SM parameters from Lattice QCD

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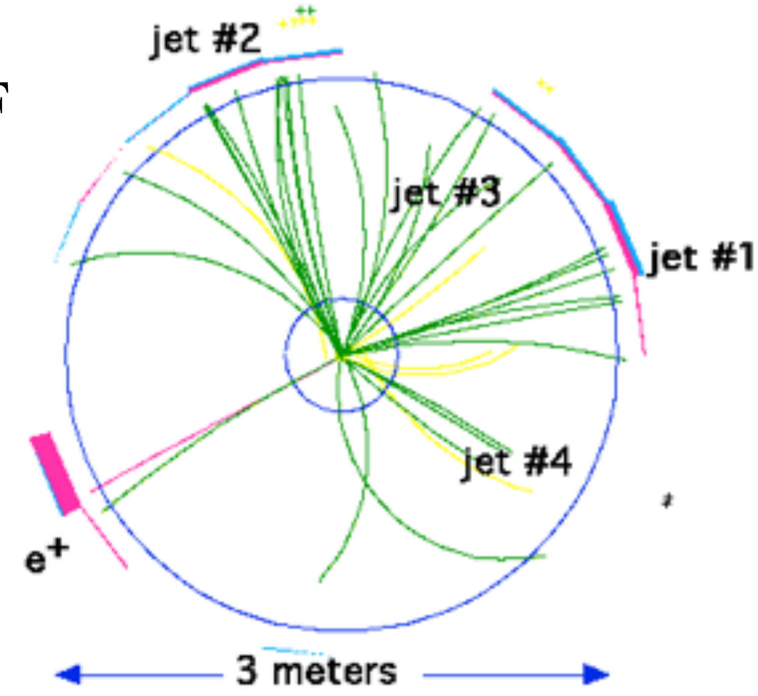
EFT & LGT
Munich, May 2016

Quark masses and strong coupling are fundamental parameters of the SM but cannot be directly determined from experiment.

CDF



Well-defined m_q/α_s are scheme and scale-dependent.

Convention to use $\overline{MS}$

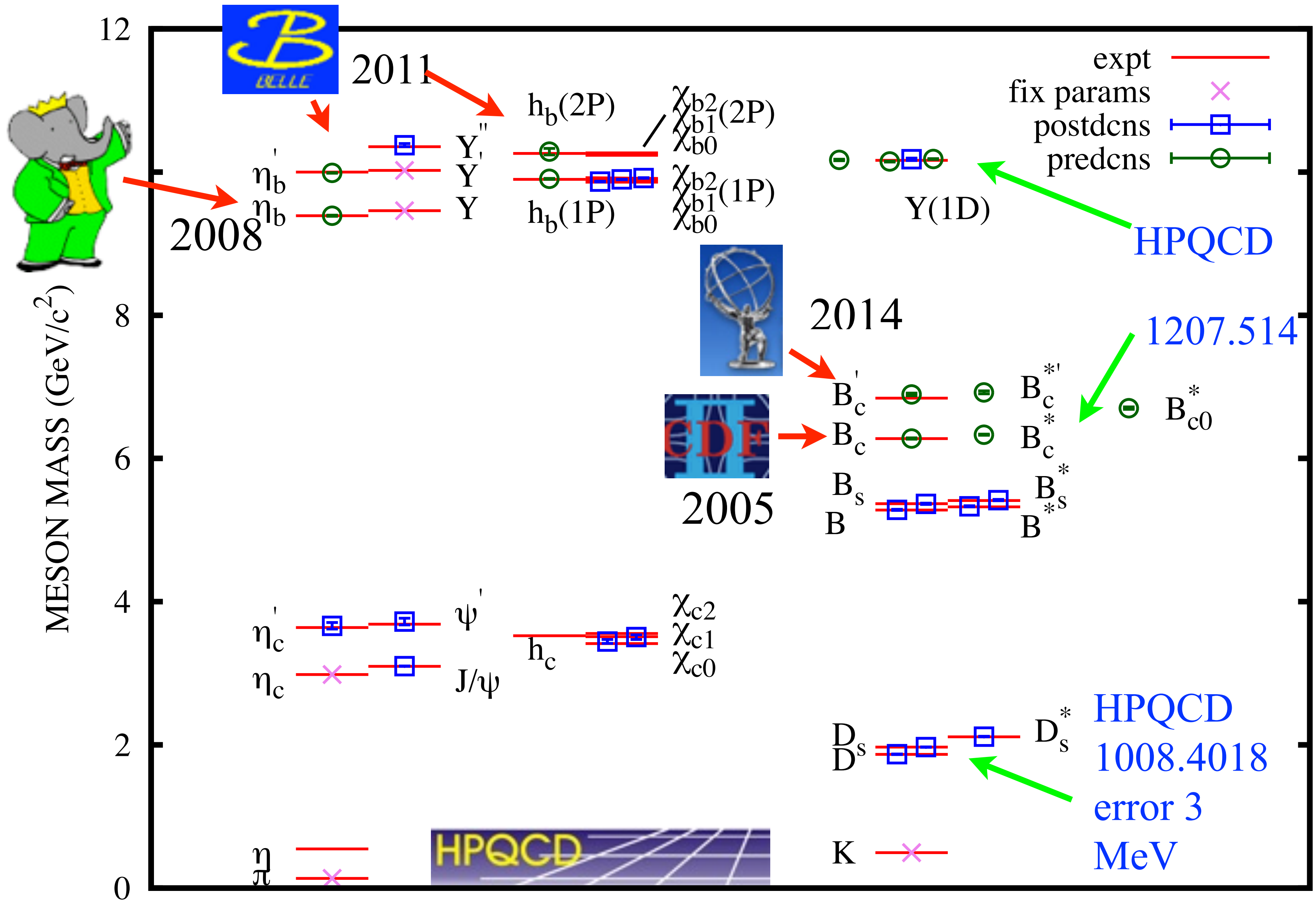
Compare results from multiple approaches for strong test of QCD.

Masses are input to theoretical expressions for SM cross-sections e.g. $H \to c\bar{c}$

	Higgs X-Section WG	PDG	lattice	Karlsruhe (e^+e^-)	world non-lattice
$\delta\ \alpha_s$	0.002	0.0007	0.0007		0.0012
$\delta\ m_c$ (GeV)	0.03	0.025	0.006	0.013	
$\delta\ m_b$ (GeV)	0.06	0.03	0.023	0.016	

P. Mackenzie, Snowmass 2013

Lattice QCD works directly with the QCD Lagrangian. Can tune bare mass parameters very accurately using experimentally very well-determined hadron masses.



Conversion of lattice quark masses to $\overline{MS}$ scheme

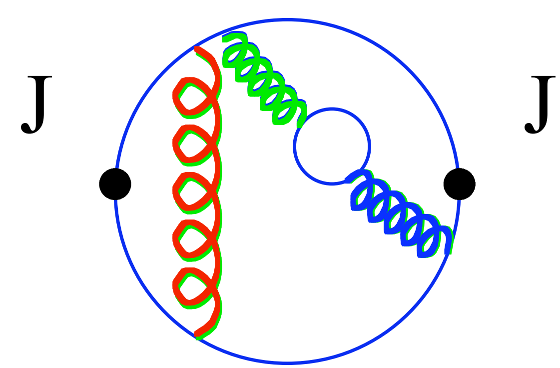
- Direct methods: Determine $m_{q,latt}$ in lattice QCD.

$$m_{\overline{MS}}(\mu) = Z_m(\mu a) m_{latt}$$

Calculate Z in lattice QCD pert. th. or use 'nonpert' RI-MOM lattice matching.
Error dominated by that of Z and continuum extrapolation.
Note: Z cancels in mass ratios.

- Indirect methods: (after tuning m_{latt}) match a quantity calculated in lattice QCD to continuum pert. th. in terms of $\overline{MS}$ quark mass

e.g. Current-current correlators for heavy quarks known through α_s^3 .



Chetyrkin et al, Maier et al

Issues with handling 'heavy' quarks on the lattice:

$$L_q = \overline{\psi}(\not{D} + m)\psi \rightarrow \overline{\psi}(\gamma \cdot \Delta + ma)\psi$$

Δ is a finite difference on the lattice - leads to discretisation errors. What sets the scale for these?

For light hadrons the scale is Λ_{QCD} = few hundred MeV

For heavy hadrons the scale can be m_Q

$$E(a) = E(a=0) \times (1 + A(m_Q a)^2 + B(m_Q a)^3 + \ldots)$$

$m_c a \approx 0.4, m_b a \approx 2$ for $a \approx 0.1\text{fm}$

➡ need good discretisation of Dirac equation and multiple values of a for accurate continuum extrapolation.

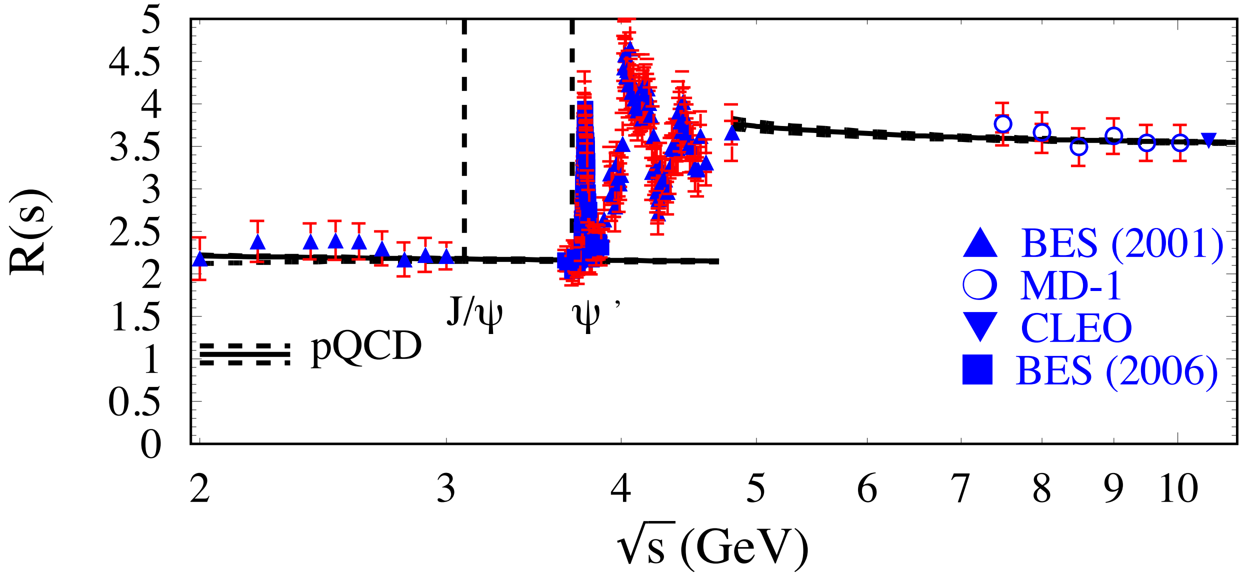
Highly Improved Staggered Quarks (HISQ) formalism has errors improved to $\alpha_s(am)^2, (am)^4$ Follana et al, HPQCD, hep-lat/0610092

Current-current correlator method for m_c

Continuum: extract charm piece of:

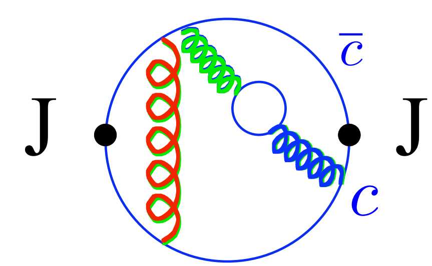
e.g. Kuhn et al, hep-ph/0702103

$$R_{e^+e^-}(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{4\pi\alpha^2/(3s)}$$ from experiment, then



$$\mathcal{M}_k \equiv \int \frac{ds}{s^{k+1}} R_{e^+e^-}(s)$$

$$= \frac{12\pi^2}{n!}\left(\frac{d}{dq^2}\right)^k \Pi_c(q^2)|_{q^2=0}$$



vector coupling to photon

$$\Pi_c(q^2) = \frac{3}{16\pi^2} e_c^2 \sum_{k\geq 0} C_k^V \left(\frac{q^2}{4(m_c(\mu))^2}\right)^k$$

C_k a power series in $\alpha_s(\mu)$, known through α_s^3 for first few values of k

Use k=1: $m_c(m_c) = 1.279(13)\text{GeV}$

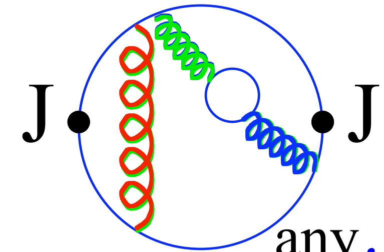
errors: expt + α_s

Chetyrkin et al, 0907.2110

Current-current correlator method for lattice m_c

HPQCD + Chetyrkin et al, 0805.2999, C. Mcneile et al, HPQCD,1004.4285

• Substitute time-moment of lattice charmonium correlator for experiment. In principle can use any current J now.



any J^{PC} current now

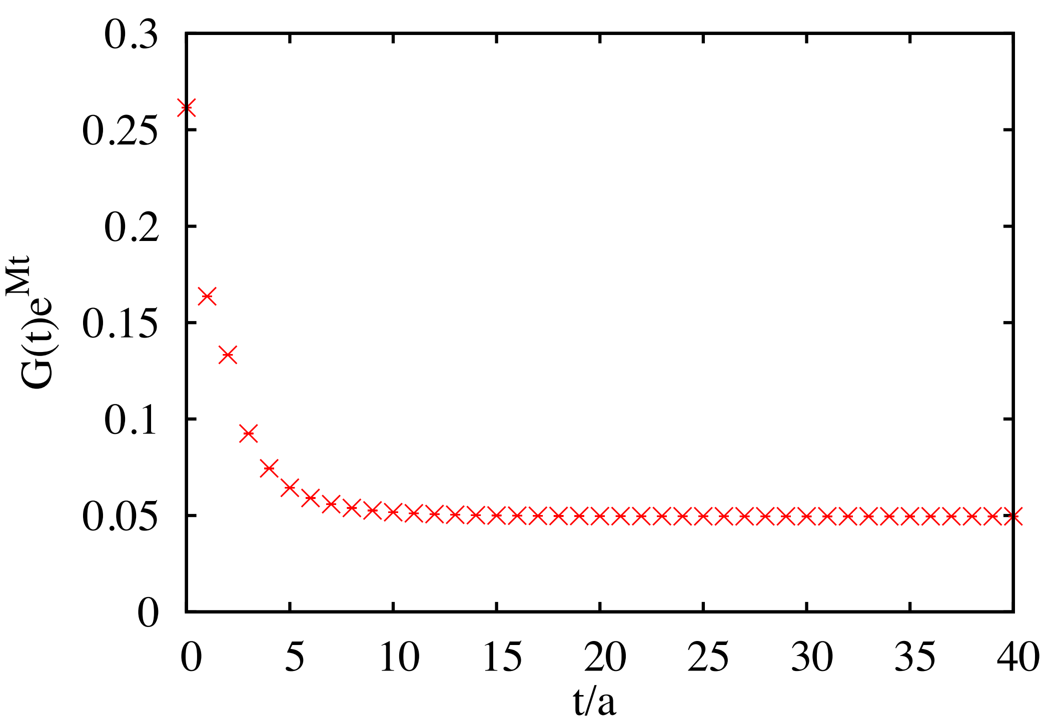
• For HISQ quarks pseudoscalar η_c correlator is most accurate. J is absolutely normalised.

step 1: calculate η_c correlators by combining lattice charm quark propagators

step 2: large time - fit to exponential, gives η_c mass

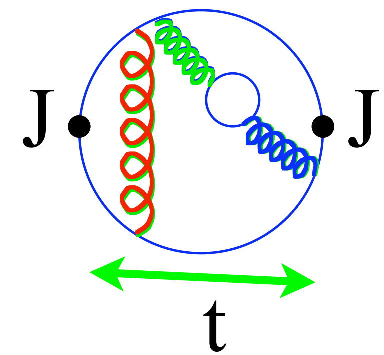
step 3: tune lattice quark mass so η_c mass correct.

step 4: calculate time moments to compare to QCD pert. theory. Emphasises short-time contribns.



Correlator time-moments:

$$G(t) = a^6 \sum_{\vec{x}} (am_c)^2 < 0|j_5(\vec{x},t) j_5(0,0)|0 >$$



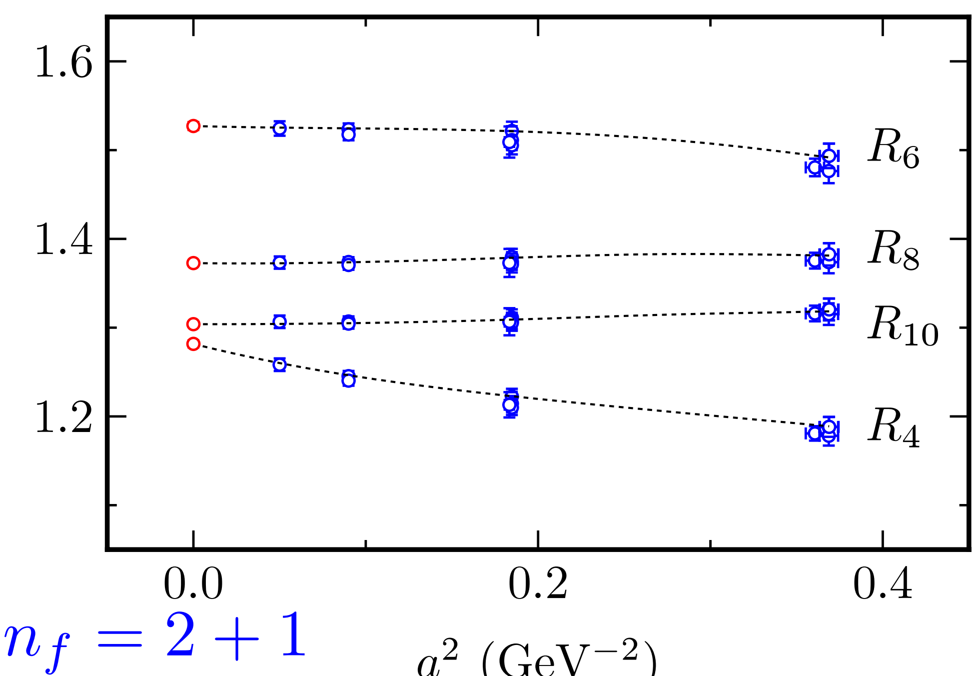
$$G_n = \sum_t (t/a)^n G(t)$$

ratio to results with no gluon field improves disc. errors

$$R_{n,latt} = G_4/G_4^{(0)} \qquad n = 4$$

$$= \frac{am_{\eta_c}}{2am_c}(G_n/G_n^{(0)})^{1/(n-4)} \qquad n = 6, 8, 10 \ldots$$

(match k = 2, 3, 4 ...)



$n_f = 2 + 1$

extrapolate to a=0 and compare to contnm pert. th.

$$R_{n,cont} = \frac{m_{\eta_c}}{2m_c(\mu)} \frac{C_k^P}{C_k^{P,0}} \qquad n = 2k + 2$$

$$\frac{C_k^P}{C_K^{P,0}} = 1 + \sum c_i \alpha_s^i(\mu)$$

Fit first 4 moments simultaneously, gives $\frac{m_{\eta_c}}{2m_c(\mu)}$ AND $\alpha_s(\mu)$

$\mu = 3m_c$

Result:

$$m_c(m_c) = 1.273(6)\mathrm{GeV}$$

error dominated by unknown higher orders in pert. th.

C. McNeile et al, HPQCD,1004.4285

Further check:

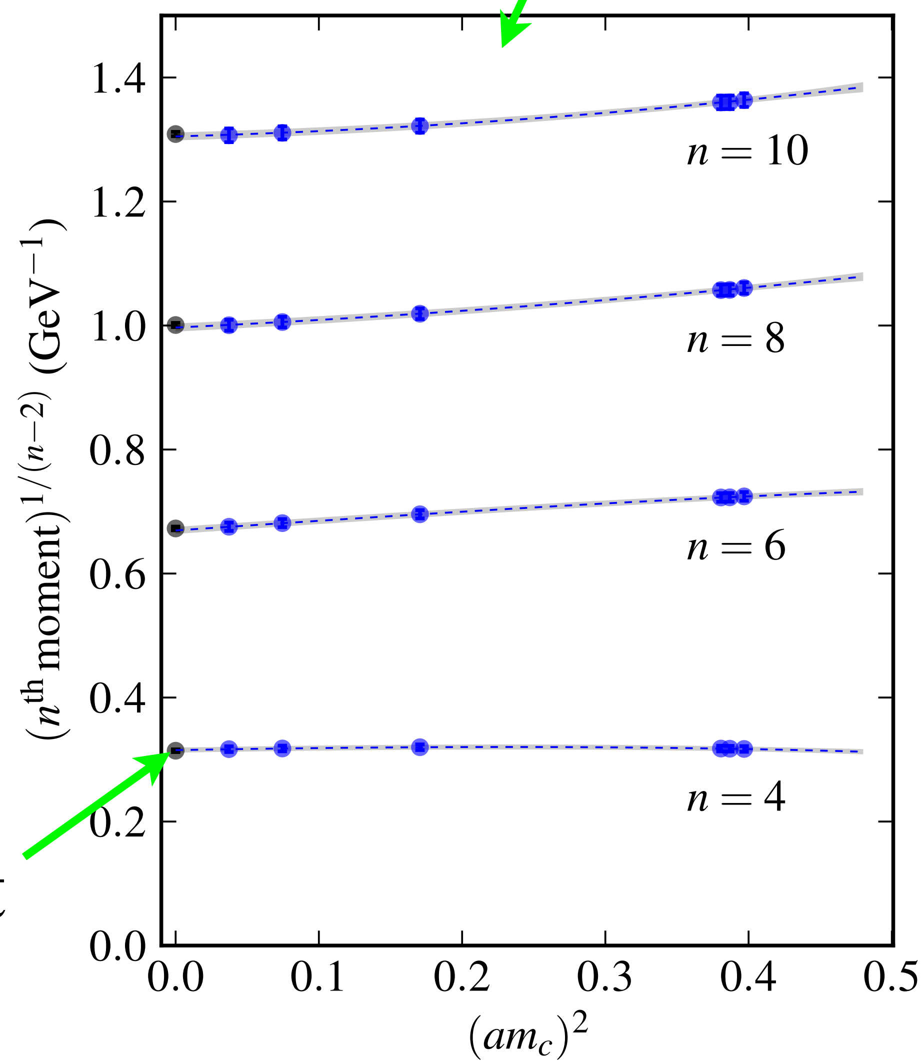
compare vector moments (after normalising current) to those extracted from $R_{e^+e^-}$

Agreement is a 1% test of (lattice) QCD. Gives:

$$a_\mu^c = 14.4(4) \times 10^{-10}$$

HPQCD, 1208.2855, 1403.1778

lattice and expt errors similar size



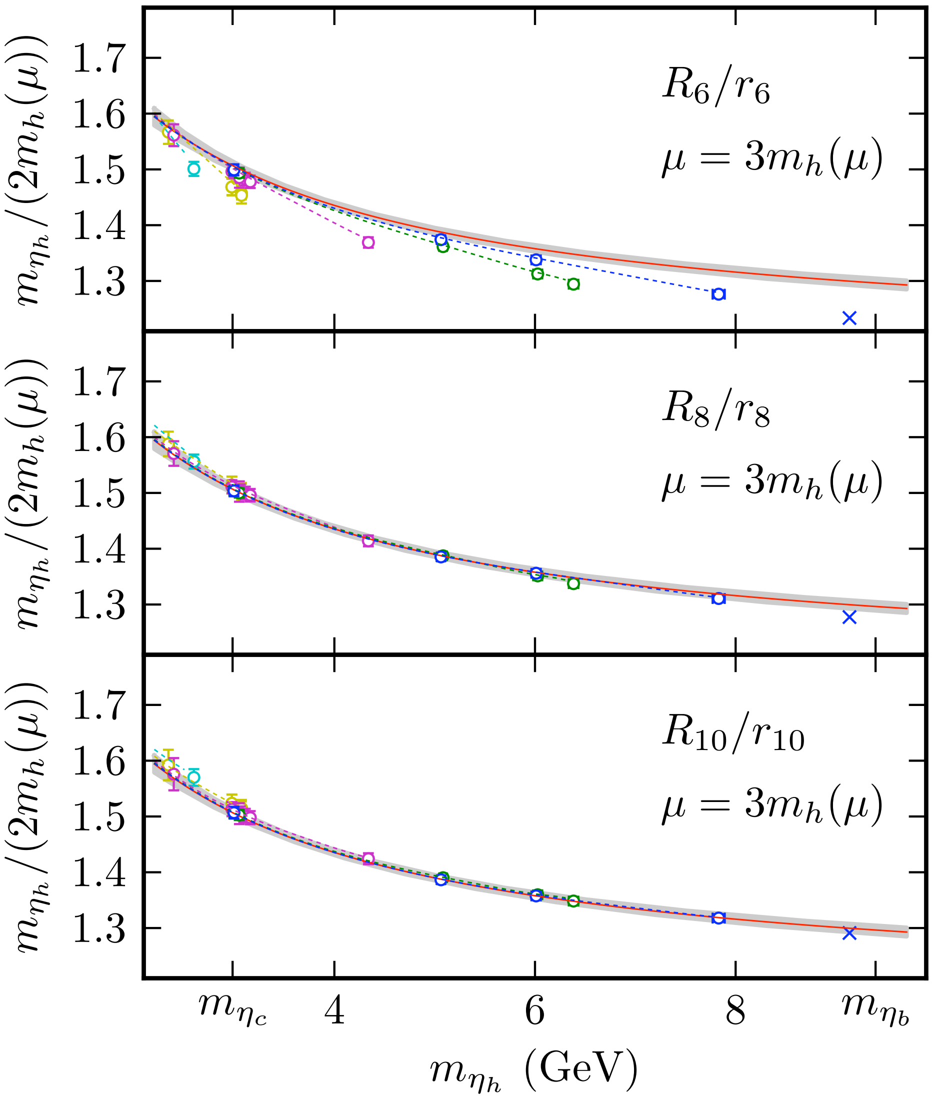
expt

see also ETMC, 1111.5252

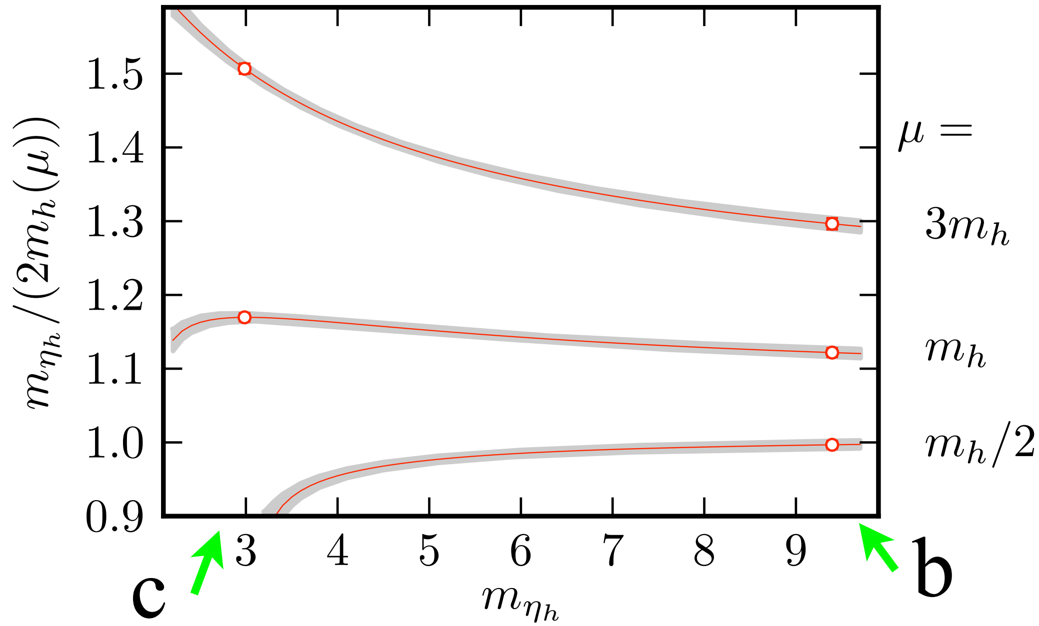
Current-current correlator method -HISQ

HPQCD, 1004.4285

- Repeat calcln for $m_q \geq m_c$ inc. ultrafine lattices



Agrees well with contnm results using $R_{e^+e^-}$



Can determine m_h/m_{η_h} for heavy quarks - extrapolate (slightly) to b.

$$\overline{m}_b^{n_f=5}(\overline{m}_b) = 4.164(23)\text{GeV}$$

key error is now extrapoln in a

Update and improved method

HPQCD, 1408.4169

Use improved $n_f = 2+1+1$ gluon field configs, more accurate lattice spacing determination etc etc.

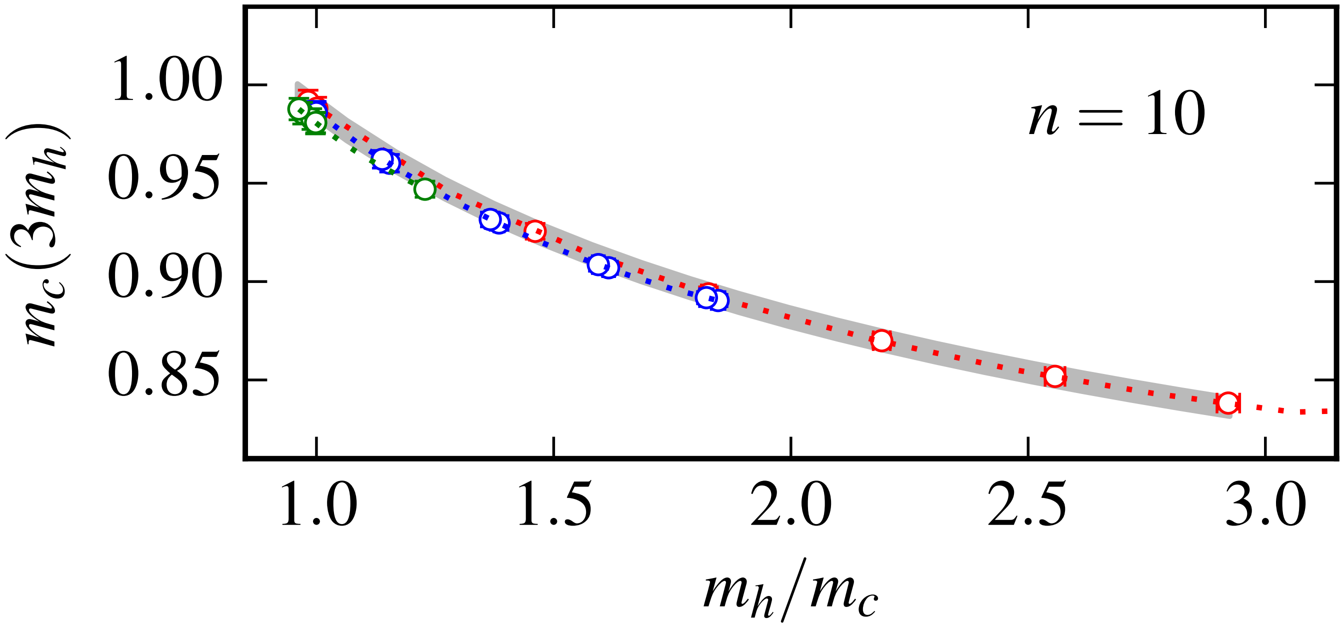
Determine m_c at higher scales by using multiple m_h

$$\tilde{R}_n = \frac{1}{m_c}\left(G_n/G_n^{(0)}\right)^{1/(n-4)} \rightarrow \frac{1}{m_c(\mu)}\frac{C_k^P}{C_k^{P.0}}$$

tuned m_c

G_n at m_h

$\mu = 3m_h$

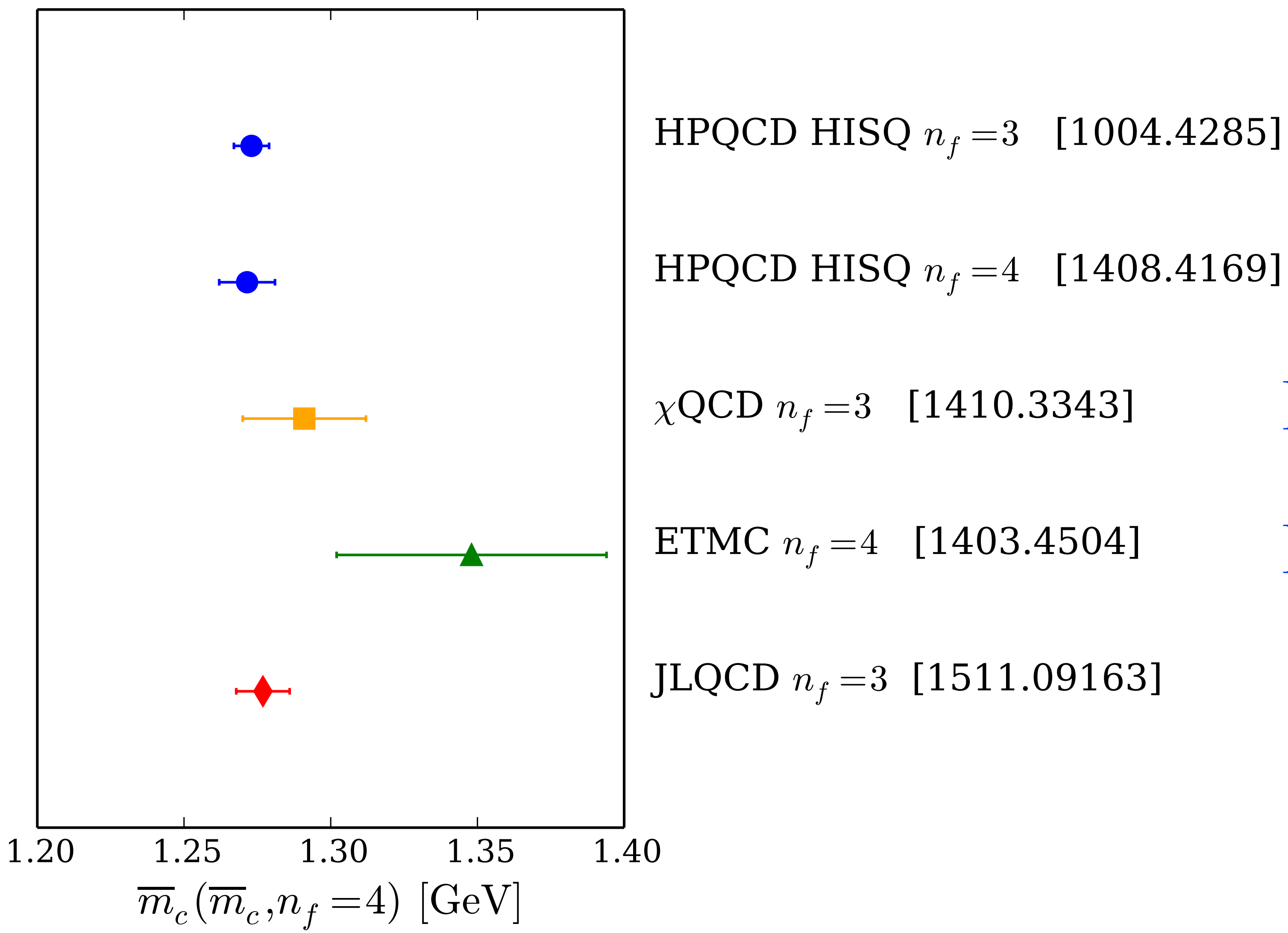


$$\frac{m_{0h}}{m_{0c}} = \frac{m_h(\mu)}{m_c(\mu)}$$

$$m_c(m_c) = 1.2715(95)\mathrm{GeV}$$

m_c summary

Good consistency between lattice methods and actions



JJ

JJ

RI-MOM

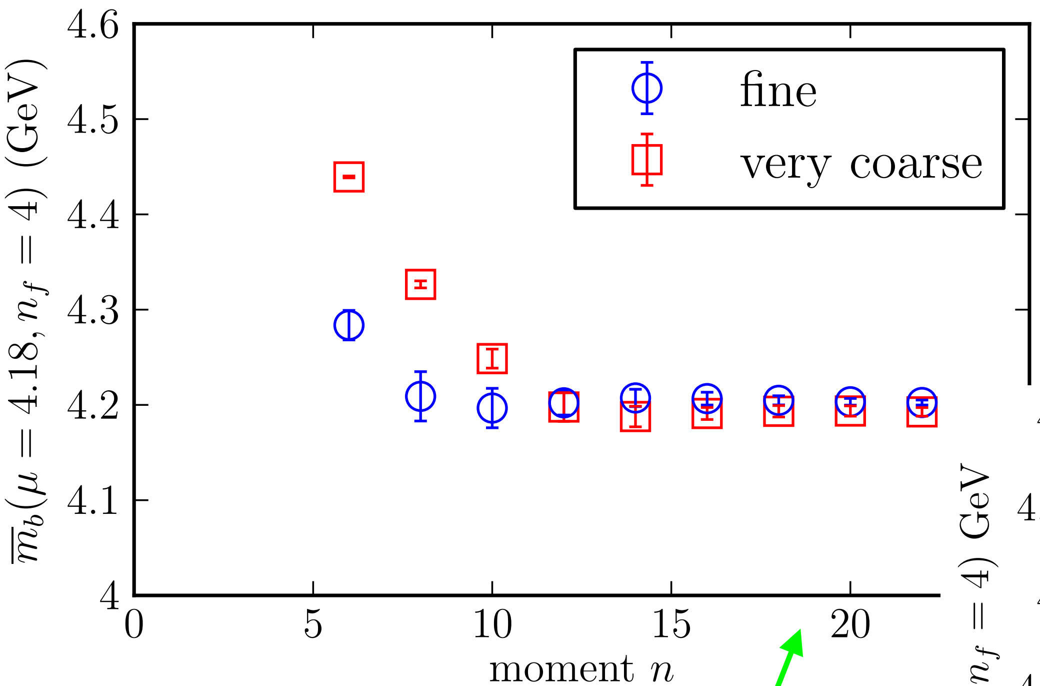
RI-MOM

JJ

Alternative determinations of m_b

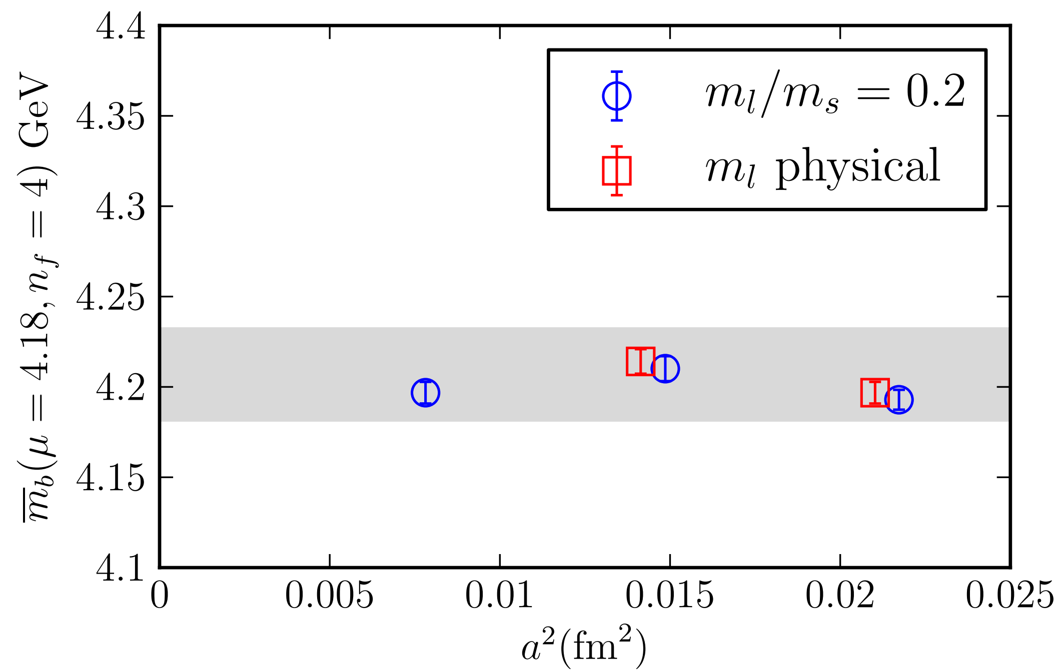
HPQCD, 1408.4768

Current-current correlator method using vector bottomonium correlators calculated with improved NRQCD b quarks



n_f = 2+1+1 HISQ sea quarks

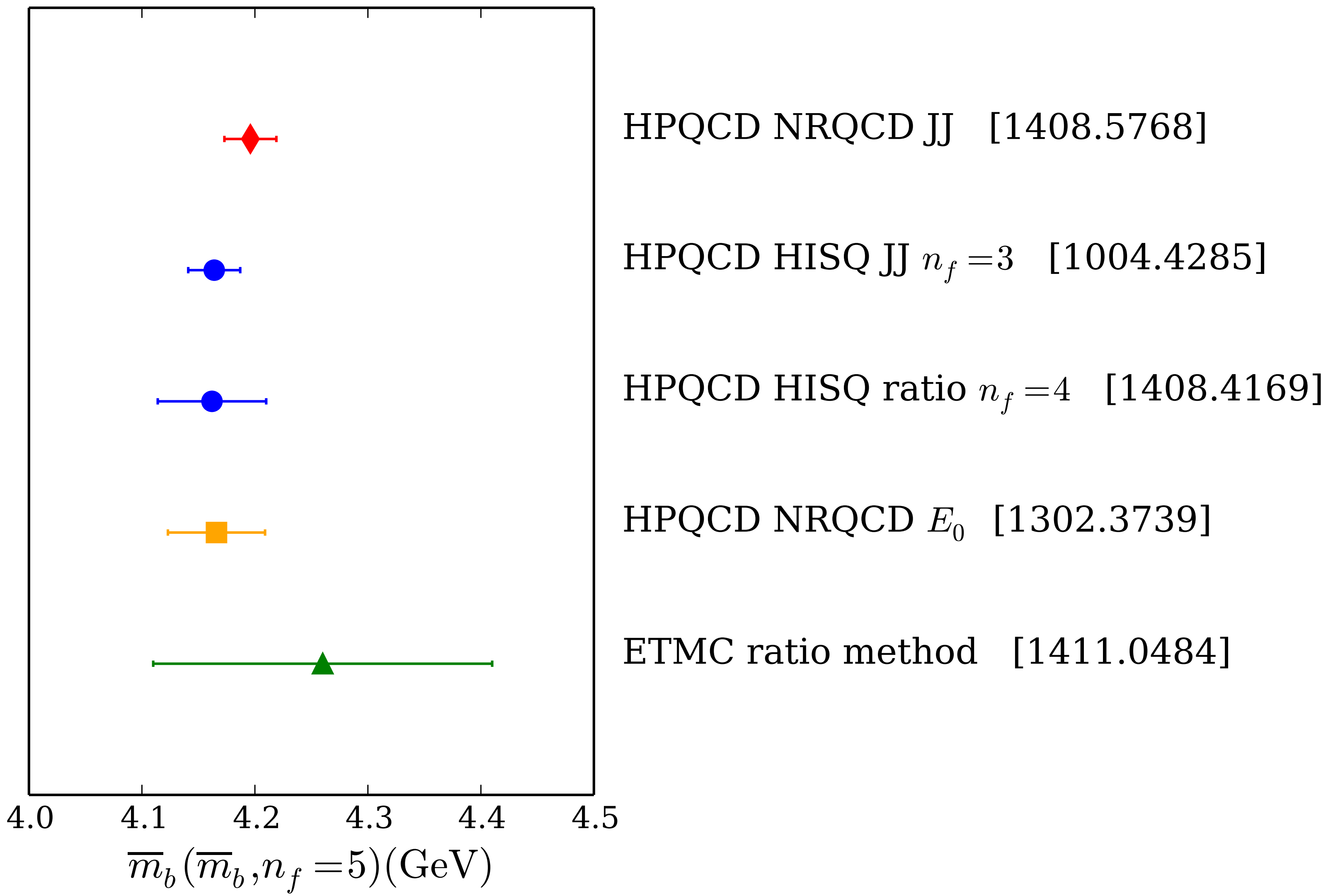
$$\overline{m}_b(\overline{m}_b, n_f = 5) = 4.196(25)\text{GeV}$$



Larger moment numbers more nonrelativistic - use 18

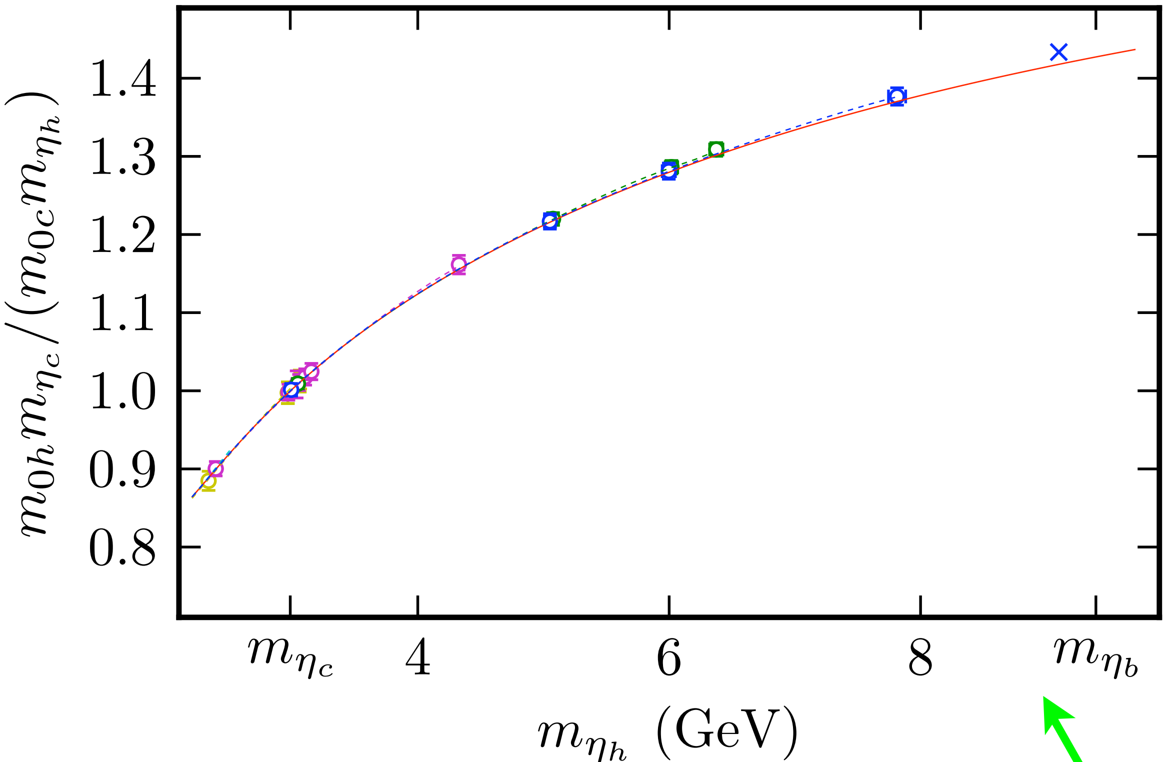
m_b summary

Again, good consistency between different lattice methods



m_b/m_c from lattice QCD

HPQCD, 1004.4285



$$\left(\frac{m_{q1,latt}}{m_{q2,latt}}\right)_{a=0}$$

$$= \frac{m_{q1,\overline{MS}}(\mu)}{m_{q2,\overline{MS}}(\mu)}$$

completely nonperturbative determination of ratio gives:

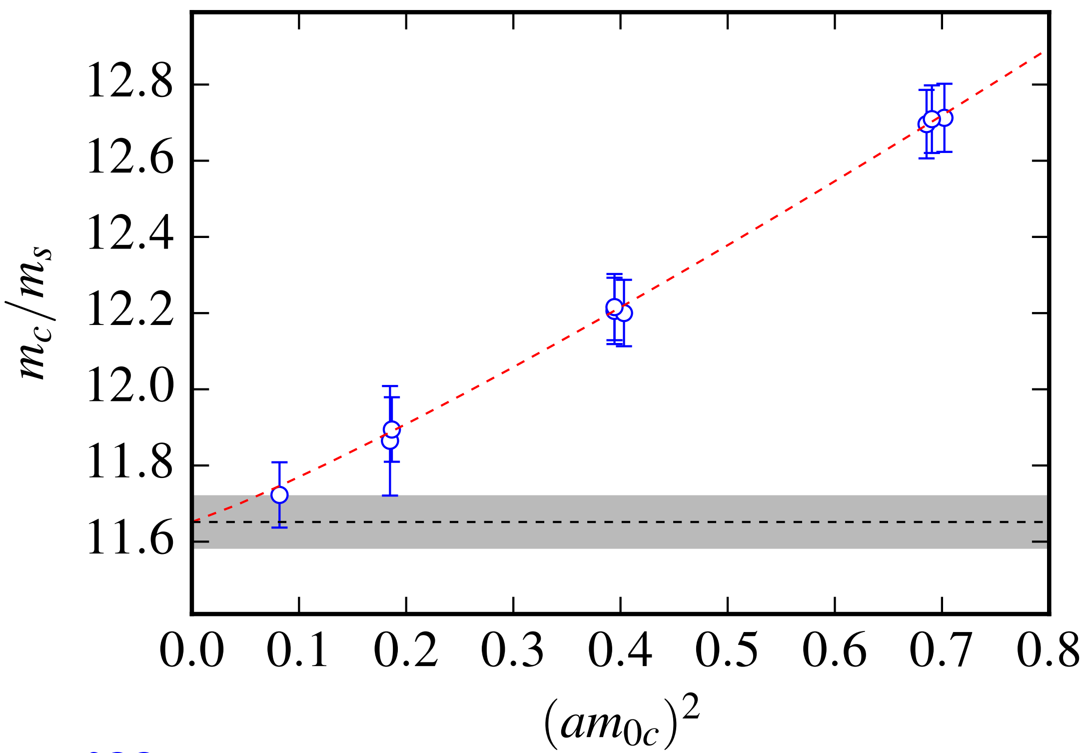
$$\frac{m_b}{m_c} = 4.49(4)$$

Agrees with that from current-current correlator method - test of pert. th. . Also tested n_f=2+1+1

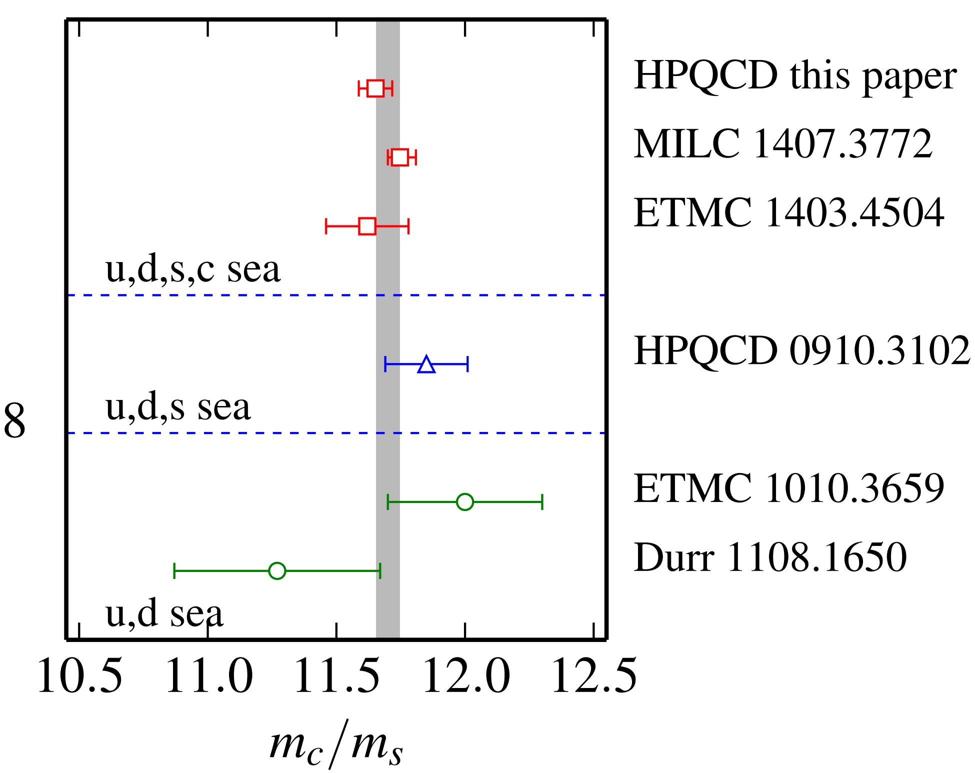
HPQCD, 1408.4169

m_c/m_s

Mass ratio can be obtained directly from lattice QCD if same quark formalism is used for both quarks.
Not possible with any other method ...



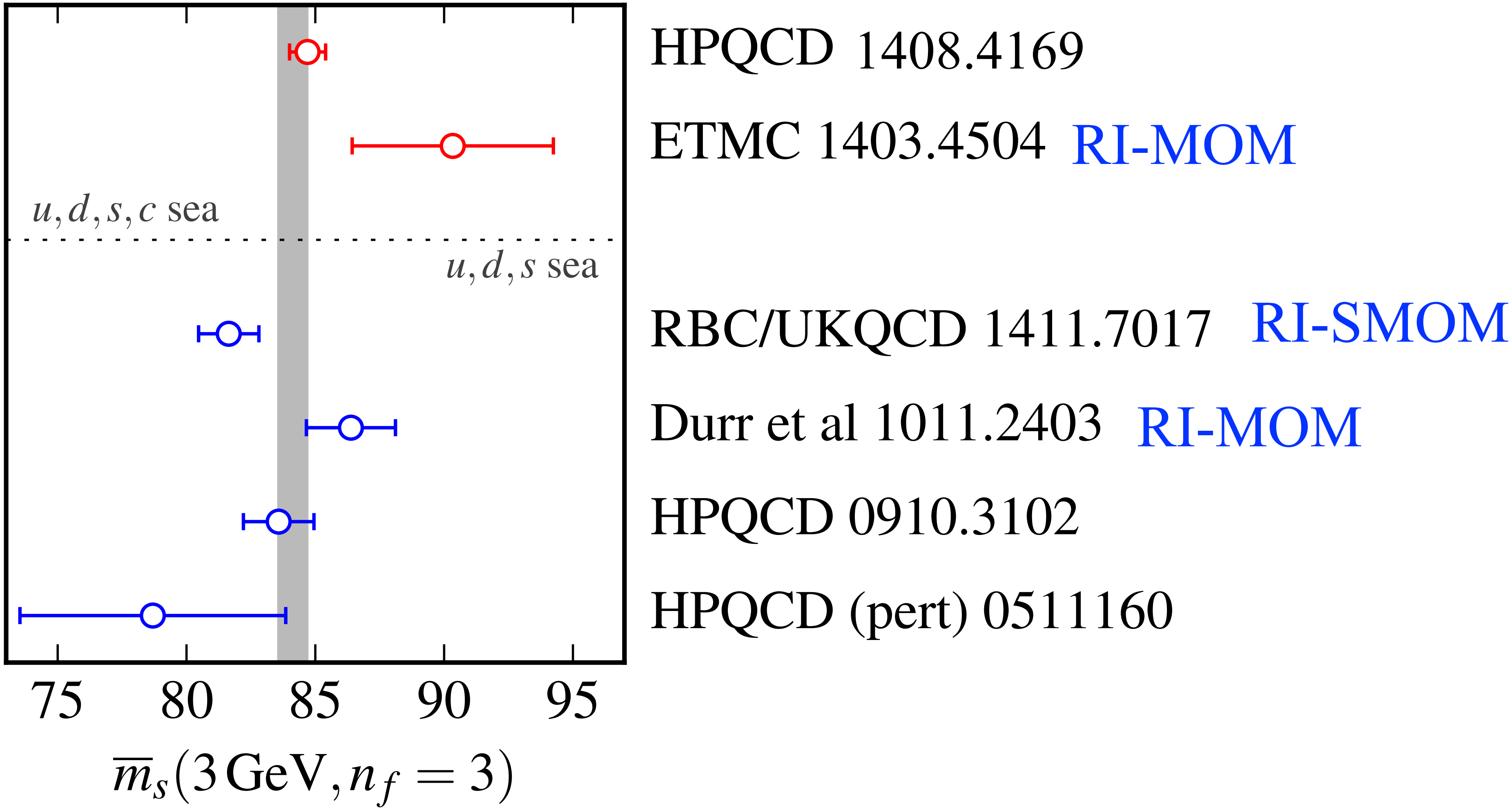
Good consistency between different lattice actions



$$\frac{m_c}{m_s} = 11.652(65)$$

HPQCD, 1408.4169

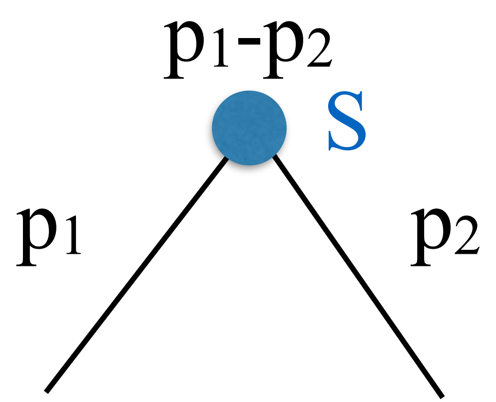
Combining m_c and m_c/m_s leads to 1% accuracy in m_s



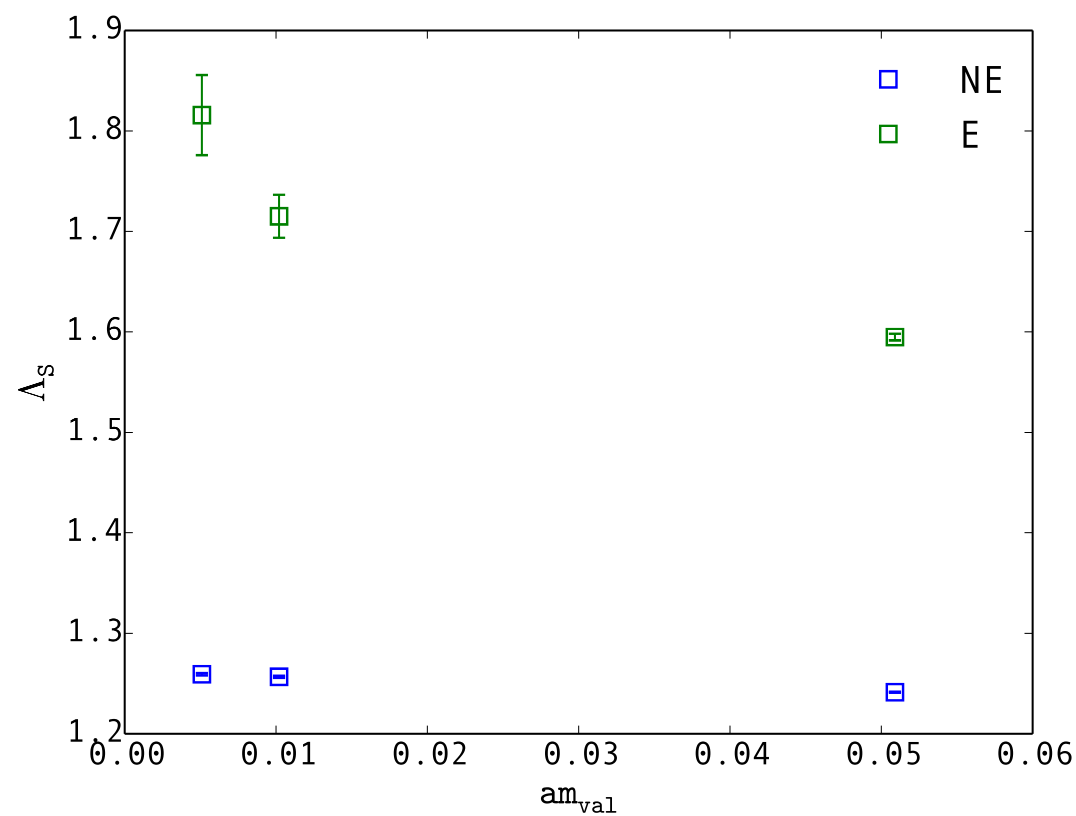
$$\overline{m}_s(3\mathrm{GeV}, n_f = 3) = 84.1(5)\mathrm{MeV}$$

New HPQCD results for m using RI-SMOM scheme

Direct test of mass determination using different method to current-current correlators but the same formalism



Lytle, HPQCD, 1511.06547



Use RI-MOM method, fixing Landau gauge Green's function of quark bilinear to tree-level value. Match to $\overline{MS}$ perturbatively.

$$Z_m = Z_S^{-1}$$

'Non-exceptional' kinematics:

$$p_1^2 = p_2^2 = (p_1 - p_2)^2$$

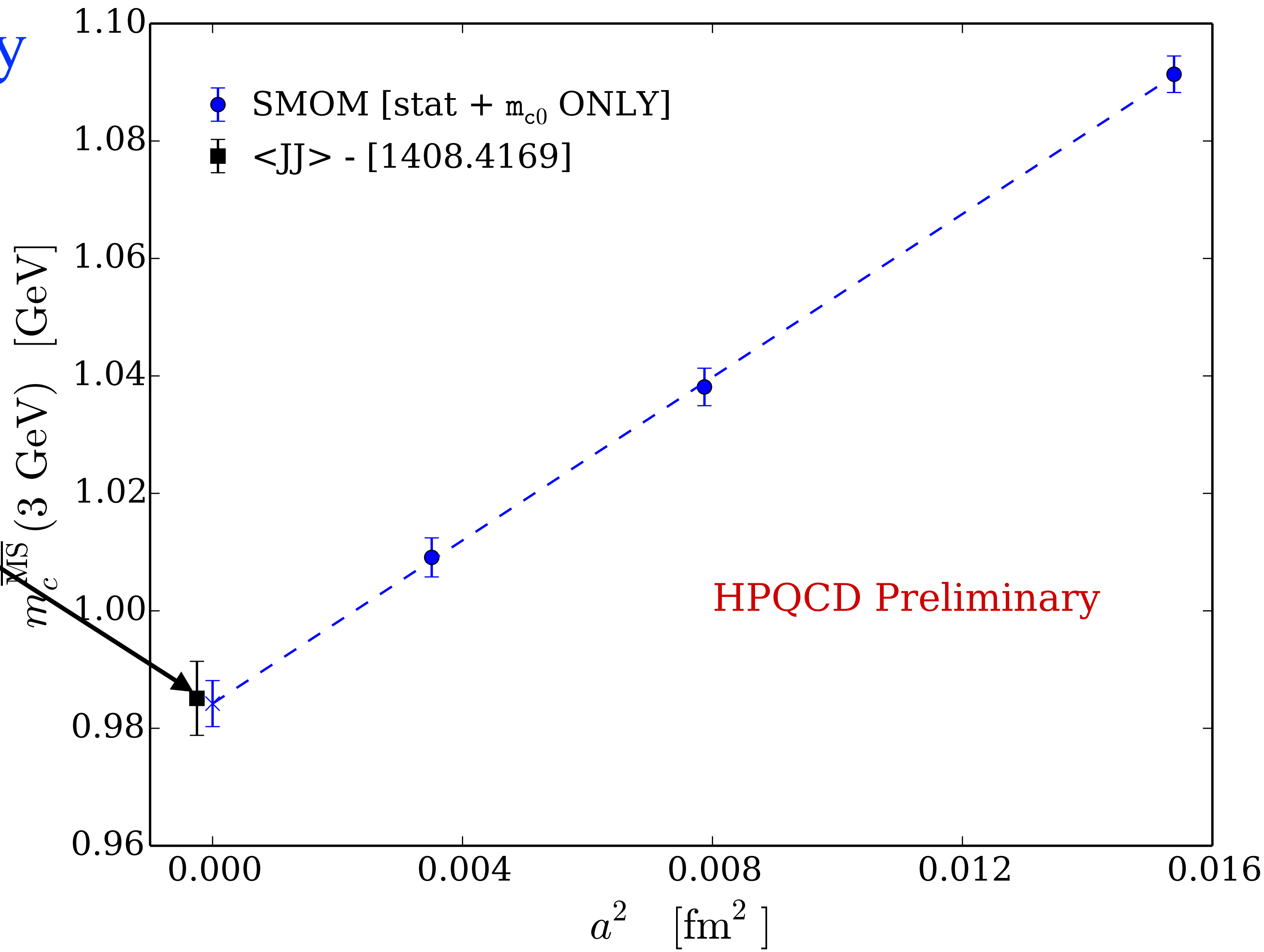
much improved results

Sturm et al, 0901.2599

Infrared sensitivity much reduced and perturbative matching factor (known through α_s^2) very close to 1.

Multiply tuned bare masses by Z_m:

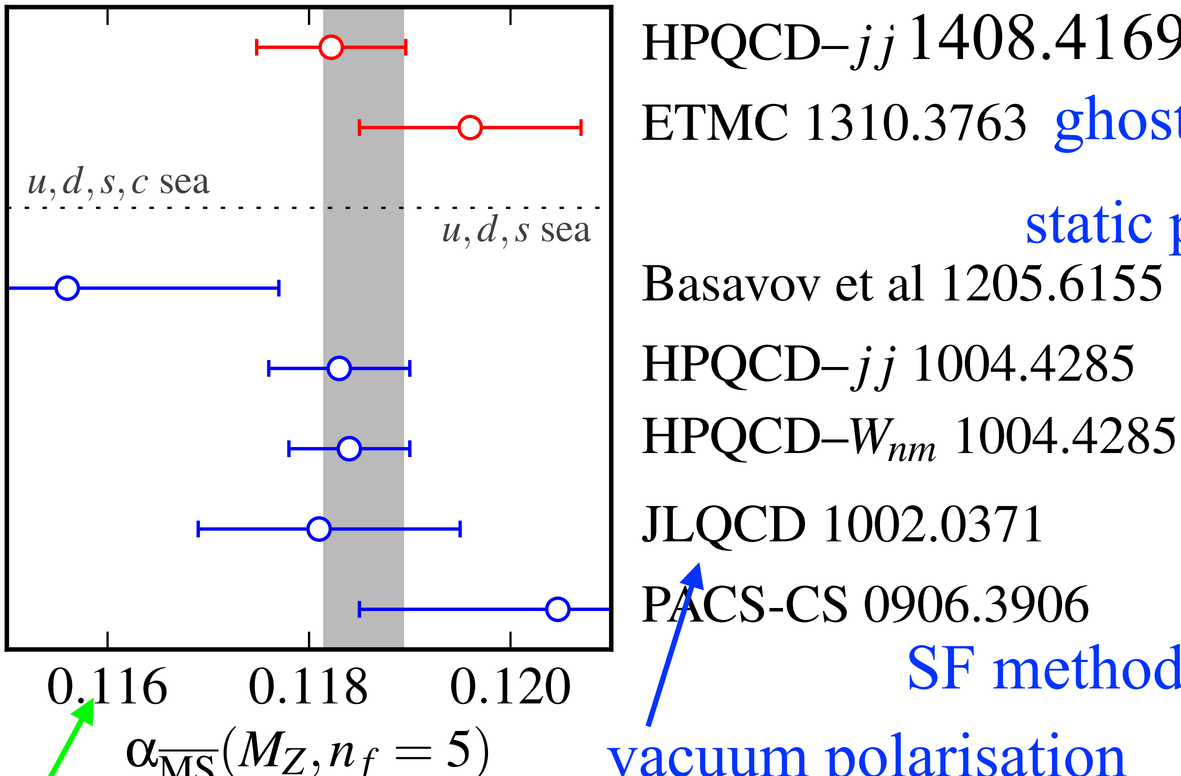
Good agreement with results from current-current correlators



still to do: error budget

α_s summary

α_s is also well determined in current-current correlator method (particularly lowest moment)



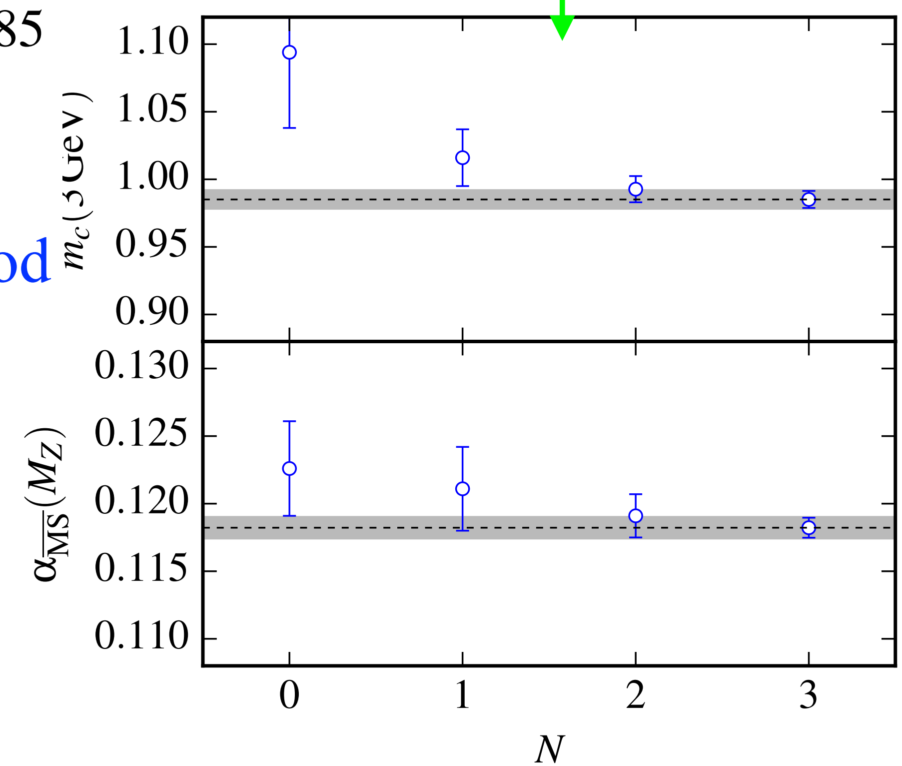
ghost-gluon

static potential

SF method

vacuum polarisation

Effect of high-order pert. theory in JJ method



Good consistency from very different lattice methods

$$\alpha_{\overline{MS}}(M_Z, n_f = 5) = 0.1185(4)$$

Conclusions

$m_c(m_c)$ is determined to 1% and
$m_b(m_b)$ to 0.5% from continuum and lattice methods.
$\alpha_S(M_Z)$ to 0.5% from lattice - multiple methods

1% accurate m_c/m_s ratio allows 1% in m_s also, along with RI-MOM methods

Tests of perturbation theory from completely non-perturbative mass ratios and JJ/RI-MOM comparison

Future improvements from higher order pert th. (?possible) and finer lattices to push up mu values.

Error budget for HISQ current-current method

TABLE IV. Error budget [31] for the c mass, QCD coupling, and the ratios of quark masses m_c/m_s and m_b/m_c from the $n_f = 4$ simulations described in this paper. Each uncertainty is given as a percentage of the final value. The different uncertainties are added in quadrature to give the total uncertainty. Only sources of uncertainty larger than 0.05% have been listed.

HPQCD, 1408.4169

	$m_c(3)$	$\alpha_{\overline{\rm MS}}(M_Z)$	m_c/m_s	m_b/m_c
Perturbation theory	0.3	0.5	0.0	0.0
Statistical errors	0.2	0.2	0.3	0.3
$a^2 \to 0$	0.3	0.3	0.0	1.0
$\delta m_{uds}^{\rm sea} \to 0$	0.2	0.1	0.0	0.0
$\delta m_c^{\rm sea} \to 0$	0.3	0.1	0.0	0.0
$m_h \neq m_c$ (Eq. (15))	0.1	0.1	0.0	0.0
Uncertainty in w_0, w_0/a	0.2	0.0	0.1	0.4
α_0 prior	0.0	0.1	0.0	0.0
Uncertainty in m_{η_s}	0.0	0.0	0.4	0.0
$m_h/m_c \to m_b/m_c$	0.0	0.0	0.0	0.4
δm_{η_c}: electromag., annih.	0.1	0.0	0.1	0.1
δm_{η_b}: electromag., annih.	0.0	0.0	0.0	0.1
Total:	0.64%	0.63%	0.55%	1.20%