Quantum Optics and Lattice Gauge Systems

Collaborators: J. Pachos (Leeds) B. Reznik (Tel-Aviv) A. Farace E. Zohar

MPQ
Max-Planck-Institut für Quantenoptik

Symposium in Effective Theories and Lattice Gauge Theory,
IAS/TUM, Garching, May 19th, 2016

QUANTUM SIMULATORS

Simulating Physics with Computers

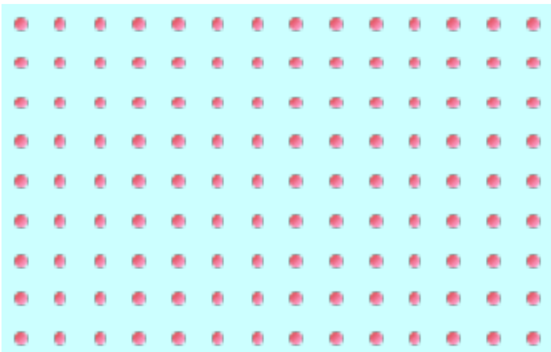
Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

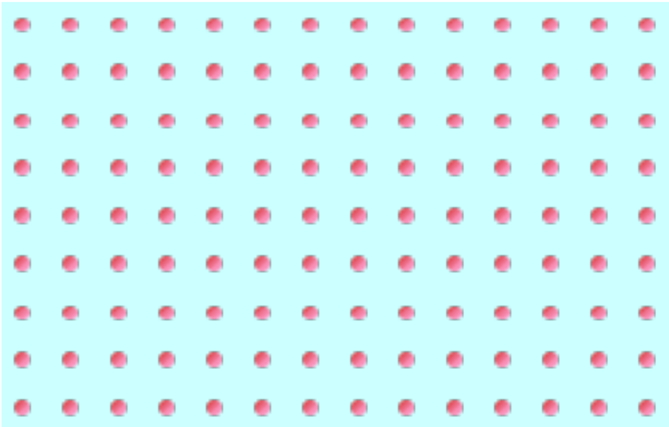
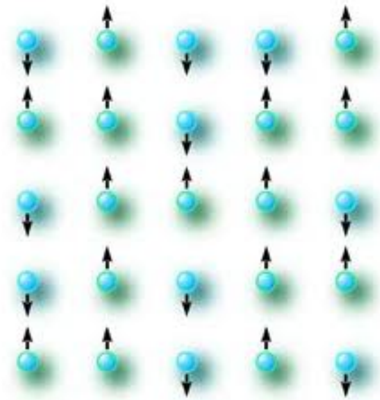
1. INTRODUCTION

On the program it says this is a keynote speech—and I don't know what a keynote speech is. I do not intend in any way to suggest what should



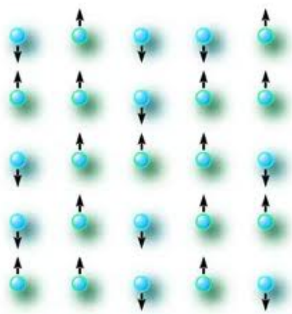
$$c_1 \,|000\ldots0\rangle + c_2 \,|000\ldots1\rangle + \ldots + c_{2^N} \,|111\ldots1\rangle$$

QUANTUM SIMULATORS
ANALOG

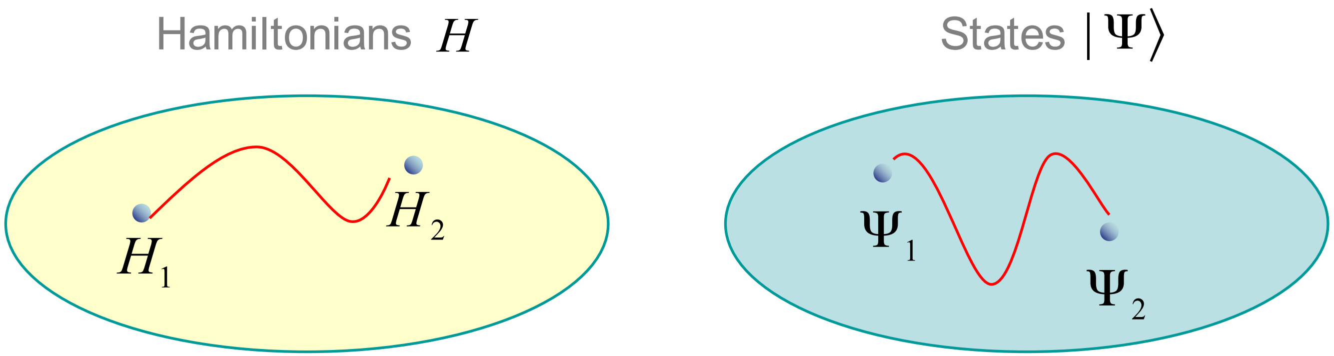
MODEL	QUANTUM SIMULATOR
	
Model Hamiltonian	Model Hamiltonian
$H = \ldots$	$H = \ldots$

QUANTUM SIMULATORS
ANALOG

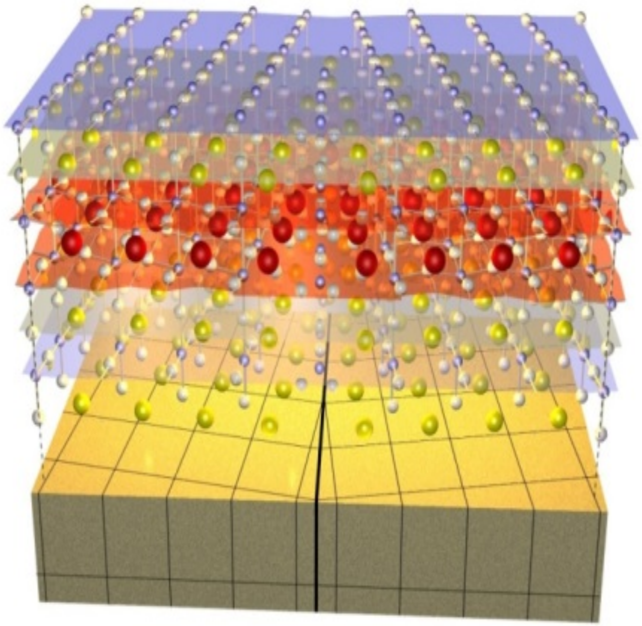
How does it work?



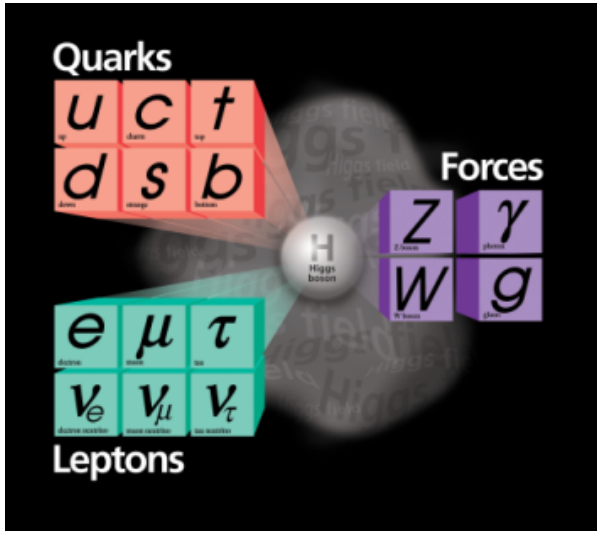
- Dynamics:
- Ground state:



QUANTUM SIMULATORS
APPLICATIONS



Material Science



HEP?

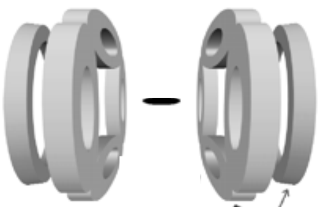
COLD ATOMS IN OPTICAL LATTICES

COLD ATOMS

- **Control:** External fields

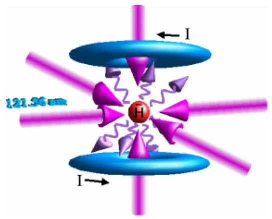
Trapping:

lasers　　Magnetic fields



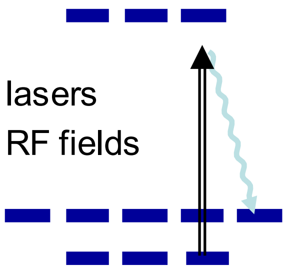
Cooling:

lasers　　evaporation



Internal manipulation

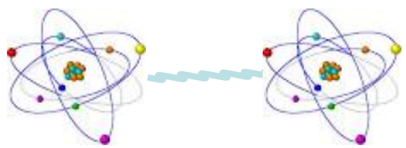
lasers
RF fields



purification
coherence
detection

Interactions

tune scattering length



COLD ATOMS

- Cold atoms are described by simple quantum field theories:

$$H = \int \Psi^{\dagger}_{\sigma} \left(-\nabla^2 + V(r) \right) \Psi_{\sigma} + u_{\sigma_i} \int \Psi^{\dagger}_{\sigma_1} \Psi^{\dagger}_{\sigma_2} \Psi_{\sigma_3} \Psi_{\sigma_4}$$

  - We can have bosons or fermions (or both).
  - We can have different internal states (spin).
  - The external potential, V, and interaction coefficients, u, can be engineered using lasers, and electric and magnetic fields.
  - In certain limits, one obtains effective theories that are interesting in other fields of Physics.

⬇

Quantum Simulations

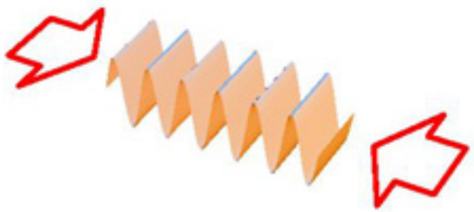
COLD ATOMS
OPTICAL LATTICES

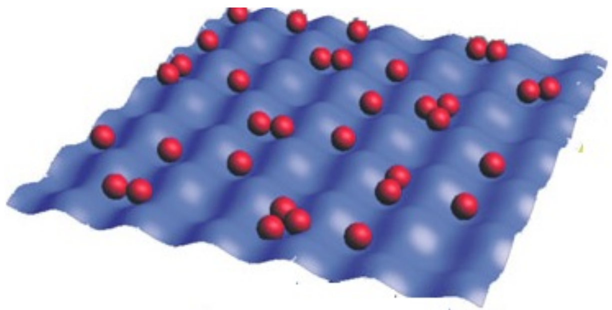
- Laser standing waves: dipole-trapping

VOLUME 81, NUMBER 15 PHYSICAL REVIEW LETTERS 12 OCTOBER 1998

Cold Bosonic Atoms in Optical Lattices

D. Jaksch,[1,2] C. Bruder,[1,3] J. I. Cirac,[1,2] C. W. Gardiner,[1,4] and P. Zoller[1,2]

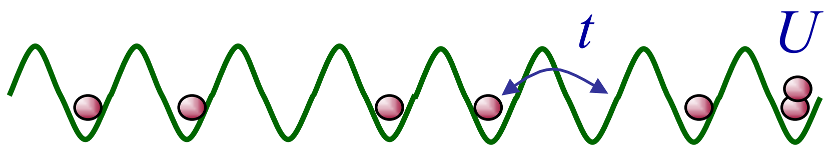




COLD ATOMS
OPTICAL LATTICES

- Laser standing waves: dipole-trapping

$$H = \int \Psi_{\sigma}^{\dagger}\left(-\nabla^2 + V(r)\right)\Psi_{\sigma} + u_{\sigma_i}\int \Psi_{\sigma_1}^{\dagger}\Psi_{\sigma_2}^{\dagger}\Psi_{\sigma_3}\Psi_{\sigma_4}$$



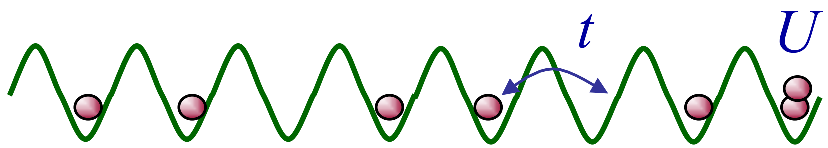
Lattice theory: Bose/Fermi-Hubbard model

$$H = -t\sum_{n}\left(a_n^{\dagger}a_{n+1} + h.c\right) + U\sum_{n} a_n^{\dagger 2}a_n^{2}$$

COLD ATOMS
OPTICAL LATTICES

- Laser standing waves: dipole-trapping

$$H = \int \Psi_\sigma^\dagger \left(-\nabla^2 + V(r) \right) \Psi_\sigma + u_{\sigma_i} \int \Psi_{\sigma_1}^\dagger \Psi_{\sigma_2}^\dagger \Psi_{\sigma_3} \Psi_{\sigma_4}$$



Lattice theory: Bose/Fermi-Hubbard model

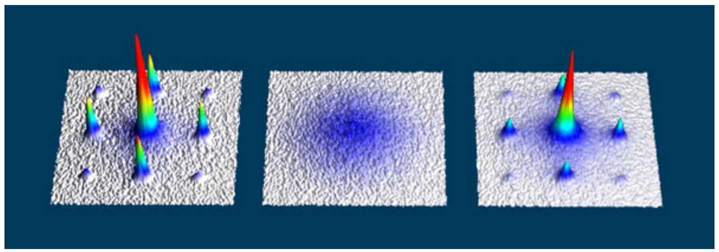
$$H = -t \sum_n \left(a_n^\dagger a_{n+1} + h.c \right) + U \sum_n a_n^{\dagger 2} a_n^2$$

articles

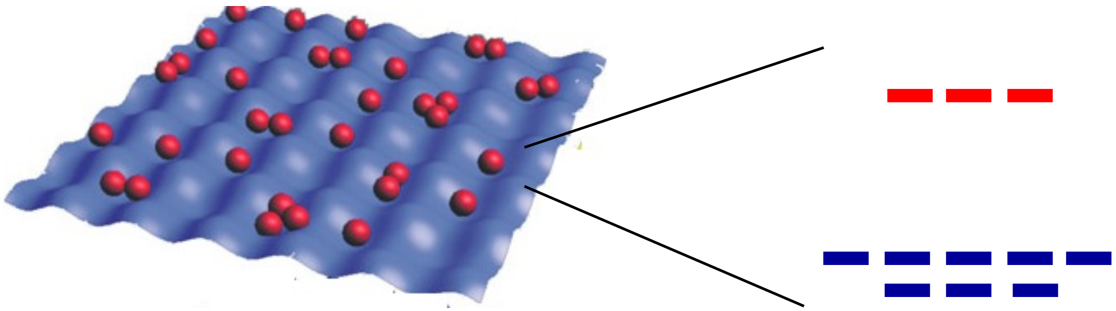
Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms

Markus Greiner*, Olaf Mandel*, Tilman Esslinger†, Theodor W. Hänsch* & Immanuel Bloch*

* Sektion Physik, Ludwig-Maximilians-Universität, Schellingstrasse 4/III, D-80799 Munich, Germany, and Max-Planck-Institut für Quantenoptik, D-85748 Garching, Germany
† Quantenelektronik, ETH Zürich, 8093 Zurich, Switzerland



COLD ATOMS
QUANTUM SIMULATION

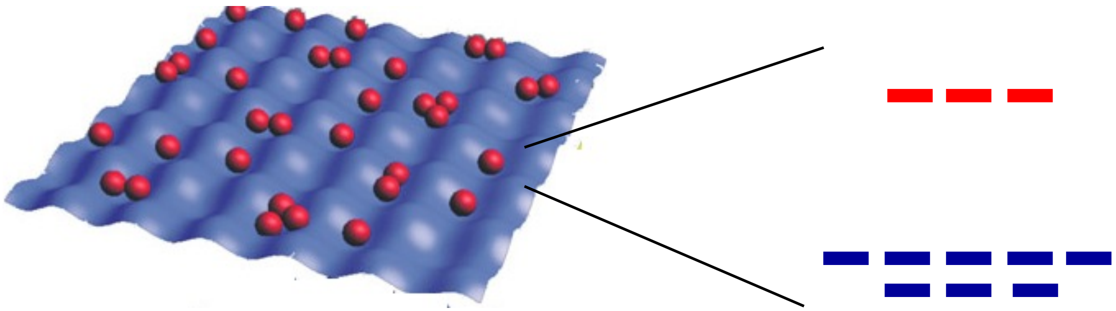


- Bosons/Fermions: $$H = -\sum_{\substack{<n,m> \\ \sigma,\sigma'}} \left(t_{\sigma,\sigma'} a^{\dagger}_{n,\sigma} a_{m,\sigma'} + h.c \right) + \sum_{\substack{n \\ \sigma,\sigma'}} U_{\sigma,\sigma'} a^{\dagger}_{n,\sigma} a^{\dagger}_{n,\sigma'} a_{n,\sigma'} a_{n,\sigma}$$

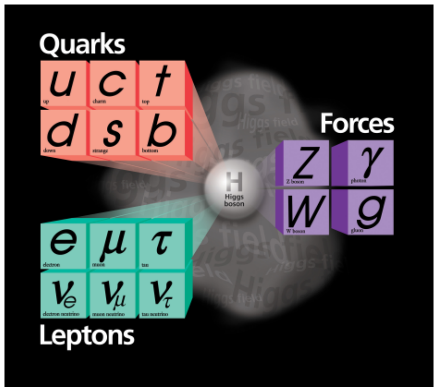
- Spins: $$H = -\sum_{\substack{<n,m> \\ \sigma,\sigma'}} \left(J_x S^x_n S^x_m + J_y S^y_n S^y_m + J_z S^z_n S^z_m \right) + \sum_{\substack{n \\ \sigma,\sigma'}} B_n S^z_n$$

→ CONDENSED MATTER PHYSICS

COLD ATOMS
QUANTUM SIMULATION



HIGH ENERGY PHYSICS?



QUANTUM SIMULATIONS OF HEP MODELS

INGREDIENTS

$$S = \int \overline{\Psi}(i\gamma^{\mu}\partial_{\mu} - m)\Psi - Q\int A_{\mu}\overline{\Psi}\gamma^{\mu}\Psi - \frac{1}{4}\int F_{\mu\nu}F^{\mu\nu} + \ldots\ldots$$

- Matter + Gauge Fields
- Relativistic theory
- Gauge invariant
- Hamitonian formulation: $i\partial_t |\Psi\rangle = H|\Psi\rangle$
  - Gauss law $G(x)|\Psi\rangle = 0$ $\quad [H, G(x)] = 0$

QUANTUM SIMULATION HEP MODELS
INGREDIENTS

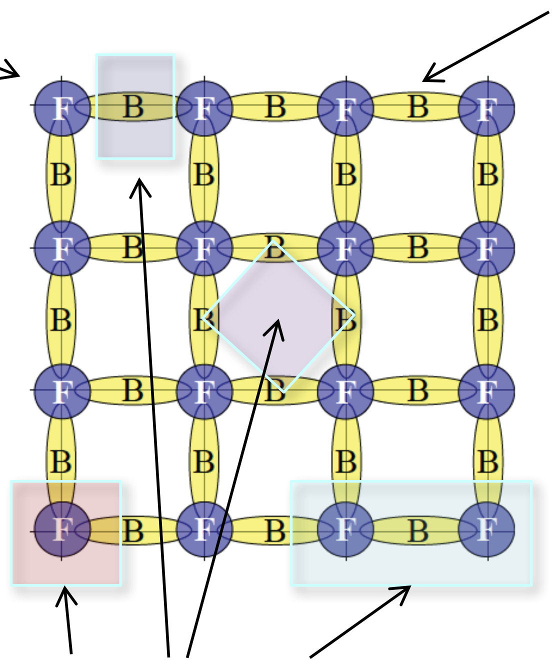
Lattice → J. Kogut and L. Susskind, Phys. Rev. D **11**, 395 (1975).
J. B. Kogut, Rev. Mod. Phys. **51**, 659 (1979).
J. B. Kogut, Rev. Mod. Phys. **55**, 775 (1983).

$$H = \int \Psi^{\dagger}_{\sigma}\left(-\nabla^2 + V(r)\right)\Psi_{\sigma} + u\int \Phi^{\dagger}_{\mu}\Phi_{\sigma'}\Psi^{\dagger}_{\sigma}\Psi_{\sigma'} + v\int \Phi^{\dagger}_{\sigma}\Phi^{\dagger}_{\sigma'}\Phi_{\sigma'}\Phi_{\sigma} + \ldots$$

Lattice — Fermion-gauge field coupling — Gauge field dynamics

Matter (Fermions): can move

Gauge fields (Bosons): Static

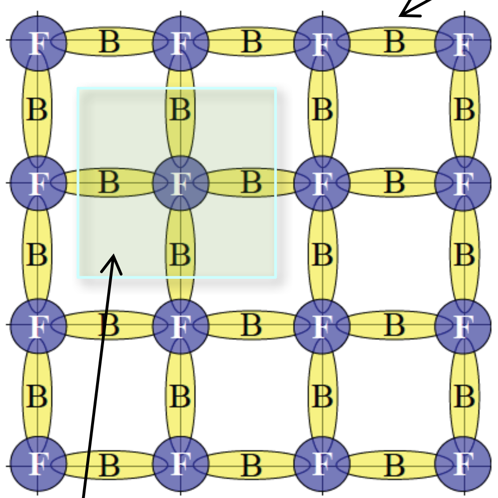


- Hamiltonian: $H = H_M + H_{KS} + H_{\text{int}}$

HEP LATTICE MODELS
HAMILTONIAN FORMULATION

Matter (Fermions): can move

Gauge fields (Bosons): Static

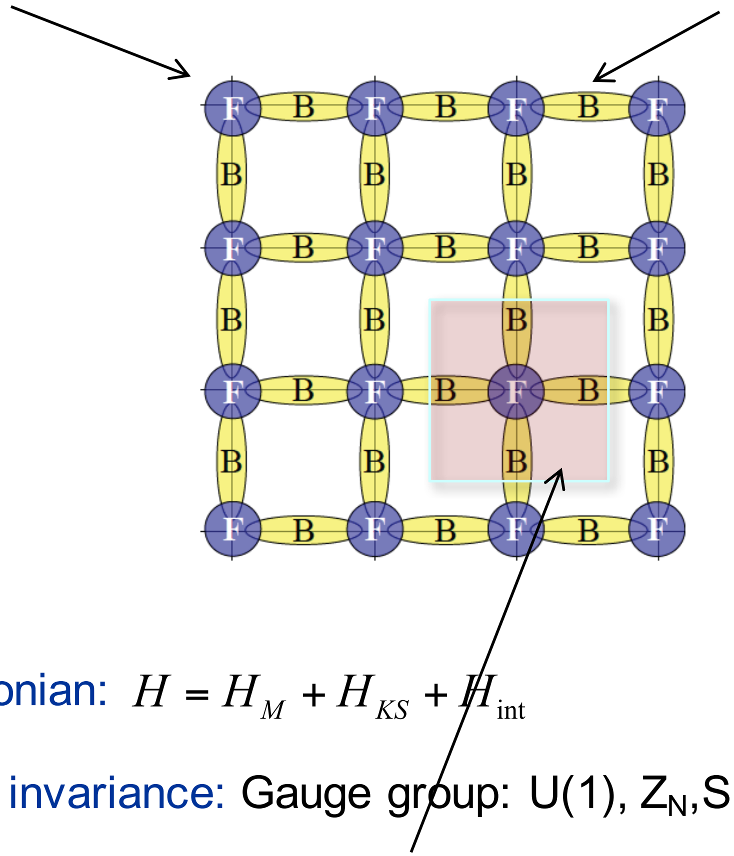


- Hamiltonian: $H = H_M + H_{KS} + H_{\text{int}}$
- Gauge invariance: Gauge group: U(1), Z_N,SU(N), etc

HEP LATTICE MODELS
HAMILTONIAN FORMULATION

Matter (Fermions): can move

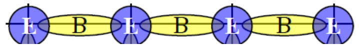
Gauge fields (Bosons): Static



- Hamiltonian: $H = H_M + H_{KS} + H_{\text{int}}$
- Gauge invariance: Gauge group: U(1), Z_N,SU(N), etc
- Gauss law: $G_{plaquette} \,|\, phys\rangle = 0$

HEP LATTICE MODELS
HAMILTONIAN FORMULATION

Example: compact-QED in 1D



- Hamiltonians:

$$H_M = \sum_{\mathbf{n}} M_{\mathbf{n}} \psi_{\mathbf{n}}^\dagger \psi_{\mathbf{n}}$$

$$H_{int} = \epsilon \sum_{\mathbf{n},k} \left(\psi_{\mathbf{n}}^\dagger e^{i\phi_{\mathbf{n},k}} \psi_{\mathbf{n}+\hat{\mathbf{k}}} + \psi_{\mathbf{n}+\hat{\mathbf{k}}}^\dagger e^{-i\phi_{\mathbf{n},k}} \psi_{\mathbf{n}} \right)$$

$$H_{KS} = H_E = \frac{g^2}{2} \sum_{n,k} E_n^2$$

$$[E_{\mathbf{n},k}, \phi_{\mathbf{m},l}] = -i\delta_{\mathbf{nm}}\delta_{kl}$$ (ie, compact)

- Gauss law: $G_n \mid phys\rangle = 0$

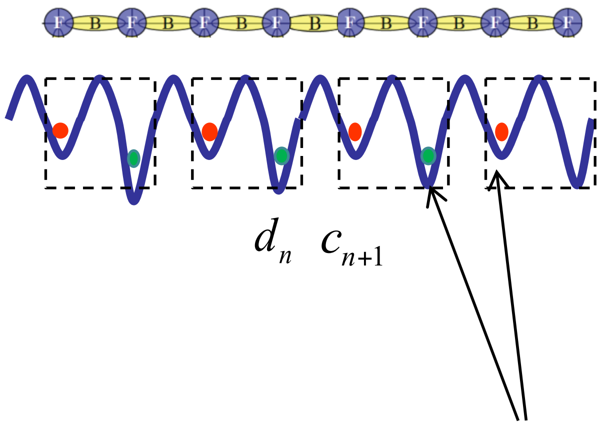
$$G_n = E_{n+1} - E_n - \psi_n^\dagger \psi_n$$

- Gauge invariance: $e^{-i\theta G_n} H e^{i\theta G_n} = H$

SCHWINGER MODEL

QUANTUM SIMULATION
SCHWINGER MODEL 1+1

- Fermions:



$$\{c_n, c_n^\dagger\} = \{d_n, d_n^\dagger\} = 1$$

$$H_M = M\sum_n (-1)^n \psi_n^\dagger \psi_n$$

$$H = \int \Psi_\sigma^\dagger \left(-\nabla^2 + V(r)\right)\Psi_\sigma + u\int \Phi_\mu^\dagger \Phi_{\sigma'} \Psi_\sigma^\dagger \Psi_{\sigma'} + v\int \Phi_\sigma^\dagger \Phi_{\sigma'}^\dagger \Phi_{\sigma'} \Phi_\sigma + \ldots$$

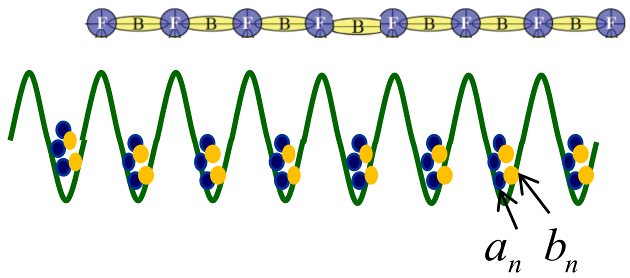
- Even sites: hole = particle
- Odd sites: fermion = antiparticle

Staggered Fermions:

L. Susskind, Phys. Rev. D **16**, 3031 (1977).
G. 't Hooft, Nucl. Phys. B **75**, 461 (1974)

QUANTUM SIMULATION
SCHWINGER MODEL 1+1

- Bosons:



internal states

$$[a_n, a_n^\dagger] = [b_n, b_n^\dagger] = 1$$

- Schwinger rep:

$$L_+ = a^\dagger b$$
$$L_z = \frac{1}{2}(N_a - N_b)$$
$$\ell = \frac{1}{2}(N_a + N_b)$$

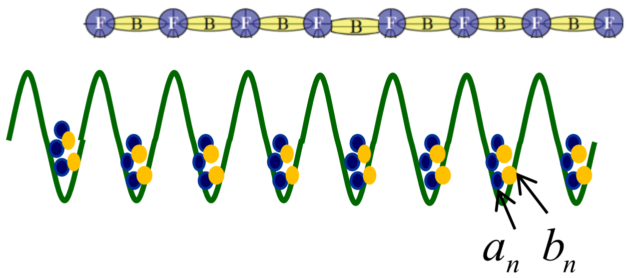
$l >> 1$ ⟹

$$L_+ \approx l\, e^{i\phi}$$
$$L_z \approx i\partial_\phi$$

- If l is small (eg 2 atoms), we obtain a truncated version
- One can also use a single atom with few internal levels (Z_M is the gauge group)

QUANTUM SIMULATION
SCHWINGER MODEL 1+1

- Bosons:



internal states

$$[a_n, a_n^\dagger] = [b_n, b_n^\dagger] = 1$$

Schwinger rep:

$$L_+ = a^\dagger b$$

$$L_z = \frac{1}{2}(N_a - N_b)$$

$$\ell = \frac{1}{2}(N_a + N_b)$$

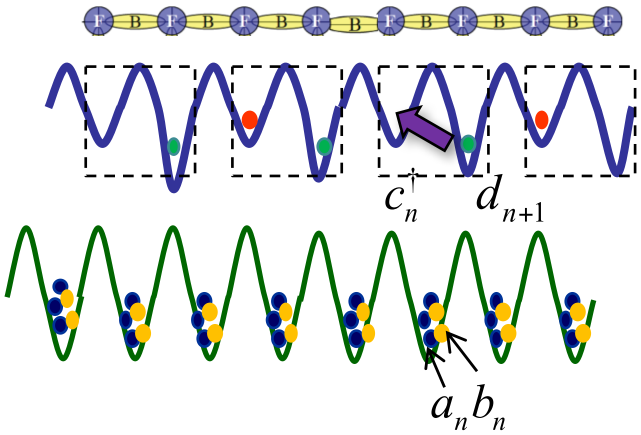
$$H_E = \frac{g^2}{2}\sum_n L_{z,n}^2$$

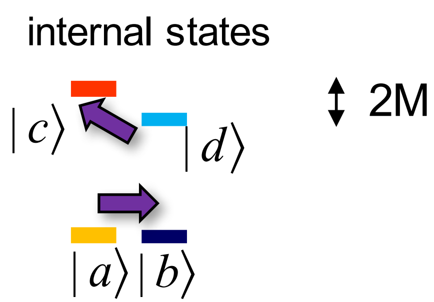
$$= \frac{g^2}{8}\sum_n \left(N_{a,n}^2 + N_{b,n}^2 - 2N_{a,n}N_{b,n}\right)$$

$$H = \int \Psi_\sigma^\dagger \left(-\nabla^2 + V(r)\right)\Psi_\sigma + u\int \Phi_\mu^\dagger \Phi_{\sigma'} \Psi_\sigma^\dagger \Psi_{\sigma'} + v\int \Phi_\sigma^\dagger \Phi_{\sigma'}^\dagger \Phi_{\sigma'} \Phi_\sigma + \ldots$$

- Interactions:

$c_n^\dagger a_n^\dagger b_n d_{n+1}$





$$H = \int \Psi_\sigma^\dagger \left(-\nabla^2 + V(r)\right)\Psi_\sigma + u\int \Phi_\mu^\dagger \Phi_{\sigma'} \Psi_\sigma^\dagger \Psi_{\sigma'} + v\int \Phi_\sigma^\dagger \Phi_{\sigma'}^\dagger \Phi_{\sigma'} \Phi_\sigma + \ldots$$

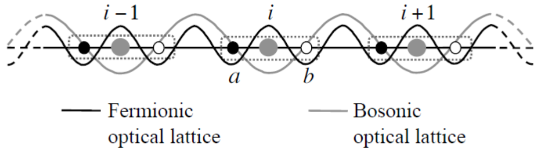
conserves angular momentum locally

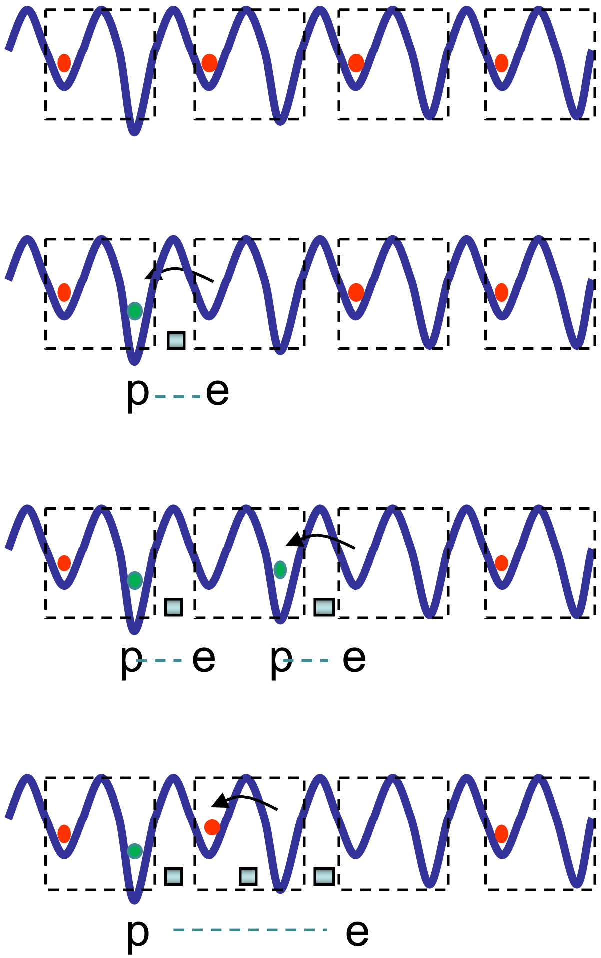
→ Gauge invariance

$$H_{int} \quad \frac{1}{\sqrt{\ell(\ell+1)}} \psi_n^\dagger a_n^\dagger b_n \psi_{n+1} \approx \psi_n^\dagger e^{i\phi_n} \psi_{n+1}$$

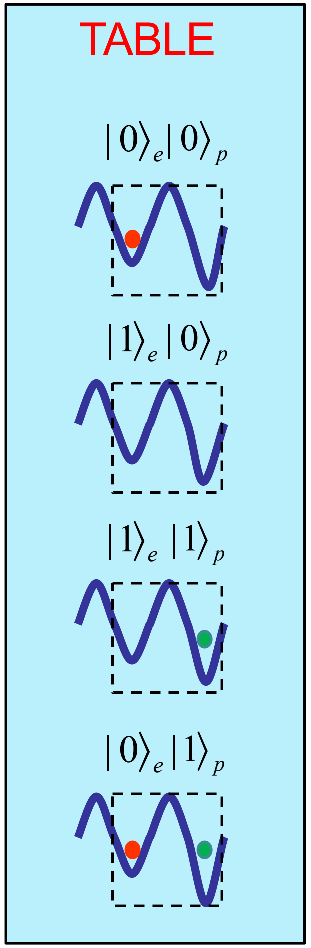
QUANTUM SIMULATION
SCHWINGER MODEL 1+1

- Physical processes:



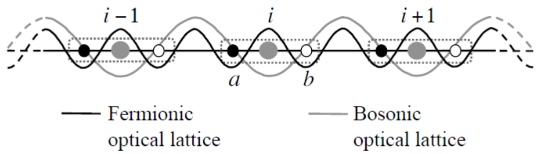


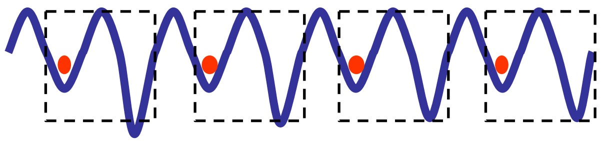
non-interacting vacuum



QUANTUM SIMULATION
SCHWINGER MODEL 1+1

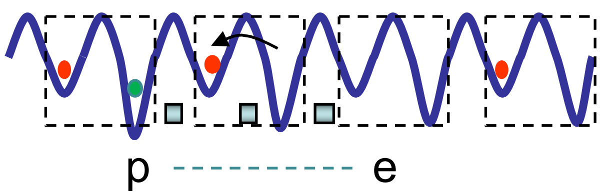
- Preparation:



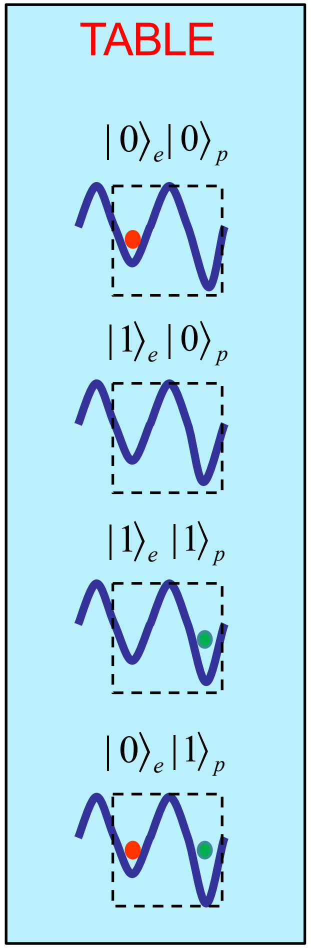


non-interacting vacuum

switch on interactions

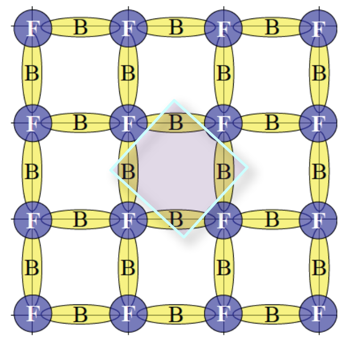


interacting vacuum



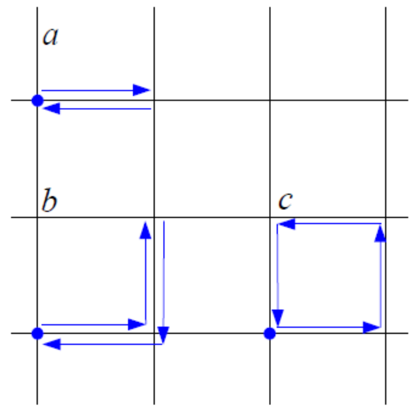
- Confinement
- Excitations: vector + scalar
- Time-dependent phenomena
- First experiments: few bosonic atoms

QUANTUM SIMULATION
HIGHER DIMENSIONS, NON-ABELIAN



- Plaquette interactions:

$$H_B = -\frac{2\epsilon^4}{\lambda^3}\sum_{\mathbf{n}}\left(U_{\mathbf{n},1}U_{\mathbf{n}+\hat{\mathbf{1}},2}U^\dagger_{\mathbf{n}+\hat{\mathbf{2}},1}U^\dagger_{\mathbf{n},2}+h.c.\right) =$$
$$-\frac{4\epsilon^4}{\lambda^3}\sum_{\mathbf{n}}\cos\left(\phi_{\mathbf{n},1}+\phi_{\mathbf{n}+\hat{\mathbf{1}},2}-\phi_{\mathbf{n}+\hat{\mathbf{2}},1}-\phi_{\mathbf{n},2}\right)$$

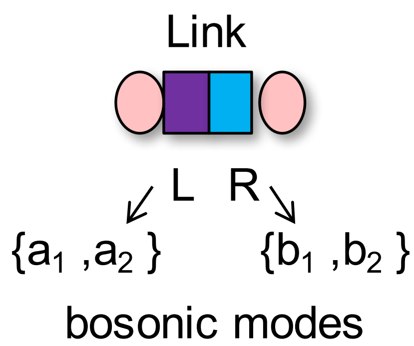


- Non-abelian gauge theories:

$$H_E = \frac{g^2}{2}\sum_{\mathbf{n},k,a}\left(E_{\mathbf{n},k}\right)_a\left(E_{\mathbf{n},k}\right)_a$$

$$H_B = -\frac{1}{g^2}\sum_{\text{plaquettes}}\left(\mathrm{Tr}\left(U_1U_2U_3^\dagger U_4^\dagger\right)+h.c.\right)$$

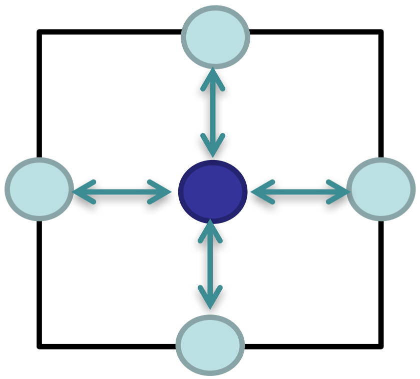
$$H_{int} = \epsilon\sum_{\mathbf{n},k}\left(\psi^\dagger_{\mathbf{n}}U^r_{\mathbf{n},k}\psi_{\mathbf{n}+\hat{\mathbf{k}}}+h.c.\right)$$



TROTTER DECOMPOSITION:

$$e^{-iHT} = \left(e^{-iH\frac{T}{N}}\right)^N = \left(e^{-i\sum_j H_j \frac{T}{N}}\right)^N \underset{N\gg1}{\approx} \left(\prod_j e^{-iH_j\frac{T}{N}}\right)^N$$

MEDIATING INTERACTIONS:



Earlier work: Taggliacozzo et al

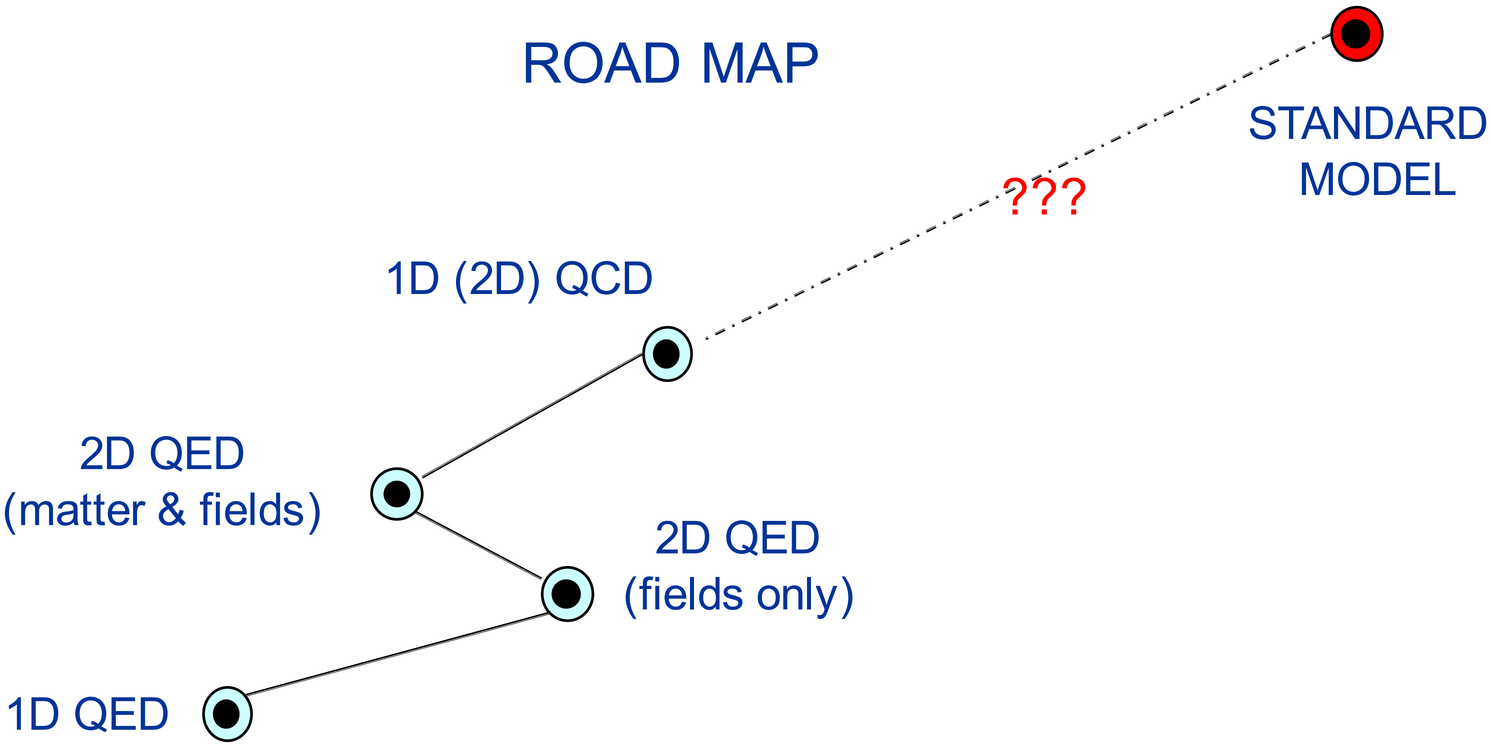
COLD ATOMS
EXPERIMENTAL CONSIDERATIONS

- Cold bosons in optical lattices
  - Mott insulator – superfluid transition
  - Exchange interaction (2nd order perturbation theory)
  - Dynamics
  - Anderson-Higgs mechanism in 2D
- Cold fermions in optical lattices
  - Mott insulator in 2D
- Cold fermions and bosons in optical lattices
  - Mean-field dynamics
- Techniques
  - Tuning of interactions: Magnetic/optical Feschbach resonances
  - Lattice geometry
  - Time of flight measurements
  - Single-site addressing: initializaton
  - Single-site measurement
- Challanges: temperature, decoherence, control …


WARNING

QUANTUM SIMULATION
HIGH ENERGY MODELS

ROAD MAP



IC, Maraner, Pachos, PRL **105**, 19403 (2010)
Zohar, IC, Reznik, PRL **107**, 275301 (2011)
Zohar, IC, Reznik, PRL **109**, 125302 (2012)
Zohar, IC, Reznik, PRL **110**, 125304 (2013)
Zohar, IC, Reznik, PRA **88**, 023617 (2013)
Zohar, Farace, Reznik, JIC, in preparation

See also:

Kapit,Mueller, PRA**83**, 033625 (2011)
Banerjee,…, Wiese, Zoller, PRL**109**, 175302 (2013)
Banerjee,…, Wiese, Zoller, PRL**110**, 125303 (2013)
Gauge fields: Lewenstein et al

INGREDIENTS

- Problem:

$$H = \int \Psi^{\dagger}_{\sigma} \left(-\nabla^2 + V(r)\right) \Psi_{\sigma} + u \int \Phi^{\dagger}_{\mu} \Phi_{\sigma'} \Psi^{\dagger}_{\sigma} \Psi_{\sigma'} + v \int \Phi^{\dagger}_{\sigma} \Phi^{\dagger}_{\sigma'} \Phi_{\sigma'} \Phi_{\sigma} + \ldots$$

choose $V(r), u, v$, etc such that (in some limit), we have

$$i\partial_t |\Psi\rangle = H |\Psi\rangle$$

$$G(x) |\Psi\rangle = 0 \qquad [H, G(x)] = 0$$

corresponding to

$$S = \int \overline{\Psi}(i\gamma^{\mu}\partial_{\mu} - m)\Psi - Q \int A_{\mu} \overline{\Psi}\gamma^{\mu}\Psi - \frac{1}{4}\int F_{\mu\nu}F^{\mu\nu} + \ldots\ldots$$

INGREDIENTS

- Matter + Gauge Fields

$$S=\int\overline{\Psi}(i\gamma^{\mu}\partial_{\mu}-m)\Psi-Q\int A_{\mu}\overline{\Psi}\gamma^{\mu}\Psi-\frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}+\ldots\ldots$$

We need bosonic and fermionic atoms

We need interactions among themselves

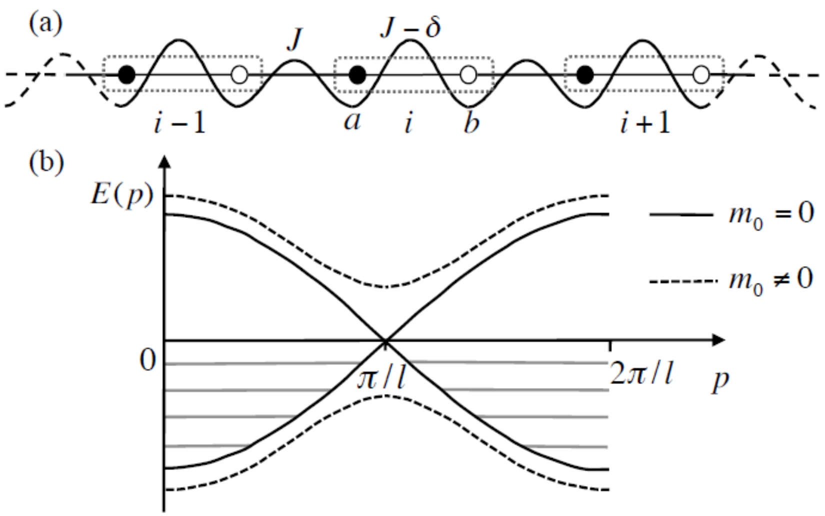
$$H=\int\Psi_{\sigma}^{\dagger}\left(-\nabla^{2}+V(r)\right)\Psi_{\sigma}+u\int\Phi_{\mu}^{\dagger}\Phi_{\sigma'}\Psi_{\sigma}^{\dagger}\Psi_{\sigma'}+v\int\Phi_{\sigma}^{\dagger}\Phi_{\sigma'}^{\dagger}\Phi_{\sigma'}\Phi_{\sigma}+\ldots$$

- Relativistic

$$S = \int \overline{\Psi}(i\gamma^{\mu}\partial_{\mu} - m)\Psi - Q\int A_{\mu}\overline{\Psi}\gamma^{\mu}\Psi - \frac{1}{4}\int F_{\mu\nu}F^{\mu\nu} + \ldots\ldots$$

$$H = \int \Psi_{\sigma}^{\dagger}\left(-\nabla^{2} + V(r)\right)\Psi_{\sigma} + u\int \Phi_{\mu}^{\dagger}\Phi_{\sigma'}\Psi_{\sigma}^{\dagger}\Psi_{\sigma'} + v\int \Phi_{\sigma}^{\dagger}\Phi_{\sigma'}^{\dagger}\Phi_{\sigma'}\Phi_{\sigma} + \ldots$$

Use a superlattice: it possesses the right limit in the continuum



(staggered fermions)