence on N_m and μ)



Renormalon-Subtracted Mass

- Use renormalon-subtracted scheme to subtracted the renormalon part
[A. Pineda hep-ph/0105008]

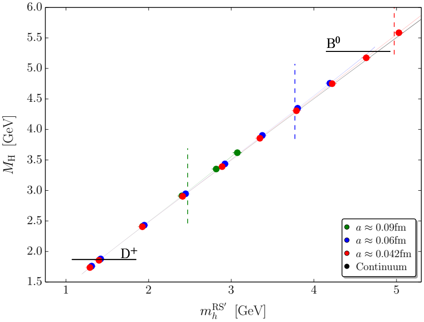
$$
\begin{aligned}
m^{\mathrm{RS}} &\equiv m^{\mathrm{pole}} - \nu_f \sum_{n=0} R_n \alpha_s^{n+1}(\nu_f) \\
&= \overline{m}\left(1 + \sum_{n=0} r_n \alpha_s^{n+1}(\overline{m})\right) - \nu_f \sum_{n=0} R_n \alpha_s^{n+1}(\nu_f) \\
&= \overline{m}\left(1 + \sum_{n=0} r_n^{\mathrm{RS}}(\overline{m}, \nu_f, \mu) \alpha_s^{n+1}(\mu)\right)
\end{aligned}
$$

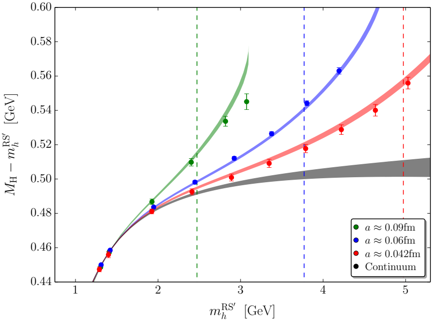
or a modified version

$$
m^{\mathrm{RS}'} \equiv m^{\mathrm{pole}} - \nu_f \sum_{n=1} R_n \alpha_s^{n+1}(\nu_f)
$$

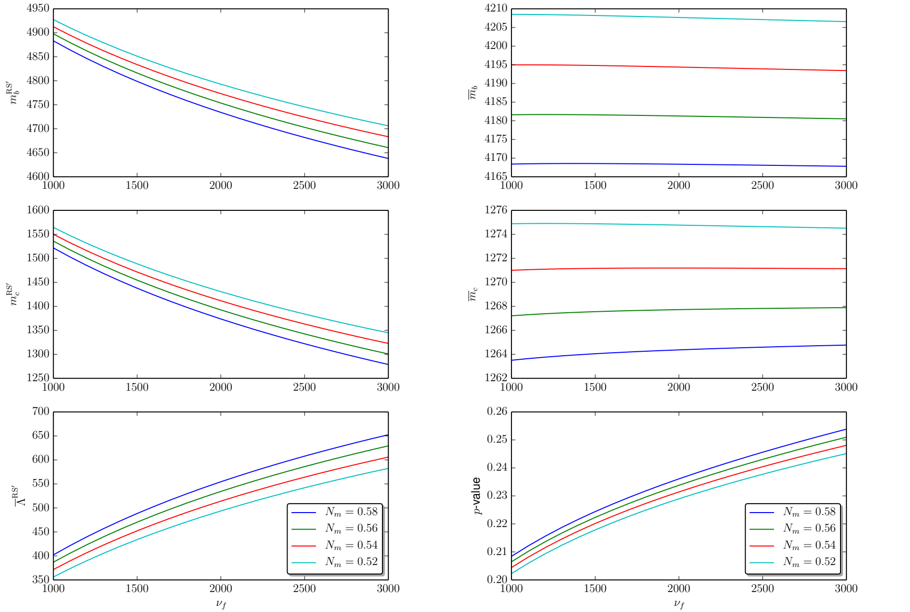
Fit to Lattice Data

- Illustration of a fit to heavy-light meson mass from HISQ ensembles of MILC collaboration
- A fit with 16 data points and 6 fit parameters
- Data points at three lattice spacings with $am'_c \leq am_h < 0.9$
- Scheme:RS', $N_m = 0.54$, $\nu_f = 2$GeV and $\mu = 2.5$GeV





Fit to Lattice Data (Dependence on N_m and ν_f)



Fit to Lattice Data (Dependence on N_m and μ)