CHIRAL PERTURBATION WITH TWISTED BOUNDARY CONDITIONS

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Overview

Chiral Perturbation Theory

A general Effective Field Theory:

- Relevant degrees of freedom
- A powercounting principle (predictivity)
- Has a certain range of validity

Chiral Perturbation Theory:

- Degrees of freedom: Goldstone Bosons from spontaneous breaking of chiral symmetry
- Powercounting: Dimensional counting in momenta/masses
- Breakdown scale: Resonances, so about M_ρ.

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Goldstone Bosons

Spontaneous breakdown

- $\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
- $SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$
- 8 generators broken $\Longrightarrow$ 8 massless degrees of freedom and interaction vanishes at zero momentum

Chiral Perturbation Theories

- Which chiral symmetry: $SU(N_f)_L \times SU(N_f)_R$, for $N_f = 2, 3, \ldots$ and extensions to (partially) quenched
- Or beyond QCD
- Space-time symmetry: Continuum or broken on the lattice: Wilson, staggered, mixed action
- Volume: Infinite, finite in space, finite T
- Which interactions to include beyond the strong one
- Which particles included as non Goldstone Bosons
- Here: restrict to standard ChPT but with extensions for the lattice

Lagrangians: Lowest order

$U(\phi) = \exp(i\sqrt{2}\Phi/F_0)$ parametrizes Goldstone Bosons

$$\Phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\,\eta_8}{\sqrt{6}} \end{pmatrix}.$$

LO Lagrangian: $\mathcal{L}_2 = \frac{F_0^2}{4}\{\langle D_\mu U^\dagger D^\mu U\rangle + \langle \chi^\dagger U + \chi U^\dagger\rangle\}\,,$

$D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu\,,$
left and right external currents: $r(l)_\mu = v_\mu + (-)a_\mu$

Scalar and pseudoscalar external densities: $\chi = 2B_0(s + ip)$ quark masses via scalar density: $s = \mathcal{M} + \cdots$

$\langle A \rangle = Tr_F\,(A)$

Lagrangians: Lagrangian structure (mesons, strong)

	2 flavour		3 flavour		PQChPT/N_f flavour	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	$\hat{L}_i^r, \hat{H}_i^r$	11+2
p^6	c_i^r	52+4	C_i^r	90+4	K_i^r	112+3

p^2: Weinberg 1966

p^4: Gasser, Leutwyler 84,85

p^6: JB, Colangelo, Ecker 99,00

- L_i LEC = Low Energy Constants = ChPT parameters
- H_i: contact terms: value depends on definition of currents/densities
- Finite volume: no new LECs
- Other effects: (many) new LECs

Mesons: which Lagrangians are known ($n_f = 3$)

Loops	$\mathcal{L}_{\rm order}$	LECs	effects included
$L = 0$	$\mathcal{L}_{p^2}$	2	strong (+ external W, γ)
	$\mathcal{L}_{e^2 p^0}$	1	internal γ
	$\mathcal{L}^{\Delta S=1}_{G_F p^2}$	2	nonleptonic weak
	$\mathcal{L}^{\Delta S=1}_{G_8 e^2 p^0}$	1	nonleptonic weak+internal γ
	$\mathcal{L}^{\rm odd}_{p^4}$	0	WZW, anomaly
$L \le 1$	$\mathcal{L}_{p^4}$	10	strong (+ external W, γ)
	$\mathcal{L}_{e^2 p^2}$	13	internal γ
	$\mathcal{L}^{\Delta S=1}_{G_8 F p^4}$	22	nonleptonic weak
	$\mathcal{L}^{\Delta S=1}_{G_{27} p^4}$	28	nonleptonic weak
	$\mathcal{L}^{\Delta S=1}_{G_8 e^2 p^0}$	14	nonleptonic weak+internal γ
	$\mathcal{L}^{\rm odd}_{p^6}$	23	WZW, anomaly
	$\mathcal{L}^{\rm leptons}_{e^2 p^2}$	5	leptons, internal γ
$L \le 2$	$\mathcal{L}_{p^6}$	90	strong (+ external W, γ)

Expand in what quantities?

- Expansion is in momenta and masses
- But is not unique: relations between masses (Gell-Mann–Okubo) exist
- Express orders in terms of physical masses and quantities (F_π, F_K)?
- Express orders in terms of lowest order masses?
- E.g. $s + t + u = 2m_\pi^2 + 2m_K^2$ in πK scattering
- Note: remaining μ dependence can occur at a given order
- Can make quite some difference in the expansion

I prefer physical masses

- Thresholds correct
- Chiral logs are from physical particles propagating
- but sometimes too many masses so very ambiguous

Extensions for the lattice

- No new parameters:
  - Finite temperature
  - Finite volume (including ϵ regime)
  - Twisted mass
  - Boundary conditions: twisted,...
- A few new parameters
  - Partially quenched ($2\to 2$,$10\to 11$, $90\to 112$)
- Many new parameters
  - Wilson ChPT ($2\to 3$,$10\to 18$)
  - Staggered ChPT ($2\to 10$,$10\to 126$ (but dependencies))
  - Mixed actions
- Other operators
  - Local object with well defined chiral properties: include via spurion techniques
  - Examples: tensor current, energy momentum tensor,...

Many LECs

- Is this too many parameters to do something?
- But if analytic in quark masses added in the fit not much extra
- Example: meson masses at NNLO have only the possible analytic quark mass dependence and the NLO meson-meson scattering parameters as input

Values of the LECs

- Recent review with more p^6 input for the standard sector: JB, Ecker,

  Ann. Rev. Nucl. Part. Sci. **64** (2014) 149 [arXiv:1405.6488]
- Review Kaon physics: Cirigliano, Ecker, Neufeld, Pich, Portoles,

  Rev.Mod.Phys. 84 (2012) 399 [arXiv:1107.6001]
- Lattice: FLAG reports:

  Colangelo et al., Eur.Phys.J. C71 (2011) 1695 [arXiv:1011.4408]

  Aoki et al., Eur. Phys. J. C **74** (2014) 9, 2890 [arXiv:1310.8555]

Program availability

Making the programs more accessible for others to use:

- Two-loop results have very long expressions
- Many not published but available from http://www.thep.lu.se/∼bijnens/chpt/
- Many programs available on request from the authors
- Idea: make a more general framework
- CHIRON:

JB,
"CHIRON: a package for ChPT numerical results at two loops,"
Eur. Phys. J. C **75** (2015) 27 [arXiv:1412.0887]
http://www.thep.lu.se/∼bijnens/chiron/

Program availability: CHIRON

- Present version: 0.54
- Classes to deal with L_i, C_i, $L_i^{(n)}, K_i$, standardized in/output, changing the scale,. . .
- Loop integrals: one-loop and sunsetintegrals
- Included so far (at two-loop order):
  - Masses, decay constants and $\langle \bar{q}q \rangle$ for the three flavour case
  - Masses and decay constants at finite volume in the three flavour case
  - Masses and decay constants in the partially quenched case for three sea quarks
  - Masses and decay constants in the partially quenched case for three sea quarks at finite volume
- A large number of example programs is included
- Manual has already reached 94 pages
- I am continually adding results from my earlier work

Finite volume

- Lattice QCD calculates at different quark masses, volumes boundary conditions,. . .
- A general result by Lüscher: relate finite volume effects to scattering (1986)
- Chiral Perturbation Theory is also useful for this
- Start: Gasser and Leutwyler, Phys. Lett. B184 (1987) 83, Nucl. Phys. B 307 (1988) 763
  M_π, F_π, $\langle\bar{q}q\rangle$ one-loop equal mass case
- I will stay with ChPT and the p regime ($M_\pi L >> 1$)
- $1/m_\pi = 1.4$ fm
  may need to go beyond leading $e^{-m_\pi L}$ terms
- Convergence of ChPT is given by $1/m_\rho \approx 0.25$ fm

Finite volume: selection of ChPT results

- masses and decay constants for π, K, η one-loop
  Becirevic, Villadoro, Phys. Rev. D 69 (2004) 054010
- M_π at 2-loops (2-flavour)
  Colangelo, Haefeli, Nucl.Phys. B744 (2006) 14 [hep-lat/0602017]
- $\langle \bar{q}q \rangle$ at 2 loops (3-flavour)
  JB, Ghorbani, Phys. Lett. B636 (2006) 51 [hep-lat/0602019]
- Twisted mass at one-loop
  Colangelo, Wenger, Wu, Phys.Rev. D82 (2010) 034502 [arXiv:1003.0847]
- Twisted boundary conditions
  Sachrajda, Villadoro, Phys. Lett. B 609 (2005) 73 [hep-lat/0411033]
- This talk:
  - Twisted boundary conditions: form-factors and two-point functions
  - Normal finite volume: lots of two-loop order

Twisted boundary conditions

- On a lattice at finite volume $p^i = 2\pi n^i/L$: very few momenta directly accessible
- Put a constraint on certain quark fields in some directions: $q(x^i + L) = e^{i\theta^i_q} q(x^i)$
- Then momenta are $p^i = \theta^i/L + 2\pi n^i/L$. Allows to map out momentum space on the lattice much better

  Bedaque,...
- Small note:
  - Beware what people call momentum: is θ^i/L included or not?
  - Reason: a colour singlet gauge transformation $G^S_\mu \to G^S_\mu - \partial_\mu \epsilon(x)$, $q(x) \to e^{i\epsilon(x)} q(x)$, $\epsilon(x) = -\theta^i_q x^i/L$
  - Boundary condition Twisted $\Leftrightarrow$ constant background field+periodic

Twisted boundary conditions: Drawbacks

Drawbacks:

- Box: Rotation invariance → cubic invariance
- Twisting: reduces symmetry further

Consequences:

- $m^2(\vec{p}^2) = E^2 - \vec{p}^2$ is not constant
- There are typically more form-factors
- In general: quantities depend on more (all) components of the momenta
- Charge conjugation involves a change in momentum

Integrals: underlying formulae

- Underlying formula in one dimension
  periodic boundary condition $F(x=0) = F(x=L)$

$$\int \frac{dp}{2\pi} F(p) \longrightarrow \frac{1}{L} \sum_{p_n = 2\pi n/L} F(p_n) \equiv \int_L \frac{dp}{2\pi} F(p)$$

- Poisson summation formula

$$\frac{1}{L} \sum_{p_n = 2\pi n/L} F(p_n) = \sum_{\ell = nL} \int \frac{dp}{2\pi} e^{i\ell p} F(p)$$

- If twist angle θ, $\phi(L) = e^{-i\theta}\phi(0)$: $p_n = \frac{2\pi}{L} n + \frac{\theta}{L}$
- Poisson summation formula

$$\frac{1}{L} \sum_{p_n = 2\pi n/L + \theta/L} F(p_n) = \sum_{\ell = nL} \int \frac{dp}{2\pi} e^{i(\ell p - \ell(\theta/L))} F(p)$$

One-loop tadpole

$$\langle X \rangle = \int_V \frac{d^d r}{(2\pi)^d} \frac{X}{(r^2+m^2)^n},$$

Poisson trick for three spatial dimensions:

$$\langle X \rangle = \sum_{l_r} \int \frac{d^d r}{(2\pi)^d} \frac{X\, e^{il_r\cdot r - il_r\cdot\Theta}}{(r^2+m^2)^n},$$

$l_r = (0, n_1 L, n_2 L, n_3 L)$, $\Theta = (0, \vec{\theta}/L)$
Split in infinite volume $l_r = 0$ term and rest

$$\langle X \rangle = \langle X \rangle^\infty + \langle X \rangle^V$$

Bring up denominator using 'α' parameters: $1/a = \int_0^\infty d\lambda e^{-\lambda a}$

$$\langle 1 \rangle^V = \frac{1}{\Gamma(n)} \sum_{l_r}{}' \int \frac{d^d r}{(2\pi)^d} \int_0^\infty d\lambda \lambda^{n-1}\, e^{il_r\cdot r - il_r\cdot\Theta} e^{-\lambda(r^2+m^2)}$$

$\sum_{l_r}{}'$ means sum without $l_r = 0$ (all components zero)

One-loop tadpole

Shift $r = \bar{r} + il_r/(2\lambda)$ to $\langle 1 \rangle^V = \frac{1}{\Gamma(n)} \sum_{l_r}' \int_0^\infty d\lambda \lambda^{n-1} \, e^{-\lambda m^2 - \frac{l_r^2}{4\lambda} - il_r\cdot\Theta} \int \frac{d^d\bar{r}}{(2\pi)^d} e^{-\lambda \bar{r}^2}$

Master formula for tadpoles:

$$\langle 1 \rangle^V = \frac{1}{(4\pi)^{d/2}\Gamma(n)} \sum_{l_r}' \int_0^\infty d\lambda \lambda^{n-\frac{d}{2}-1} \, e^{-\lambda m^2 - \frac{l_r^2}{4\lambda} - il_r\cdot\Theta}$$

- Do the λ integral — Gasser, Leutwyler, 1988
  leads to sums over Bessel functions
- Do the $\sum_{l_r}$ — Becirevic, Villadoro, 2003
  leads to an integral over Jacobi theta functions
- Large L Bessel needs only a few terms, otherwise Jacobi theta function more appropriate

Integrals

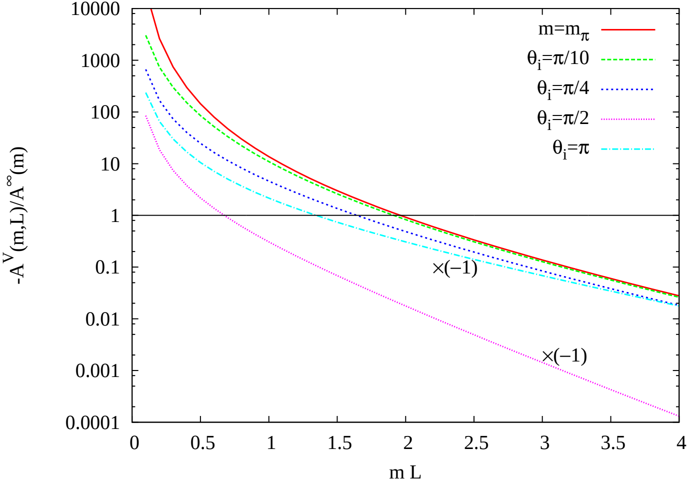
- Periodic boundary conditions: Tadpoles, Bubbles, Sunset-integrals:

  JB, Boström and Lähde, JHEP 01 (2014) 019 [arXiv:1311.3531]

- Twist: tadpoles, bubbles:

  JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]

A numerical example



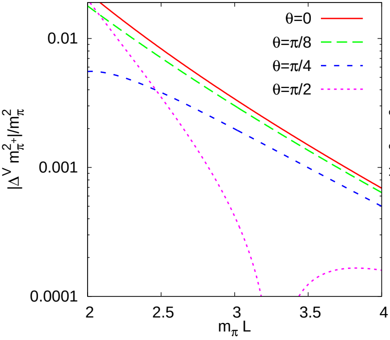
For various valus of $\theta_x = \theta_y = \theta_z = \theta_i$

For $\theta = \pi/2$ all terms with an $(l_r)_i$ odd cancel

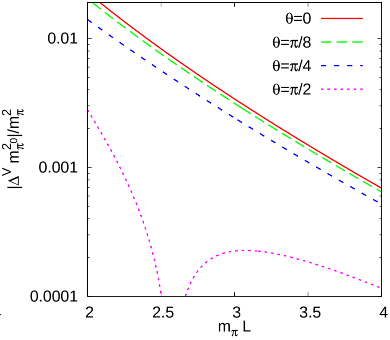
Twisted boundary conditions: volume correction masses

JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]

$m_\pi L = 2,\ \vec{\theta}_u = (\theta, 0, 0),\ \vec{\theta}_d = \vec{\theta}_s = 0$



Charged pion mass



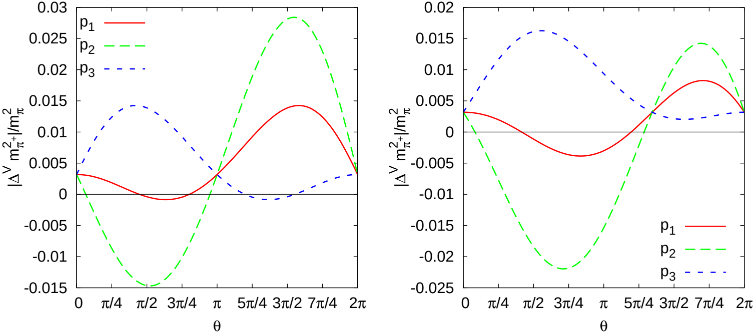
Neutral pion mass

$\Delta^V X = X^V - X^\infty$ (dip is going through zero)

Partially twisted boundary conditions: volume correction masses

Bernard, JB, Gamiz, Relefors, in preparation

$m_\pi L = 3,\ \vec{\theta}_u = (\theta,0,0),\ \vec{\theta}_d = \vec{\theta}_s = \vec{\theta}_{d\mathrm{sea}} = \vec{\theta}_{s\mathrm{sea}} = 0$



$\vec{\theta}_{u\mathrm{sea}} = 0$ $\qquad$ $\vec{\theta}_{u\mathrm{sea}} = (\pi/3,0,0)$

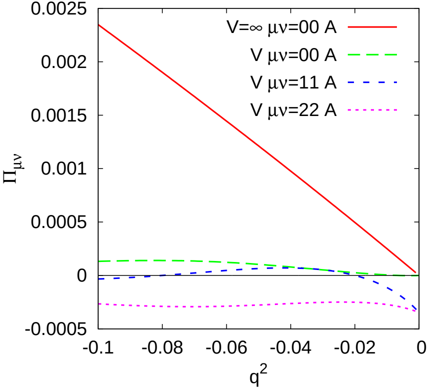
$\vec{p}_1 = (\theta,0,0)/L,\ \vec{p}_2 = (\theta+2\pi,0,0)/L,\ \vec{p}_3 = (\theta-2\pi,0,0)/L,$

Twisted boundary conditions: Two-point function

JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]

- $\int_V \frac{d^d k}{(2\pi)^d} \frac{k_\mu}{k^2 - m^2} \neq 0$
- $\langle \bar{u}\gamma^\mu u \rangle \neq 0$
- $j_\mu^{\pi^+} = \bar{d}\gamma_\mu u$
  satisfies $\partial^\mu \langle T(j_\mu^{\pi^+}(x) j_\nu^{\pi^-}(0)) \rangle = \delta^{(4)}(x) \langle \bar{d}\gamma_\nu d - \bar{u}\gamma_\nu u \rangle$
- $\Pi^a_{\mu\nu}(q) \equiv i \int d^4x e^{iq\cdot x} \langle T(j^a_\mu(x) j^{a\dagger}_\nu(0)) \rangle$
  Satisfies WT identity. $q^\mu \Pi^{\pi^+}_{\mu\nu} = \langle \bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d \rangle$
- ChPT at one-loop satisfies this
  see also Aubin et al, Phys.Rev. D88 (2013) 7, 074505 [arXiv:1307.4701]
- two-loop in partially quenched in progress JB, Relefors

Twisted boundary conditions: Two-point function



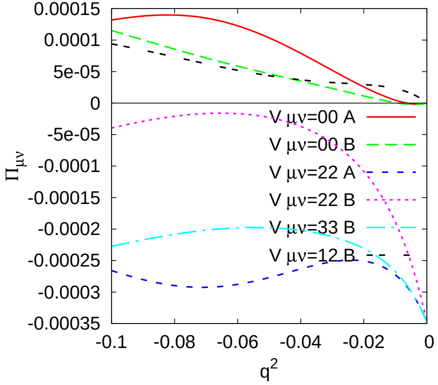
Case A:
$q = \left(0, \sqrt{-q^2}, 0, 0\right)$

$V = \infty$:
$\Pi_{00} = -\Pi_{22} = -\Pi_{33}$
$\Pi_{11} = 0$

ΔV:
$\Pi_{22} = \Pi_{33}$

$\vec{\theta}_u = L\, q$ $\qquad m_\pi L = 3$

Twisted boundary conditions: Two-point function



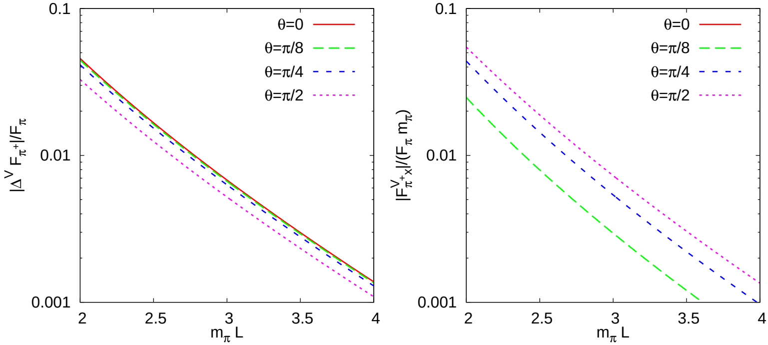
Case A:
$$q = \left(0, \sqrt{-q^2}, 0, 0\right)$$

Case B:
$$q = \left(0, \sqrt{-q^2/2}, \sqrt{-q^2/2}, 0\right)$$

$\vec{\theta}_u = L\, q$ $\qquad m_\pi L = 3$

Volume correction decay constants: F_{π^+}

- JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]
- $\langle 0|A^M_\mu|M(p)\rangle = i\sqrt{2}F_M p_\mu + i\sqrt{2}F^V_{M\mu}$
- Extra terms are needed for Ward identities



relative for F_π — Extra for $\mu = x$

Volume correction electromagnetic formfactor

- JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]
  earlier two-flavour work:
  Bunton, Jiang, Tiburzi, Phys.Rev. D74 (2006) 034514 [hep-lat/0607001]
- $\langle M'(p')|j_\mu|M(p)\rangle = f_\mu = f_+(p_\mu + p'_\mu) + f_- q_\mu + h_\mu$
- Extra terms are again needed for Ward identities
- Note that masses have finite volume corrections
  - q^2 for fixed $\vec{p}$ and $\vec{p}'$ has corrections
    small effect
  - This also affects the ward identities, e.g.
    $q^\mu f_\mu = (p^2 - p'^2) f_+ + q^2 f_- + q^\mu h_\mu = 0$
    is satisfied but all effects should be considered

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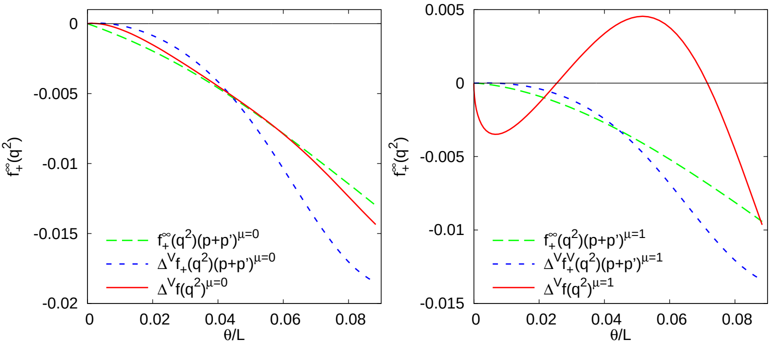
Volume correction electromagnetic formfactor

- $f_\mu = -\frac{1}{\sqrt{2}}\langle\pi^0(p')|\bar{d}\gamma_\mu u|\pi^+(p)\rangle$
  $= \left(1 + f_+^\infty + \Delta^V f_+\right)(p+p')_\mu + \Delta^V f_- q_\mu + \Delta^V h_\mu$
- Pure loop plotted: $f_+^\infty(p+p'), \Delta^V f_+(p+p')$ and $\Delta^V f_\mu$



$\mu = t$ $\mu = x$

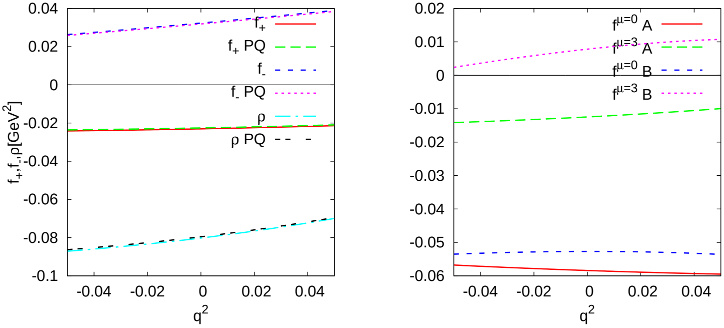
Finite volume corrections large, different for different μ

$K_{\ell 3}$

- $q = p - p'$
  $\langle \pi^-(p')|\bar{s}\gamma_\mu u(0)|K^0(p)\rangle = f_+(p_\mu + p'_\mu) + f_- q_\mu + h_\mu\,.$
- $\langle \pi^-(p')|(m_s - m_u)\bar{s}u(0)|K^0(p)\rangle = \rho\,.$
- Ward identity: $(p^2 - p'^2)f_+ + q^2 f_- + q^\mu h_\mu = \rho$
- ChPT:
  - p^4 Isopin conserving and breaking Gasser, Leutwyler, 1985
  - p^6 Isospin conserving JB, Talavera, 2003
  - p^6 Isospin breaking JB, Ghorbani, 2007
  - p^4 partially quenched, staggered Bernard, JB, Gamiz, 2013
  - p^4 Finite volume Ghorbani, Ghorbani, 2013 ($q^2 = 0$)
  - p^4 twisted, partiallyquenched, staggered
    Bernard, JB, Gamiz, Relefors, in preparation
  - Rare decays: p^4 Mescia, Smith 2007, p^6 JB, Ghorbani, 2007

$K_{\ell 3}$

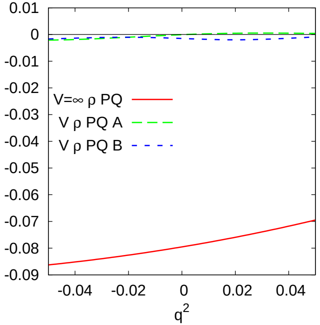
- Masses: finite volume masses with twist effect included.
- $p = \left(\sqrt{m_K^2(\vec{p}) + \vec{p}^2}, \vec{p}\right)$
- $p' = \left(\sqrt{m_\pi^2(\vec{p}') + \vec{p}'^2}, \vec{p}'\right)$
- q^2 calculated with m_K^2 and m_π^2 at $V = \infty$ will also have volume corrections (small efect)
- remaining plots: p^4 (neglecting the $L_9^r q^2$ term)
- Valence masses with $m_\pi = 135$ GeV and $m_K = 0.495$ GeV
- PQ case with $\hat{m}_{\rm sea} = 1.1\hat{m}$, $m_{s{\rm sea}} = 1.1 m_s$.
- case A: $\vec{p} = 0$, case B: $\vec{p}' = 0$



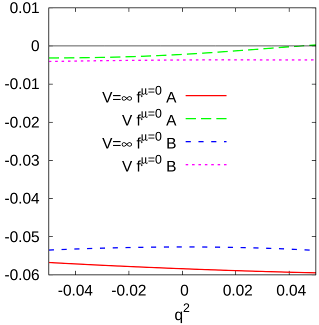
PQ case

The components are the well defined ones

$K_{\ell 3}$

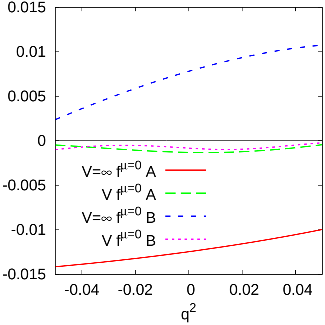


ρ



$\mu = 0$

$K_{\ell 3}$



$\mu = 3$

Calculate the volume corrections for exactly what you did

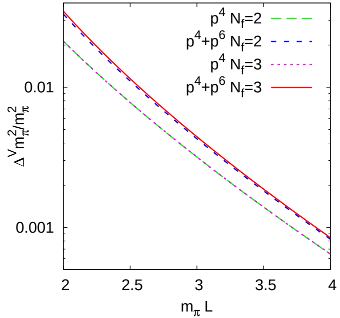
Masses at two-loop order

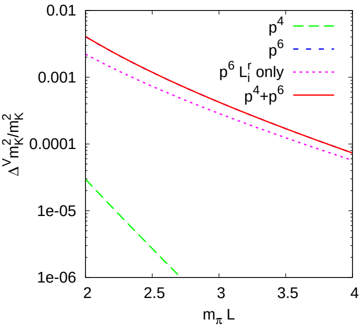
- Sunset integrals at finite volume done

  JB, Boström and Lähde, JHEP 01 (2014) 019 [arXiv:1311.3531]

- Loop calculations:

  JB, Rössler, JHEP **1501** (2015) 034 [arXiv:1411.6384]





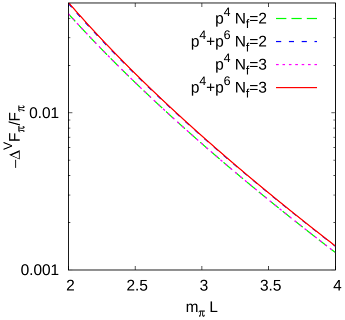
- Agreement for $N_f = 2, 3$ for pion
- K has no pion loop at LO

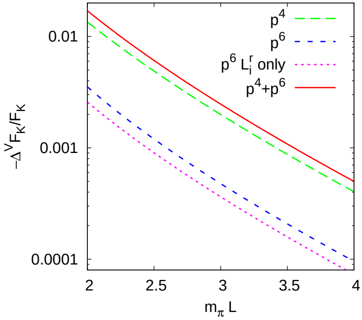
Decay constants at two-loop order

- Sunset integrals at finite volume done

  JB, Boström and Lähde, JHEP 01 (2014) 019 [arXiv:1311.3531]
- Loop calculations:

  JB, Rössler, JHEP **1501** (2015) 034 [arXiv:1411.6384]





- Agreement for $N_f = 2, 3$ for pion
- K now has a pion loop at LO

Other p^6

Masses and decay constants at finite volume:

- Finite volume for PQ three flavour (all cases) JB, Rössler, JHEP **1511** (2015) 097, [arXiv:1508.07238]
- QCD-like theories, normal and PQ (one valence mass, one sea mass) JB, Rössler, JHEP **1511** (2015) 017, [arXiv:1509.04082]
    - $SU(N) \times SU(N)/SU(N)$
    - $SU(N)/SO(N)$ (including Majorana case)
    - $SU(2N)/Sp(2N)$

Conclusions

- Showed you some of the pitfalls
- Be careful: ChPT must exactly correspond to your lattice calculation
- Programs available (for published ones) via CHIRON