Massive Photons:

An IR regularization scheme for LQCD+QED

Michael Endres, Andrea Shindler, Brian Tiburzi,
André Walker-Loud*, [redacted]

Symposium on Effective Field Theories and Lattice Gauge Theory
TUM IAS
18-21 May, 2016

* speaker

Massive Photons: Motivation

- Isospin breaking is small but plays a very important role in our universe

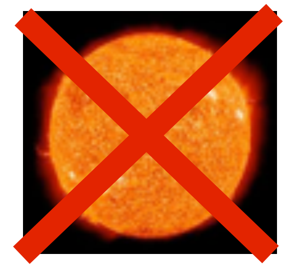
$$\frac{m_n - m_p}{m_n + m_p} \simeq 0.0007$$

- Multiple IR regulators are useful/important (as are multiple UV regulators)
- Our method is formulated with a local QFT
- $\mathrm{QED}_{\{L,TL\}}$ (zero-mode subtracted) scheme has large power-law FV corrections: EFT estimates for nuclei indicate large volumes are necessary with standard methods
- Our method trades (1/L) corrections for $e^{-m_\gamma L}$ corrections
- Our method is numerically less expensive (will show/argue)
- Our method allows for calculations involving multiple charged-particles in the in/out states a la Lüscher/Lellouch-Lüscher as m_γ introduces mass gap to hadron+γ states

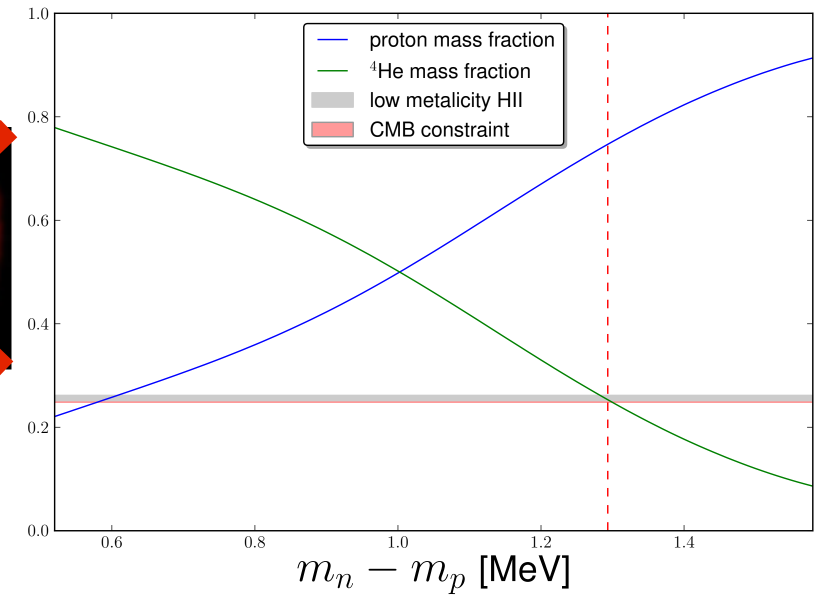
Massive Photons: Motivation

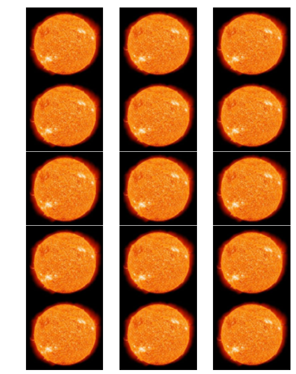
- Isospin breaking is small but plays a very important role in our universe

$$\frac{m_n - m_p}{m_n + m_p} \simeq 0.0007$$



No Sun!





Many suns!

- Multiple IR regulators are useful/important (as are multiple UV regulators)
- Our method is formulated with a local QFT
- QED$_{\{L,TL\}}$ (zero-mode subtracted) scheme has large power-law FV corrections: EFT estimates for nuclei indicate large volumes are necessary with standard methods
- Our method trades (1/L) corrections for $e^{-m_\gamma L}$ corrections
- Our method is numerically less expensive (will show/argue)
- Our method allows for calculations involving multiple charged-particles in the in/out states a la Lüscher/Lellouch-Lüscher as m_γ introduces mass gap to hadron+γ states

Massive Photons: Analytic Considerations

- m_γ breaks QED gauge invariance:
  - these effects can be systematically included in an EFT framework
  - introduces need for a new extrapolation: $m_\gamma \to 0$
  - corrections for small $m_\gamma L$ must be handled carefully. We explored and were able to control corrections for $1 \lesssim m_\gamma L$
- Modification to charged particle correlation functions leading to non-standard time dependence, must be handled carefully for small $m_\gamma L$
- For Wilson fermions, must worry about the shift in additive quark mass renormalization induced by the QED effects

Massive Photons: Analytic Considerations

Introducing a photon mass:

$$\mathcal{L}_\gamma = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{1}{2\xi} (\partial_\mu A_\mu)^2 + \frac{1}{2} m_\gamma^2 A_\mu^2 \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- For this work, we chose Landau gauge $\xi = 0$
  - preserves rotational invariance
  - complete gauge in Euclidean space: no flat directions as $m_\gamma \to 0$ except for zero-mode
  - the zero-mode results in a very mild signal/noise problem as $m_\gamma \to 0$

Massive Photons: Analytic Considerations

Introducing a photon mass:

$$\mathcal{L}_\gamma = \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{2\xi}(\partial_\mu A_\mu)^2 + \frac{1}{2}m_\gamma^2 A_\mu^2 \qquad\qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- For this work, we chose Landau gauge $\xi = 0$
  - preserves rotational invariance
  - complete gauge in Euclidean space: no flat directions as $m_\gamma \to 0$ except for zero-mode
  - the zero-mode results in a very mild signal/noise problem as $m_\gamma \to 0$

$V = 13 \times 11 \times 7 \times 5$

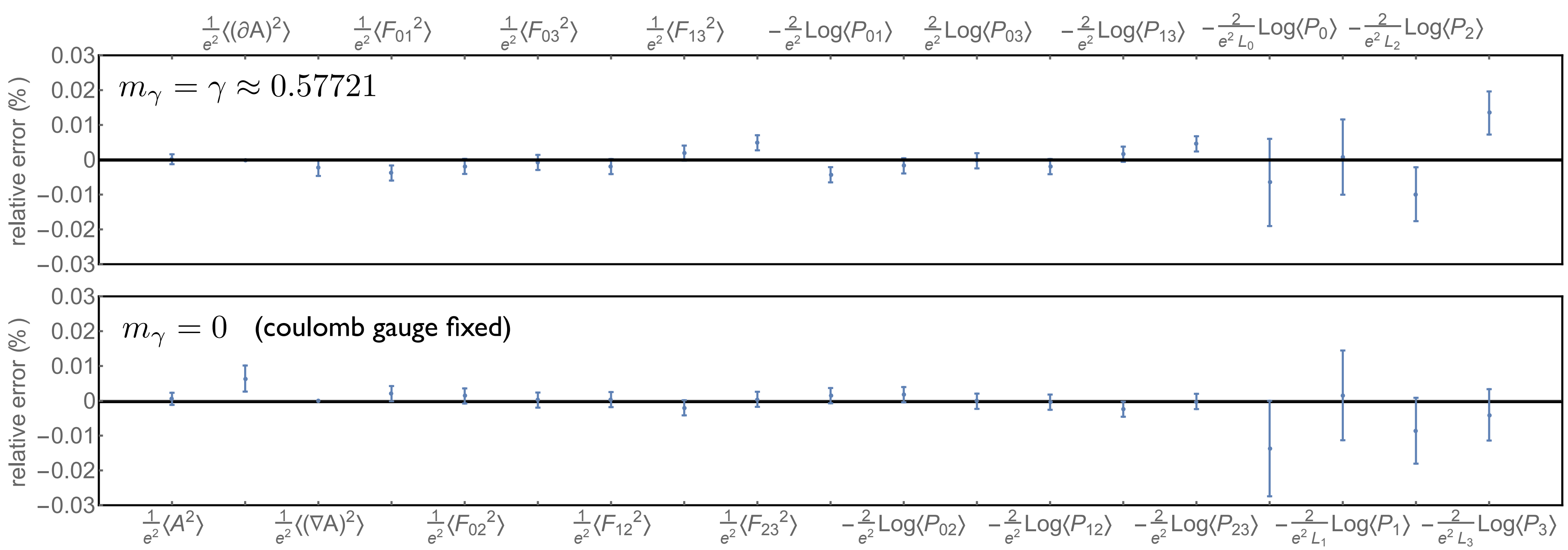
∂ = 4-divergence

∇ = 3-divergence

$P_{\mu\nu}$ = plaquette

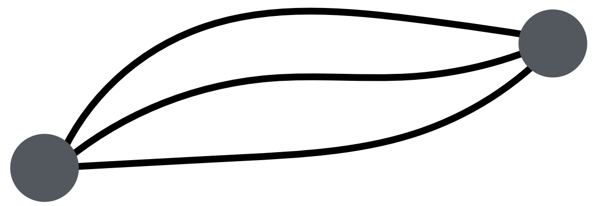
P_μ = Polyakov loop

$N_{conf} = 10^6 \quad e = \pi/3$



Massive Photons: Analytic Considerations

Massive Zero mode effects: charged particle correlation function

$C_{\tilde{A}}(t) =$  $\propto e^{iQt\tilde{A}_0(0)/V}$ $\qquad V = L^3T$

- The leading zero-mode, $\tilde{A}_\mu(0)$, contribution can be determined from quark-level diagrams
  - each charged fermion line is accompanied by a Wilson line (path independent, depends on source/sink locations)
  - Integrating over the zero-mode component of the Wilson line leads to non-standard time dependence

$$C(t) = \frac{1}{Z_{A_0}} \int d\tilde{A}_0(0) e^{-\frac{1}{Ve^2}\frac{m_\gamma^2}{2}\tilde{A}_0(0)^2} C_{\tilde{A}}(t) \propto e^{-Q^2e^2t^2/(2m_\gamma^2 V)}$$

$$\Rightarrow \quad C_{H_Q}(t) \propto e^{-M_H t - xt^2} \qquad x = \frac{4\pi\alpha}{2m_\gamma^2 L^3 T}$$

  - This effect becomes important for small $m_\gamma L$
  - We will return to this explicitly later

Massive Photons: Analytic Considerations

Finite m_γ and Effective Field Theory

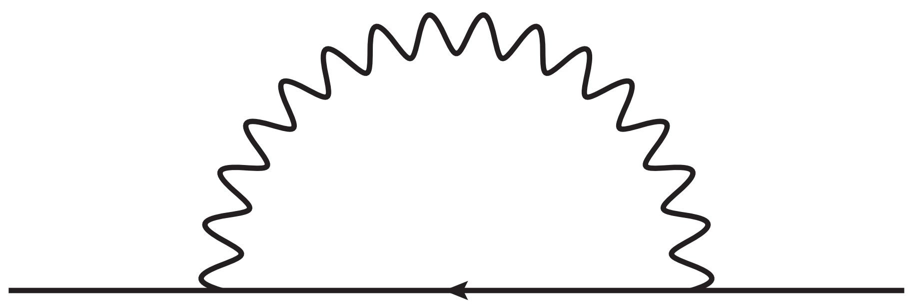
- The explicit contributions from a finite photon mass can be incorporated in an EFT framework.
  - There will be new operators that break gauge invariance and will proportional to the photon mass (gauge invariance is recovered as $m_\gamma \to 0$)
  - There are also the regular operators giving rise to QED corrections
- For simplicity, we will restrict ourselves to $m_\gamma \leq m_\pi$
  - We will not need to consider virtual corrections with both pions and photons propagating
  - We can construct an NRQED Lagrangian of point hadrons to determine the QED corrections

$$\begin{aligned}\mathcal{L}_{H\gamma} \supset\, & e^2 \frac{m_\gamma^2}{M} \left[C_0 \mathrm{tr}(Q^2) N^\dagger N + C_1 \mathrm{tr}(Q) N^\dagger Q N + C_2 N^\dagger Q^2 N\right] \\ & + C_m e^2 m_\gamma^2 A_\mu A_\mu N^\dagger N + \cdots \\ & + \text{similar operators for mesons}\end{aligned}$$

Massive Photons: Analytic Considerations

Finite m_γ and Effective Field Theory

- The leading correction comes from the photon-sunset graph
  - we can determine the UV finite correction induced by the photon mass

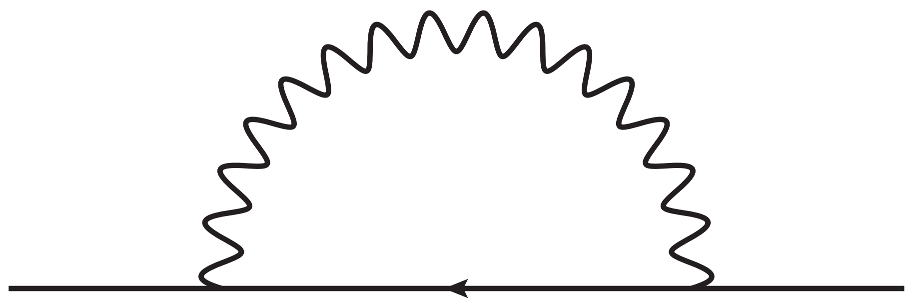


$$\Delta_\gamma M \equiv M(\alpha, m_\gamma) - M(\alpha, 0)$$
$$= -\frac{\alpha M}{2\pi} \int_0^1 dx(1+x) \ln\left(1 + \frac{1-x}{x^2}\frac{m_\gamma^2}{M^2}\right)$$

- Notice, the correction is non-analytic in m_γ^2/M^2, the hall-mark signature of IR corrections from light virtual particles.
  - consequently, expanding for small m_γ^2/M^2 and integrating produces unphysical singularities. One must either integrate, then expand, or use the method of regions (similar to steepest decent - B. Tiburzi)

Massive Photons: Analytic Considerations

Finite m_γ and Effective Field Theory



- We arrive at

$$\frac{\Delta_\gamma M}{M} = -\frac{1}{2}\alpha\left[\frac{m_\gamma}{M} - \frac{3}{2\pi}\frac{m_\gamma^2}{M^2} + \cdots\right]$$

- The first term is from non-analytic IR behavior, and therefore the coefficient is a prediction. The second term is analytic in m_γ^2/M^2 and so there will be accompanying effects from local operators - the standard EFT scenario.

- All together, one has

$$\Delta_\gamma M^{LO} = -\frac{\alpha}{2}Q^2 m_\gamma$$

$Q =$ hadron charge

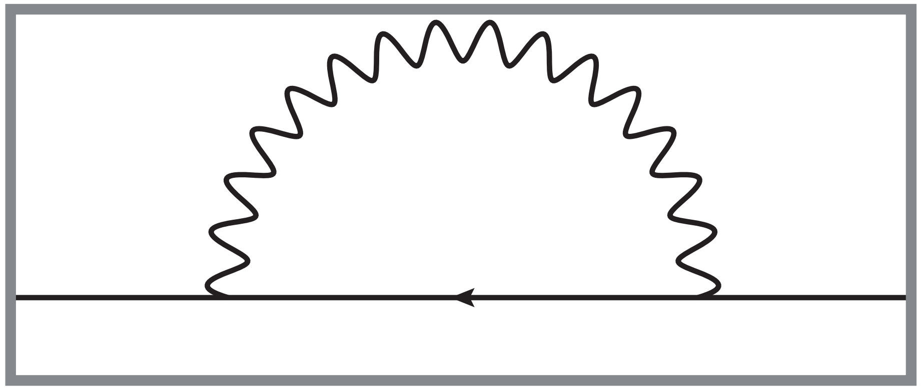
$$\Delta_\gamma M^{NLO} = \left(Ce^2 - \frac{\alpha}{4\pi}Q^2\right)\frac{m_\gamma^2}{M}$$

$$\Delta_\gamma M^{NNLO} = \frac{\alpha}{4}\left(\frac{3}{4}Q^2 - \frac{1}{2}Qc_D - c_F^2\right)\frac{m_\gamma^3}{M^2}$$

	baryons	mesons
$c_D =$	$Q + \frac{4}{3}M^2\langle r_E^2\rangle$	$\frac{4}{3}M^2\langle r_E^2\rangle$
$c_F =$	$Q + \kappa$	0

Massive Photons: Analytic Considerations

Finite Volume Corrections



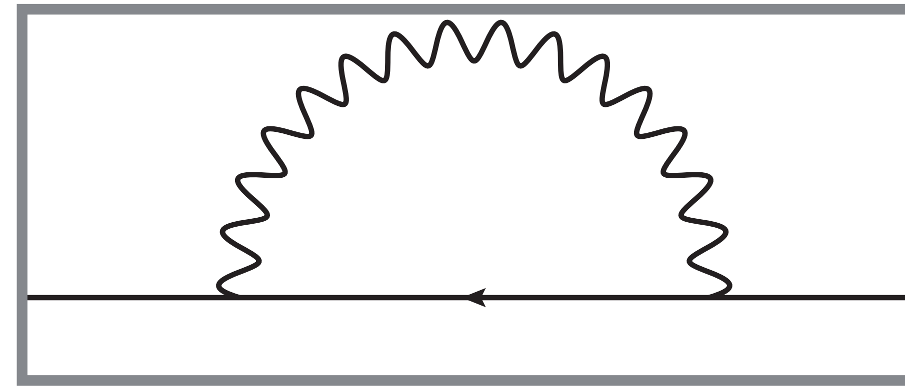
- The finite volume corrections we are interested in arise from well behaved integrands:
  - Poisson Summation Formula
- We define: $\delta_L M \equiv M(\alpha, m_\gamma, L) - M(\alpha, m_\gamma, \infty)$
- For large $m_\gamma L$, the corrections are as expected, exponentially suppressed

$$\frac{\delta_L M}{M} = 3\alpha \frac{e^{-m_\gamma L}}{ML} \left[1 + \sqrt{\frac{m_\gamma L}{2\pi}} \left(1 + \frac{2}{m_\gamma L} \right) \frac{m_\gamma}{M} - \frac{4}{\sqrt{2\pi M L}} e^{-(M - m_\gamma)L} + \cdots \right]$$

- We are interested in $m_\gamma \lesssim m_\pi$ so we would like to understand these FV corrections for small $m_\gamma L$ as well.

Massive Photons: Analytic Considerations

Finite Volume Corrections: Zero Modes



- The volume corrections can be expanded in a winding number expansion:

$$\delta_L M^{LO} = 2\pi\alpha Q^2 m_\gamma I_1(m_\gamma L)$$

$$\delta_L M^{NLO} = \pi\alpha Q^2 \frac{m_\gamma^2}{M}\left[2I_{1/2}(m_\gamma L) + I_{3/2}(m_\gamma L)\right]$$

$$I_n(z) = \frac{1}{2^{n+\frac{1}{2}}\pi^{\frac{3}{2}}\Gamma(n)} \sum_{\nu\neq 0} \frac{K_{\frac{3}{2}-n}(z|\nu|)}{(z|\nu|)^{\frac{3}{2}-n}}$$

- The $L \to \infty$ and $m_\gamma \to 0$ limits do not commute due to the photon zero-mode, which has the form

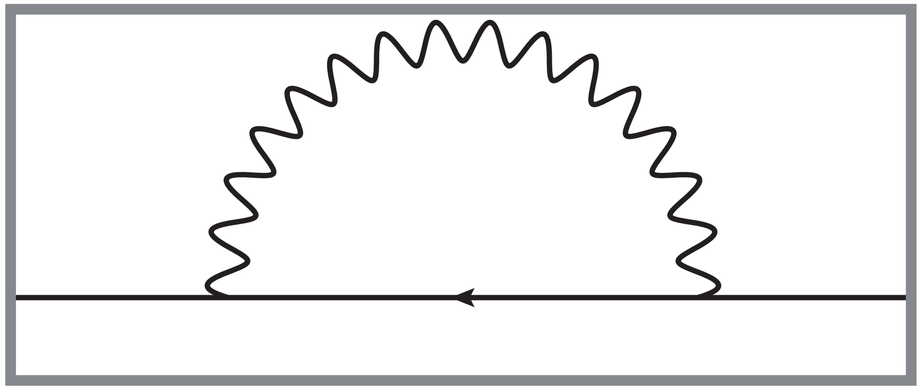
$$\delta_L M^0 = \frac{2\pi\alpha Q^2}{m_\gamma^2 L^3}\left[1 + \frac{3}{2}\frac{m_\gamma}{M}\right]$$

- These zero-modes can be treated exactly - this was mentioned a few slides earlier with the Wilson-lines accompanying charged particle correlators

Massive Photons: Analytic Considerations

Finite Volume Corrections: Zero Modes

$$C_{H_Q}(t) \propto e^{-M_H t - x t^2} \qquad x = \frac{4\pi\alpha}{2m_\gamma^2 L^3 T}$$



- This zero-mode contribution can be subtracted prior computing the mass corrections. For a heavy state (no significant backwards time components) the modified effective mass formula removes the t^2 corrections isolating M_H

$$M_{\rm eff}^{\rm exp}(t,\tau) = \frac{1}{\tau}\ln\left(\frac{C(t)}{C(t+\tau)}\right) + 2xt + x\tau$$

- When the finite T extent can not be neglected (pion/kaon) the modified cosh effective mass can be used to similarly remove the unwanted zero-mode corrections

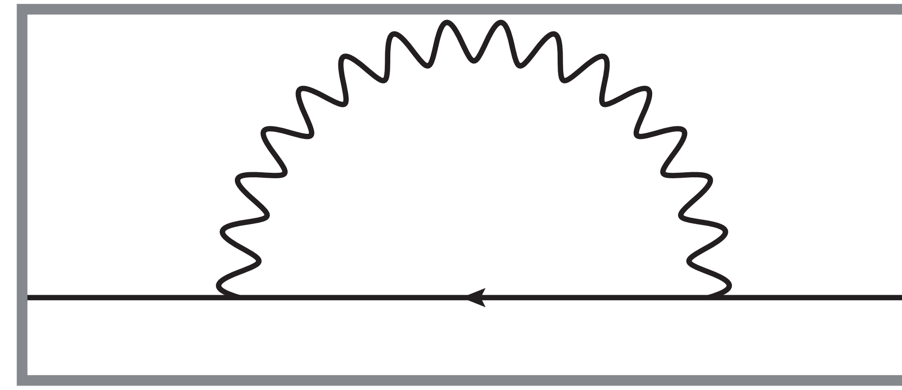
$$M_{\rm eff}^{\rm cosh}(t,\tau) = \frac{1}{\tau}\cosh^{-1}\left[\frac{e^{h(t,\tau)} + e^{h(t,-\tau)}}{2}\right] - xT$$

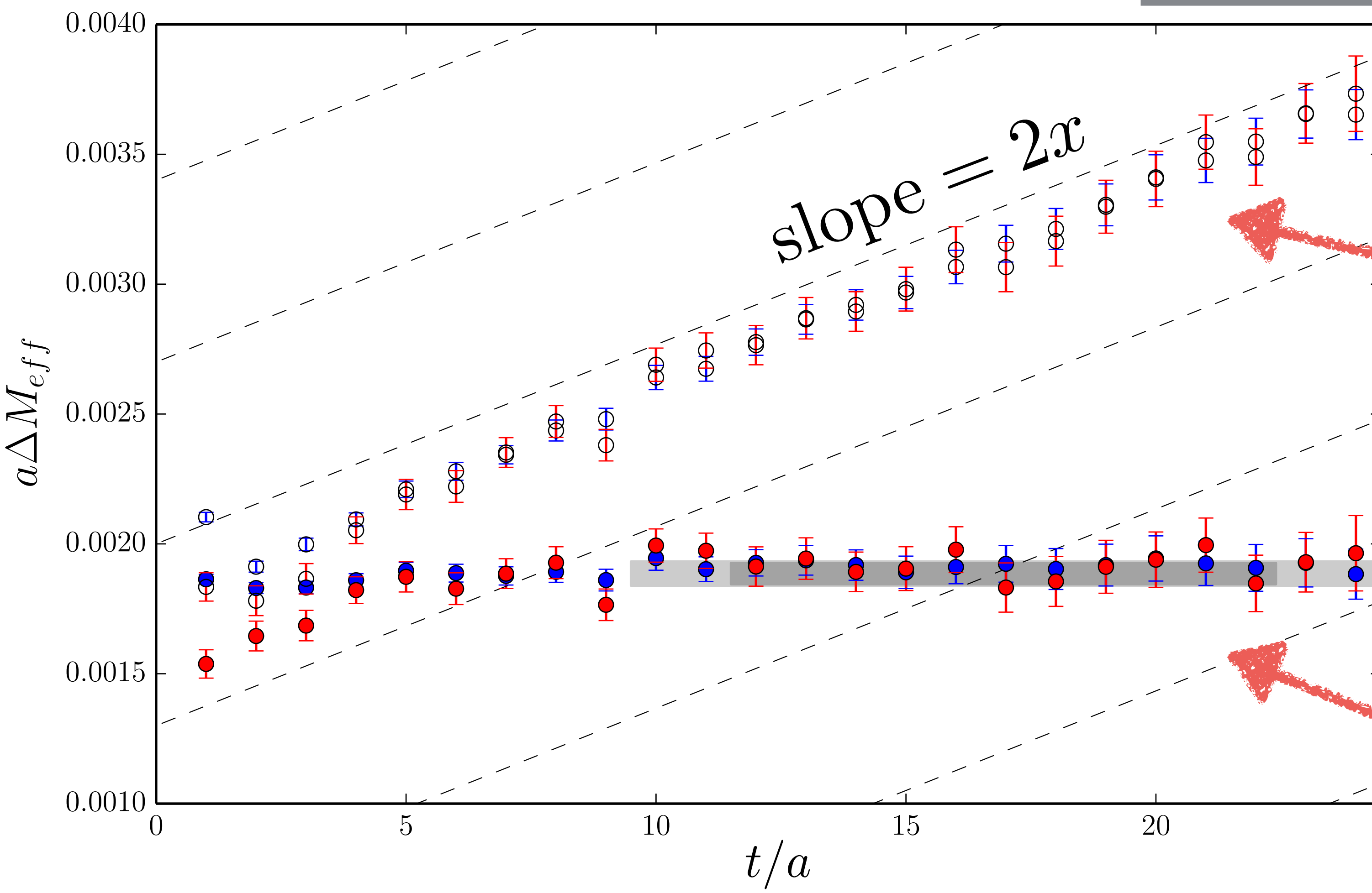
$$h(t,\tau) = x\tau(\tau - T + 2t) - \ln\left(\frac{C(t)}{C(t+\tau)}\right)$$

Massive Photons: Analytic Considerations

Finite Volume Corrections: Zero Modes

$$C_{H_Q}(t) \propto e^{-M_H t - x t^2} \qquad x = \frac{4\pi\alpha}{2 m_\gamma^2 L^3 T}$$





"worst case" scenario:
smallest L
smallest m_γ

regular cosh effective mass

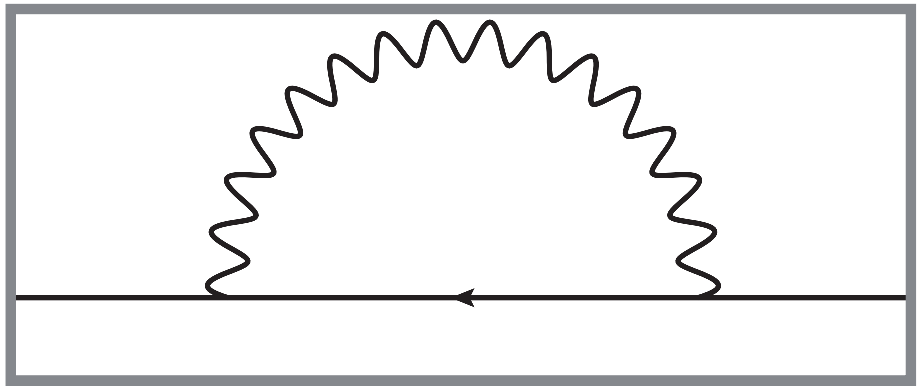
modified cosh effective mass

$$M_{\rm eff}^{\cosh}(t,\tau) = \frac{1}{\tau}\cosh^{-1}\left[\frac{e^{h(t,\tau)} + e^{h(t,-\tau)}}{2}\right] - xT$$

$$h(t,\tau) = x\tau(\tau - T + 2t) - \ln\left(\frac{C(t)}{C(t+\tau)}\right)$$

Massive Photons: Analytic Considerations

Finite Volume Corrections: Zero Modes



- After explicitly removing these zero-mode contributions, we must modify the remaining FV corrections, e.g.

$$\delta_L M^{LO} \to \delta_L M^{LO} = 2\pi\alpha Q^2 m_\gamma \left[I_1(m_\gamma L) - \frac{1}{(m_\gamma L)^3} \right]$$

Massive Photons: Numerical Results

- To study our new method - we want to isolate the finite volume corrections from QED effects, and to compare with more standard methods:
  - We chose to use new (2012) iso-clover ensembles generated by W&M/JLab
    - Lüscher-Weisz gauge action
    - single-stout smeared, tree-level tadpole improved clover fermion action
    - SU(3) flavor symmetric point, $\bar{m}_\pi = \bar{m}_K \simeq 807$ MeV $\bar{m}_n = 1634$ MeV
    - single discretization scale, $a \simeq 0.145$ fm
    - $L/a = 24$ (N_{cfg}=956) $L/a = 32$ (N_{cfg}=515) $L/a = 48$ (N_{cfg}=342)
  - We post-multiplied (electro-quenched) each ensemble by a single non-compact QED field, correlators averaged over +/- e
    - QED_{TL} ($m_\gamma = 0$, zero-mode subtracted) for comparison
    - QED_M with $\dfrac{m_\gamma}{m_\pi} = \left\{ \dfrac{1}{14}, \dfrac{1}{7}, \dfrac{1}{4}, \dfrac{1}{3}, \dfrac{5}{12}, \dfrac{1}{2}, \dfrac{7}{12}, 1 \right\}$

$m_\gamma L \simeq 1$ $\quad$ $L = 24$ $\quad$ $m_\gamma L \simeq 14$

Massive Photons: Numerical Results

- We hacked Chroma (mostly Mike) and computed the kaon and nucleon spectrum using CPU cycles from JLab (via USQCD)
- electro-quenched theory does not renormalize electromagnetic coupling
$$\alpha^{-1} = 137$$
- In SU(3) flavor limit, the errors in electro-quenched splittings are $\mathcal{O}(\alpha^2)$
- To handle the additive quark mass renormalization induced by the QED corrections, we chose to hold the mass of the neutral $\bar{q}\gamma_5 q$ mesons fixed

$$m_{\bar{q}q}^2 = x^2 + z(m_q - m_0) + Q^2\left[v(m_q - m_0) + w m_\gamma^2 + y\right]$$

$$m_0 = -0.2450 \quad \text{input quark mass for pure QCD ensembles}$$

$$x^2 = \bar{m}_\pi^2 \quad \text{we do not force fit to go through QCD point exactly}$$

- on all ensembles, we achieve $\dfrac{m_{\bar{q}q} - m_\pi}{m_\pi} \lesssim 0.001$
- while these mis-tunings are small, they can still lead to potentially large corrections to the mass splittings - these are induced strong-isospin breaking corrections

Massive Photons: Numerical Results

- We can use LO Chiral Perturbation Theory to estimate the size of the mis-tuning effects:

$$\frac{\Delta m_{K^+} - \Delta m_{K^0}}{\bar{m}_K} \simeq \frac{1}{2}\frac{\Delta m_{uu} - \Delta m_{dd}}{\bar{m}_K} \lesssim 0.0004$$

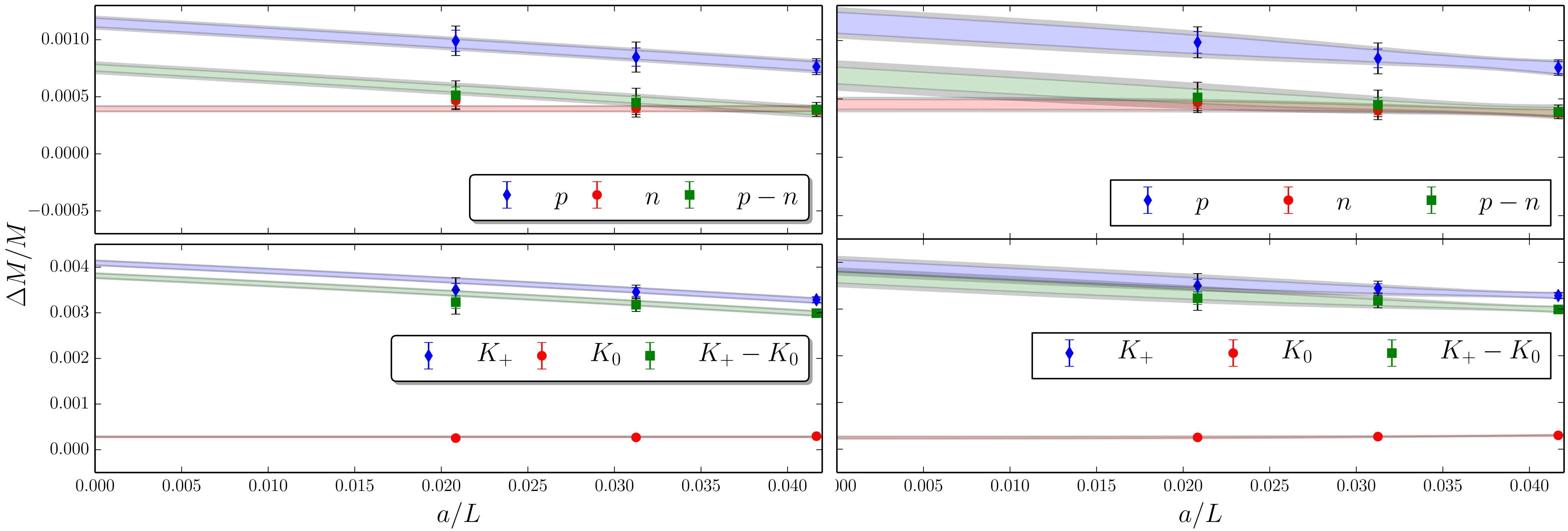
$$\frac{\Delta m_n - \Delta m_p}{\bar{m}_n} \simeq 2\alpha_{d-u}\frac{\Delta m_{uu} - \Delta m_{dd}}{\bar{m}_\pi}\frac{\bar{m}_\pi^2}{4\pi f_\pi \bar{m}_n} \lesssim 0.0002$$

- α_{d-u} is an unknown LEC which we can estimate with the LQCD determination of the m_d-m_u contribution to m_n - m_p
- While both of these mis-tuning corrections are small, they introduce potentially large uncertainties in the mass splittings. For the purpose of testing our new method, we do not need to perform the tuning more precisely.

Massive Photons: Numerical Results

QED$_{\mathrm{TL}}$

Numerical results are volume-adjusted



nucleon $$\Delta M = \Delta M(\alpha) + \frac{c_1 Q^2 \alpha}{2L}\left(1+\frac{2}{ML}\right) + \frac{\pi Q^2\alpha}{2ML}\frac{T}{L^2} + \frac{3\pi Q^2 \alpha}{(ML)^2}\frac{1}{L}\left(1-\frac{1}{2\tanh(MT)}+\frac{1}{2MT}\right)$$

kaon $$\Delta M = \Delta M(\alpha) + \frac{c_1 Q^2 \alpha}{2L}\left(1+\frac{2}{ML}\right) + \frac{\pi Q^2\alpha}{2ML}\frac{T}{L^2}$$

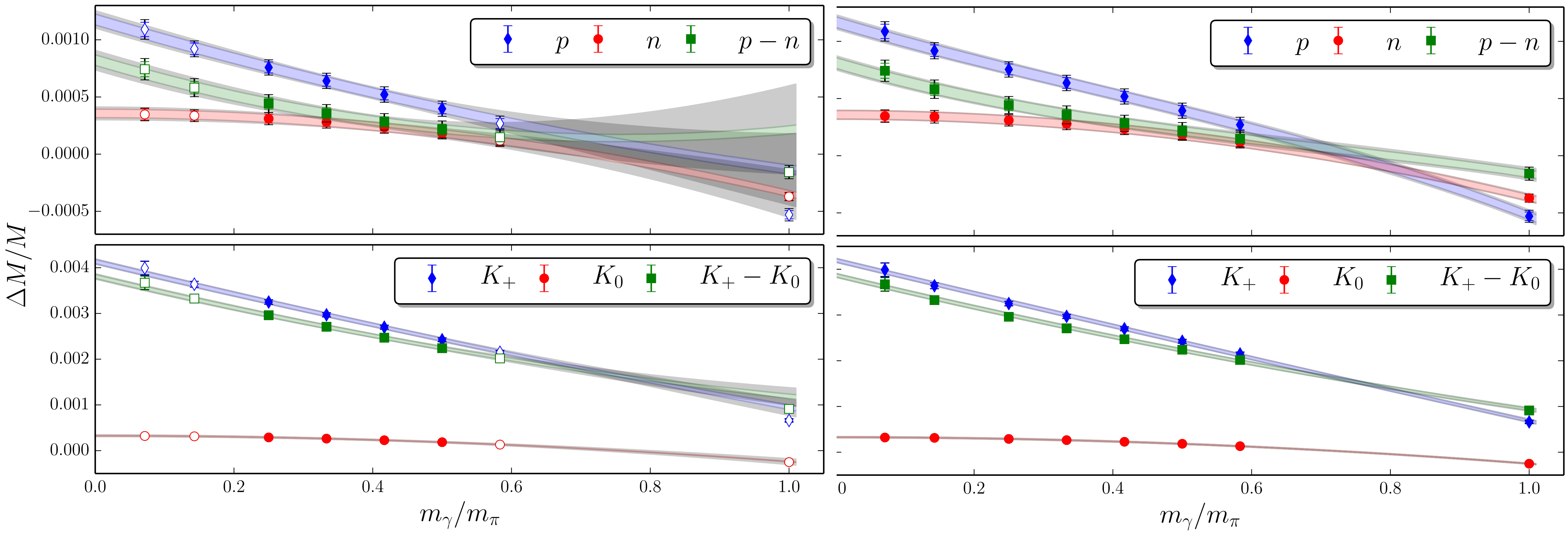
$c_1 = -2.83729$

QED$_{\mathrm{TL}}$ functions from Borsanyi et. al. Science **347** (2015)

Massive Photons: Numerical Results

QED$_\mathrm{M}$

Numerical results are volume-adjusted



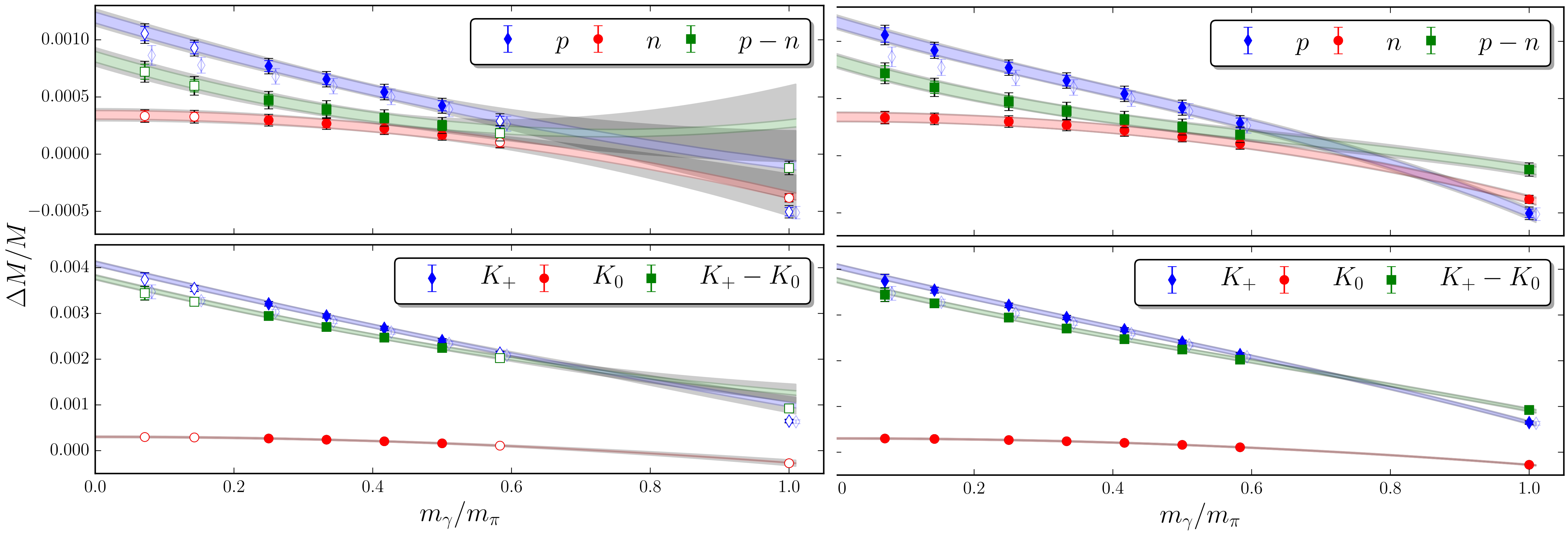
$$\Delta M = \Delta_\gamma M(\alpha) + 2\pi\alpha Q^2 m_\gamma \left[I_1(m_\gamma L) - \frac{1}{(m_\gamma L)^3} \right] + \pi\alpha Q^2 \frac{m_\gamma^2}{M} \left[2I_{1/2}(m_\gamma L) + I_{3/2}(m_\gamma L) \right]$$

$$\Delta_\gamma M(\alpha) = \Delta M(\alpha) - \frac{\alpha}{2} Q^2 m_\gamma + \left(Ce^2 - \frac{Q^2\alpha}{4\pi} \right) \frac{m_\gamma^2}{M} + \cdots$$

Massive Photons: Numerical Results

QED$_{\mathrm{M}}$

Numerical results are volume-adjusted



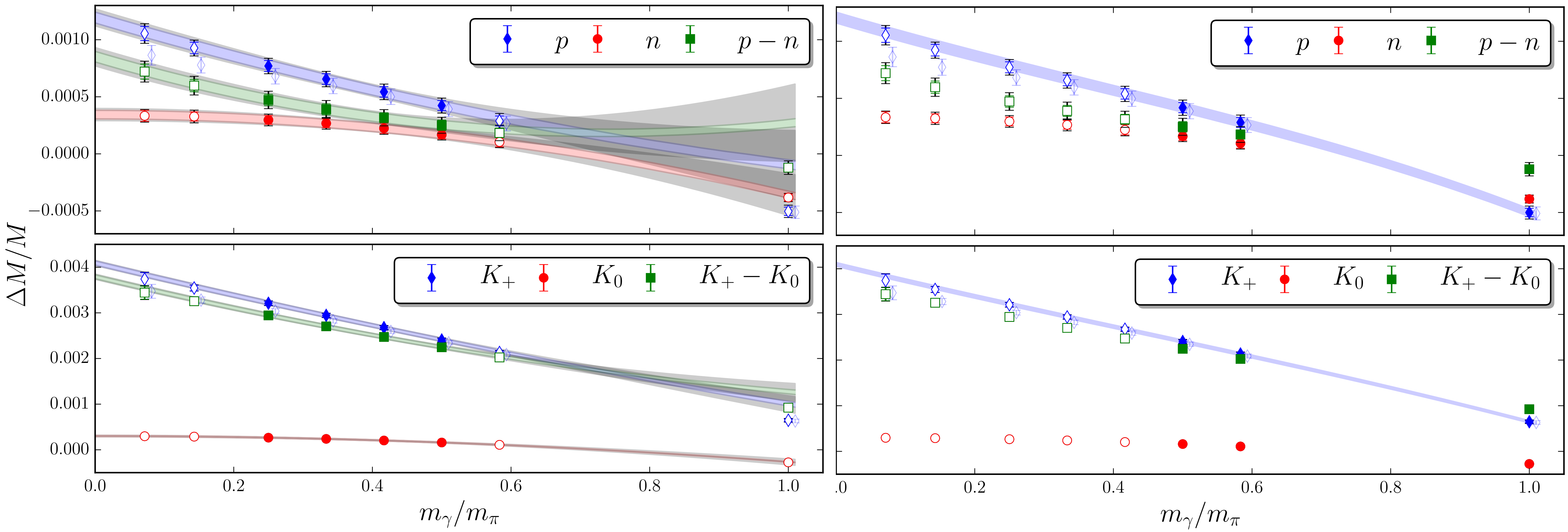
$$\Delta M = \Delta_\gamma M(\alpha) + 2\pi\alpha Q^2 m_\gamma \left[I_1(m_\gamma L) - \frac{1}{(m_\gamma L)^3} \right] + \pi\alpha Q^2 \frac{m_\gamma^2}{M} \left[2I_{1/2}(m_\gamma L) + I_{3/2}(m_\gamma L) \right]$$

$$\Delta_\gamma M(\alpha) = \Delta M(\alpha) - \frac{\alpha}{2} Q^2 m_\gamma + \left(Ce^2 - \frac{Q^2\alpha}{4\pi} \right) \frac{m_\gamma^2}{M} + \cdots$$

Massive Photons: Numerical Results

QED$_M$

Numerical results are volume-adjusted



$$\Delta M = \Delta_\gamma M(\alpha) + 2\pi\alpha Q^2 m_\gamma \left[I_1(m_\gamma L) - \frac{1}{(m_\gamma L)^3}\right] + \pi\alpha Q^2 \frac{m_\gamma^2}{M}\left[2I_{1/2}(m_\gamma L) + I_{3/2}(m_\gamma L)\right]$$

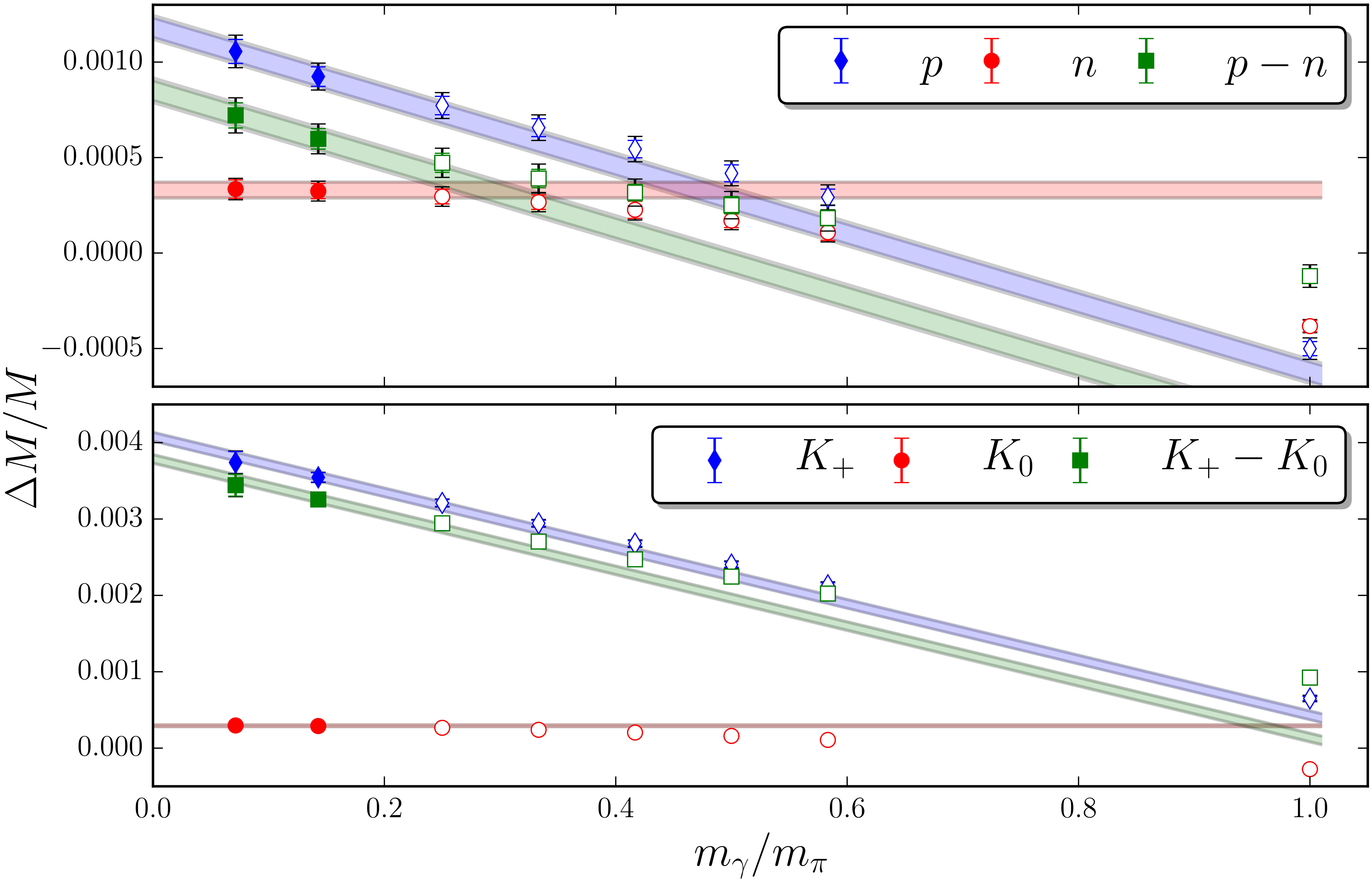
$$\Delta_\gamma M(\alpha) = \Delta M(\alpha) - \frac{\alpha}{2}Q^2 m_\gamma + \left(Ce^2 - \frac{Q^2\alpha}{4\pi}\right)\frac{m_\gamma^2}{M} + \cdots$$

Massive Photons: Numerical Results

QED_M

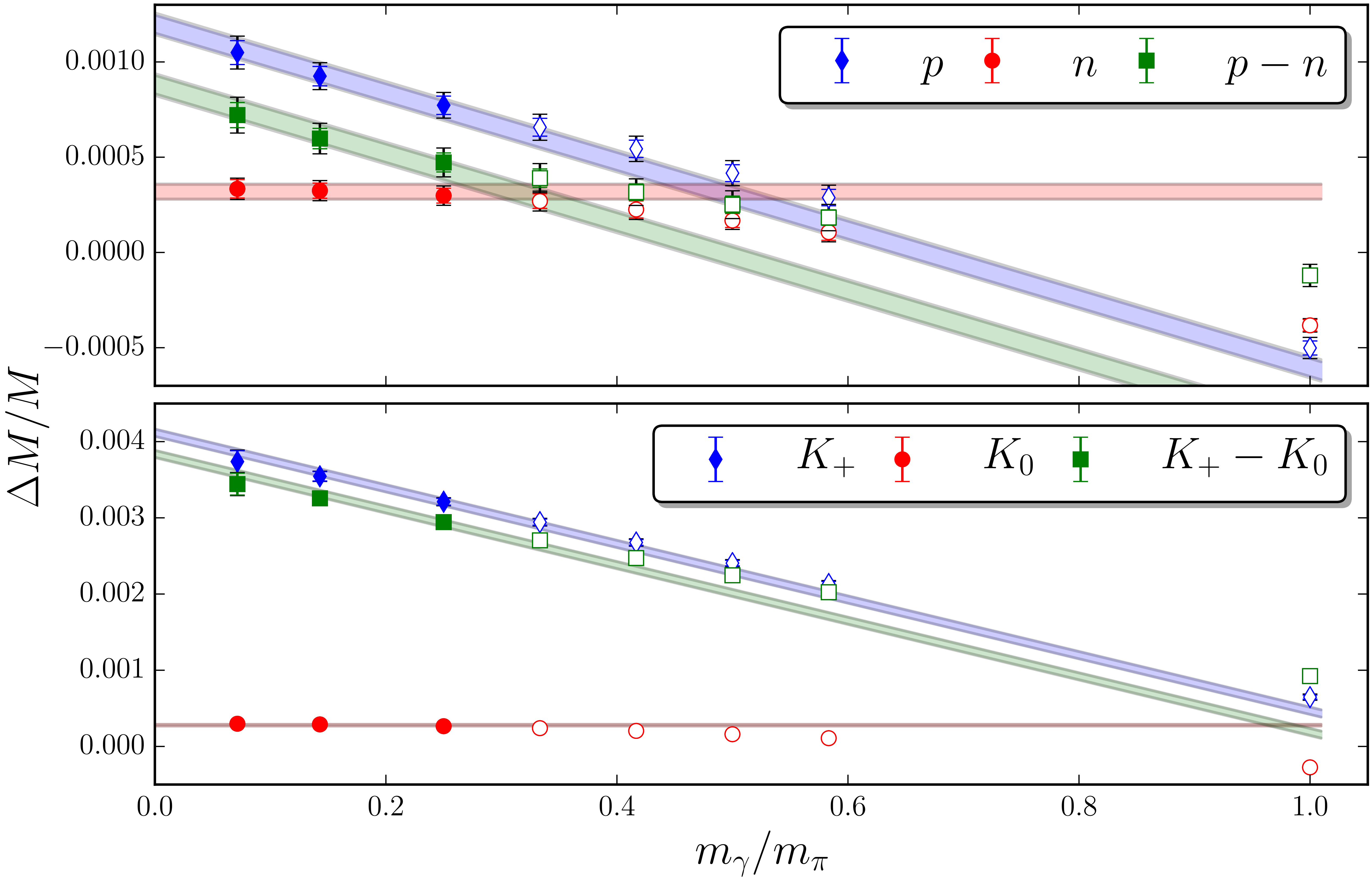
Massive Photons: Numerical Results

QED$_M$



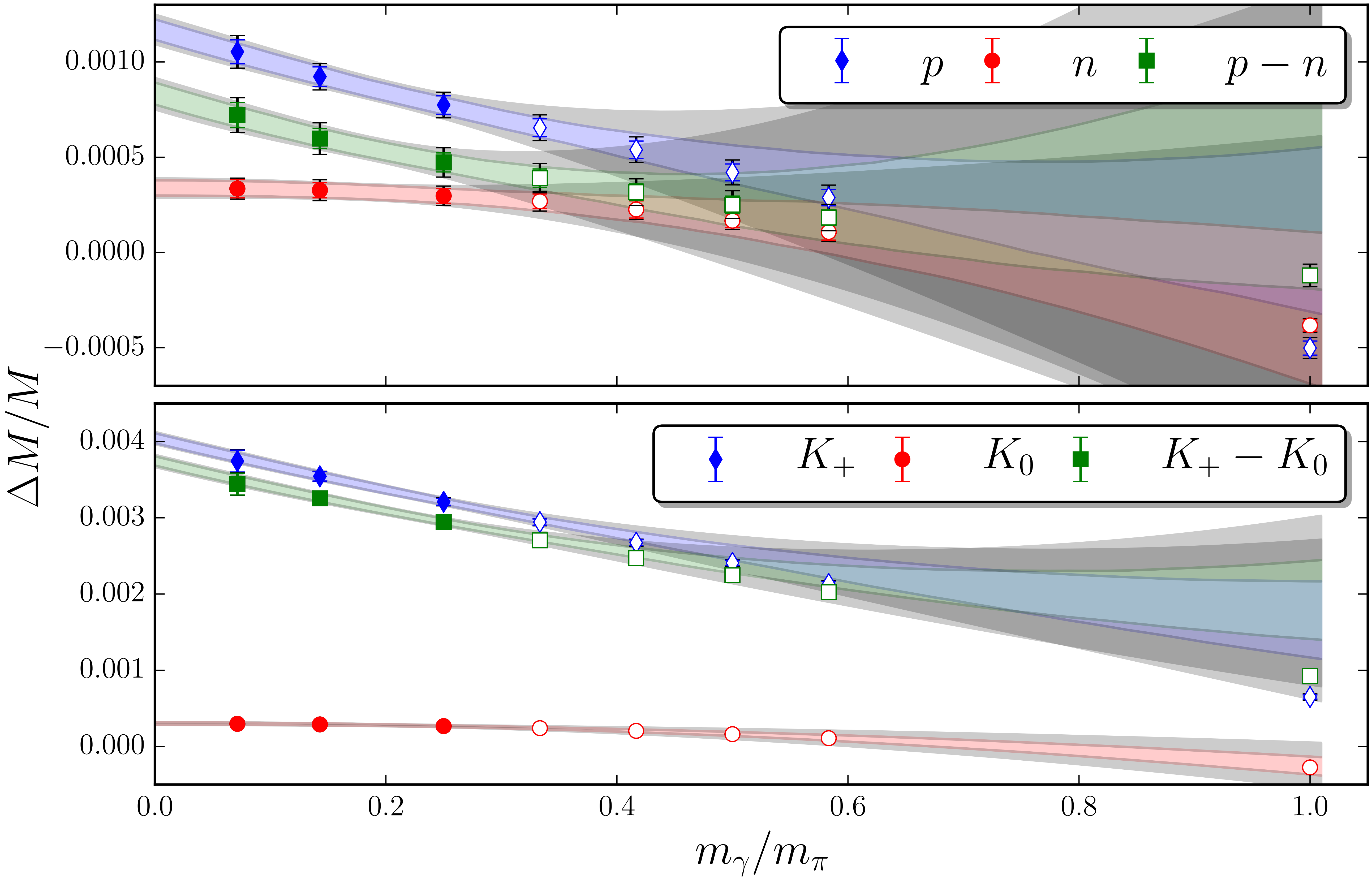
Massive Photons: Numerical Results

QED$_\mathrm{M}$



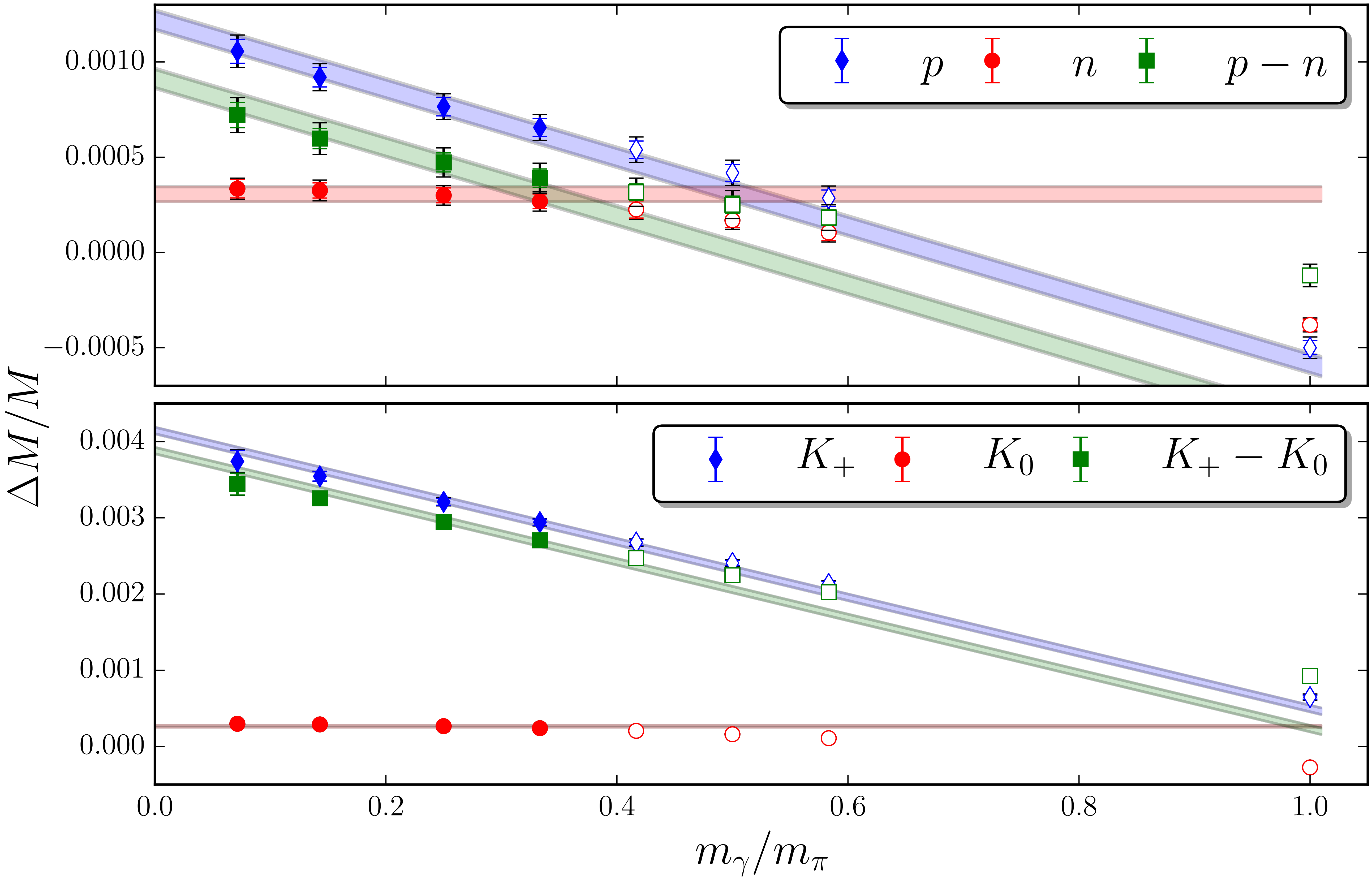
Massive Photons: Numerical Results

$\mathrm{QED_M}$



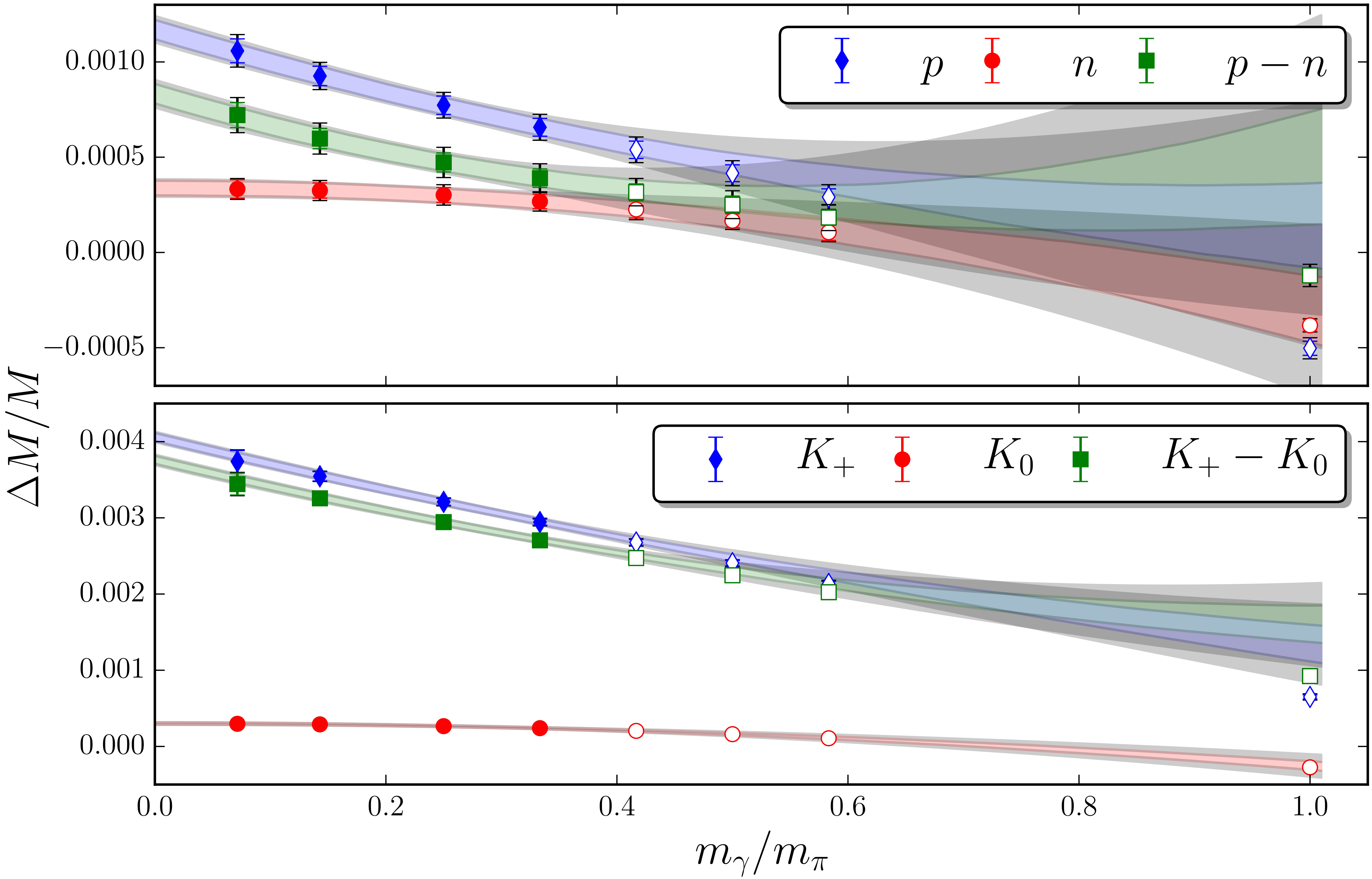
Massive Photons: Numerical Results

$\mathrm{QED_M}$



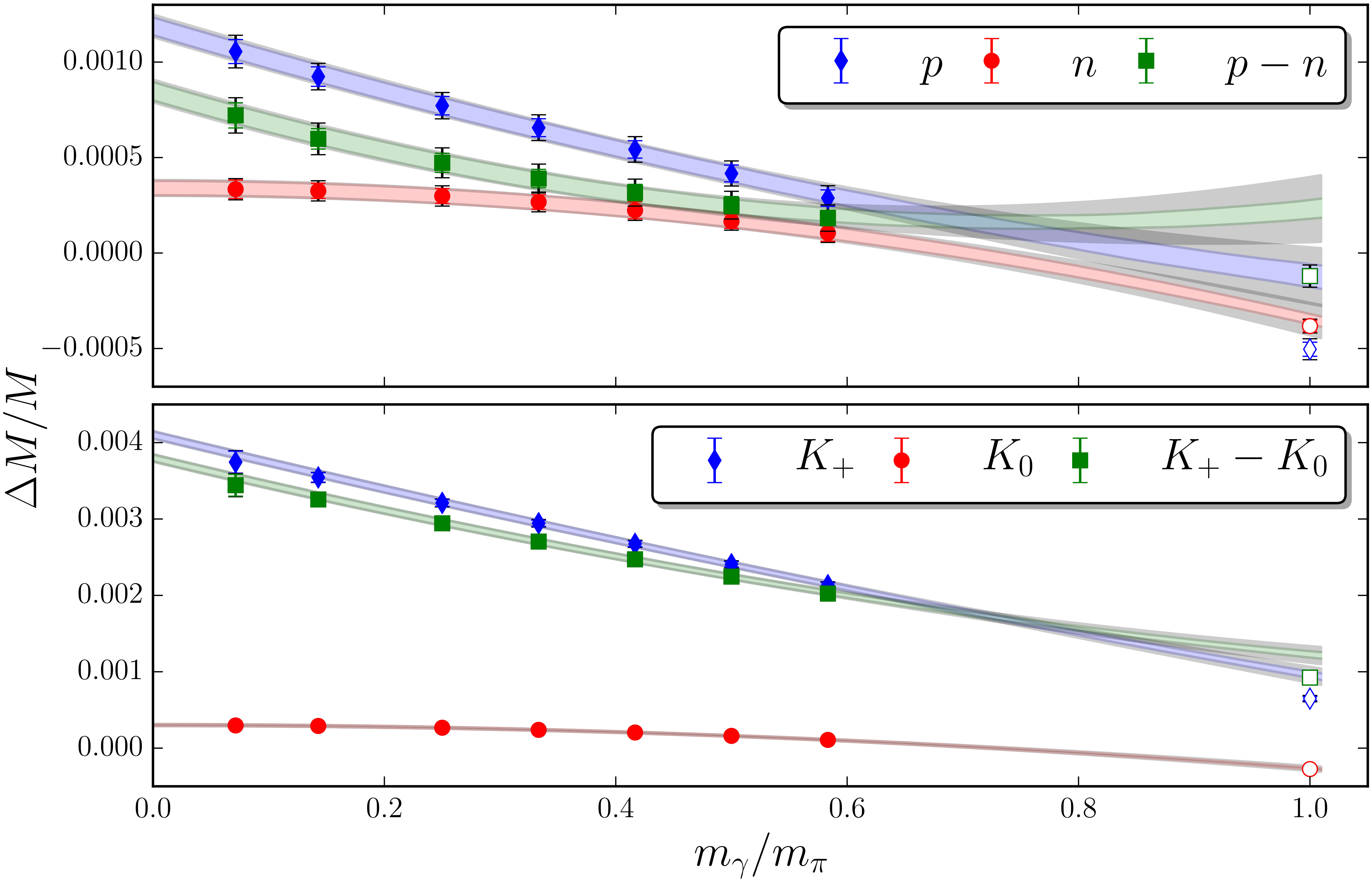
Massive Photons: Numerical Results

QED_M



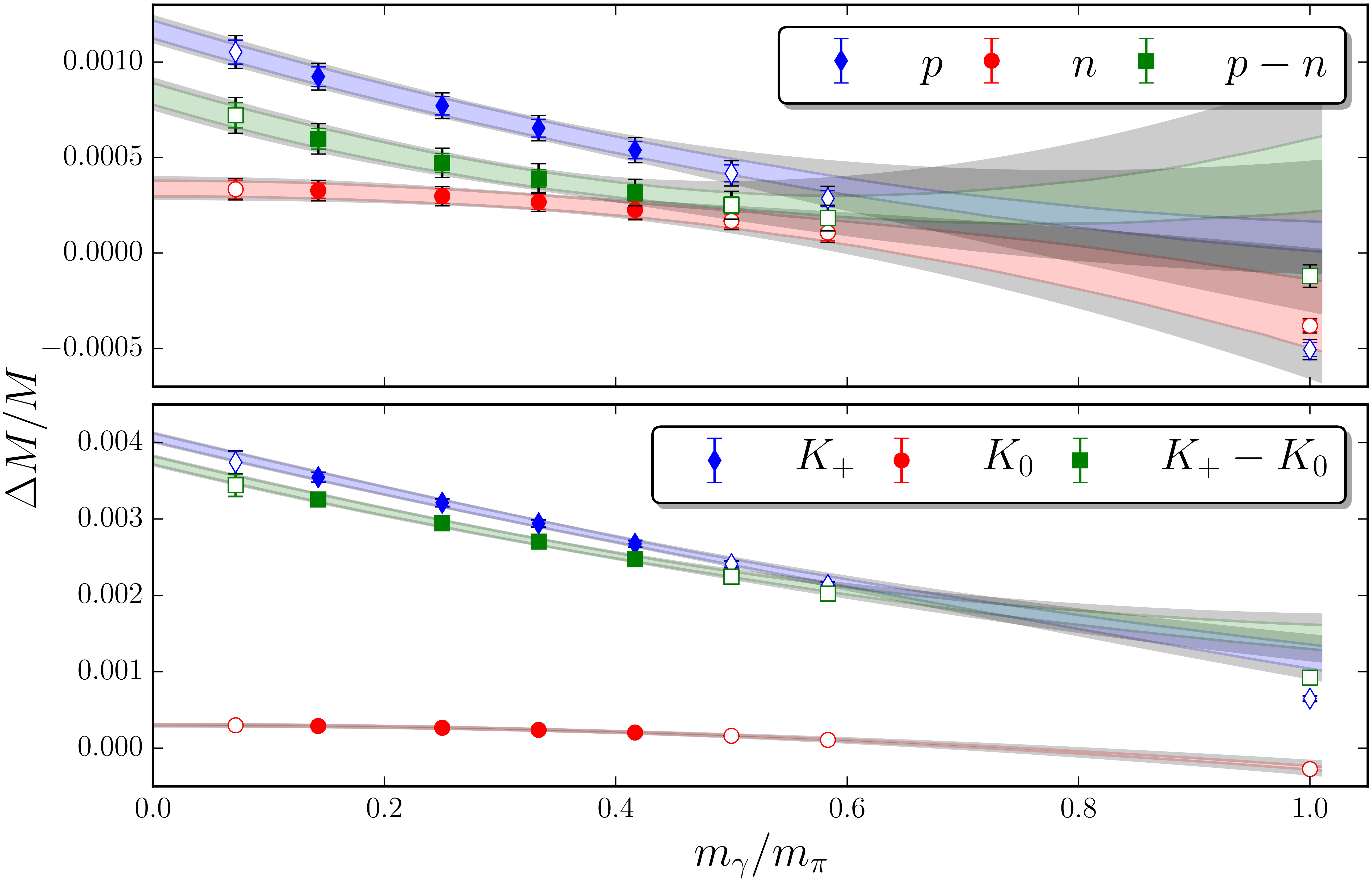
Massive Photons: Numerical Results

$\mathrm{QED_M}$



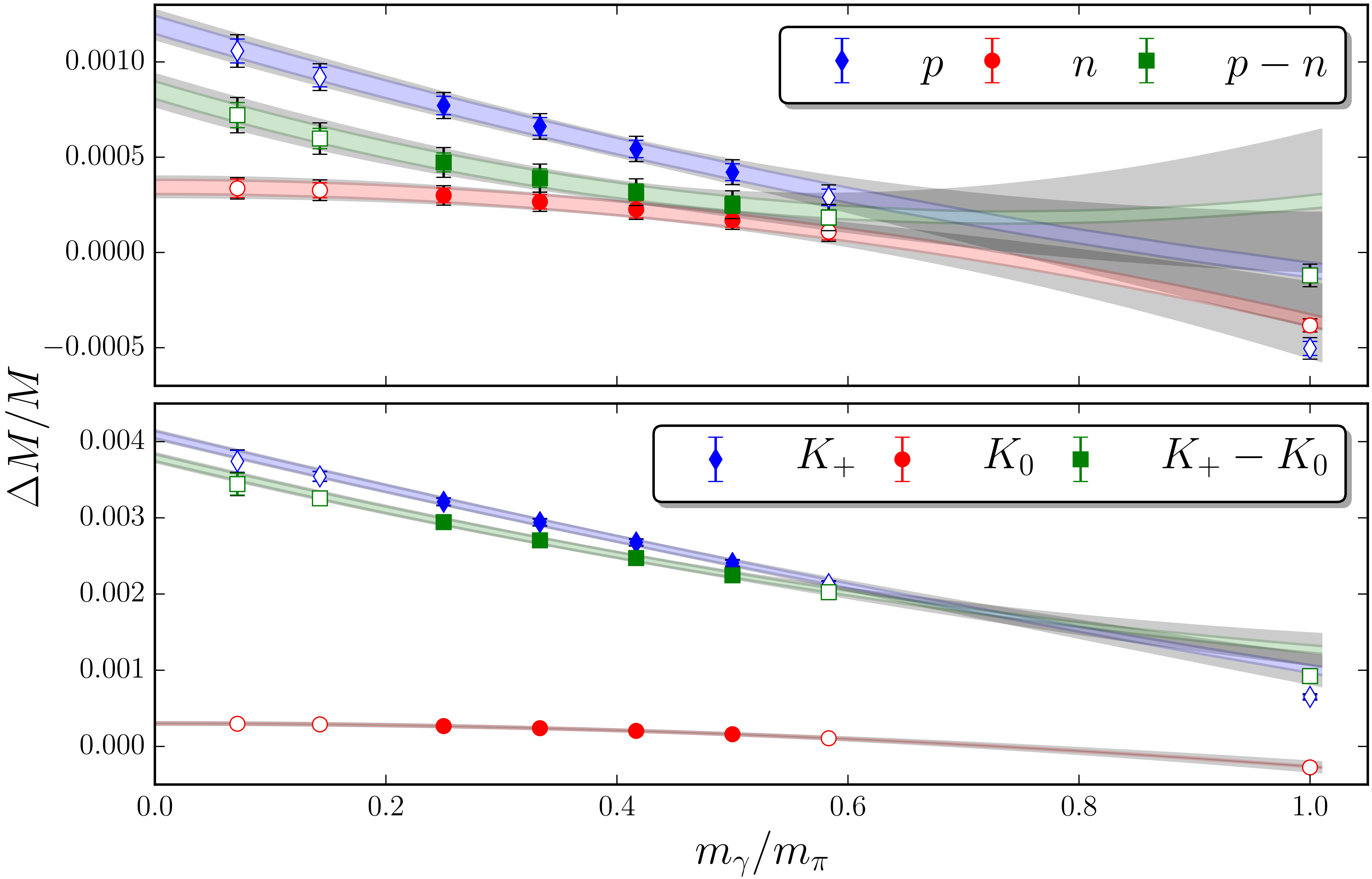
Massive Photons: Numerical Results

QED$_{\mathrm{M}}$



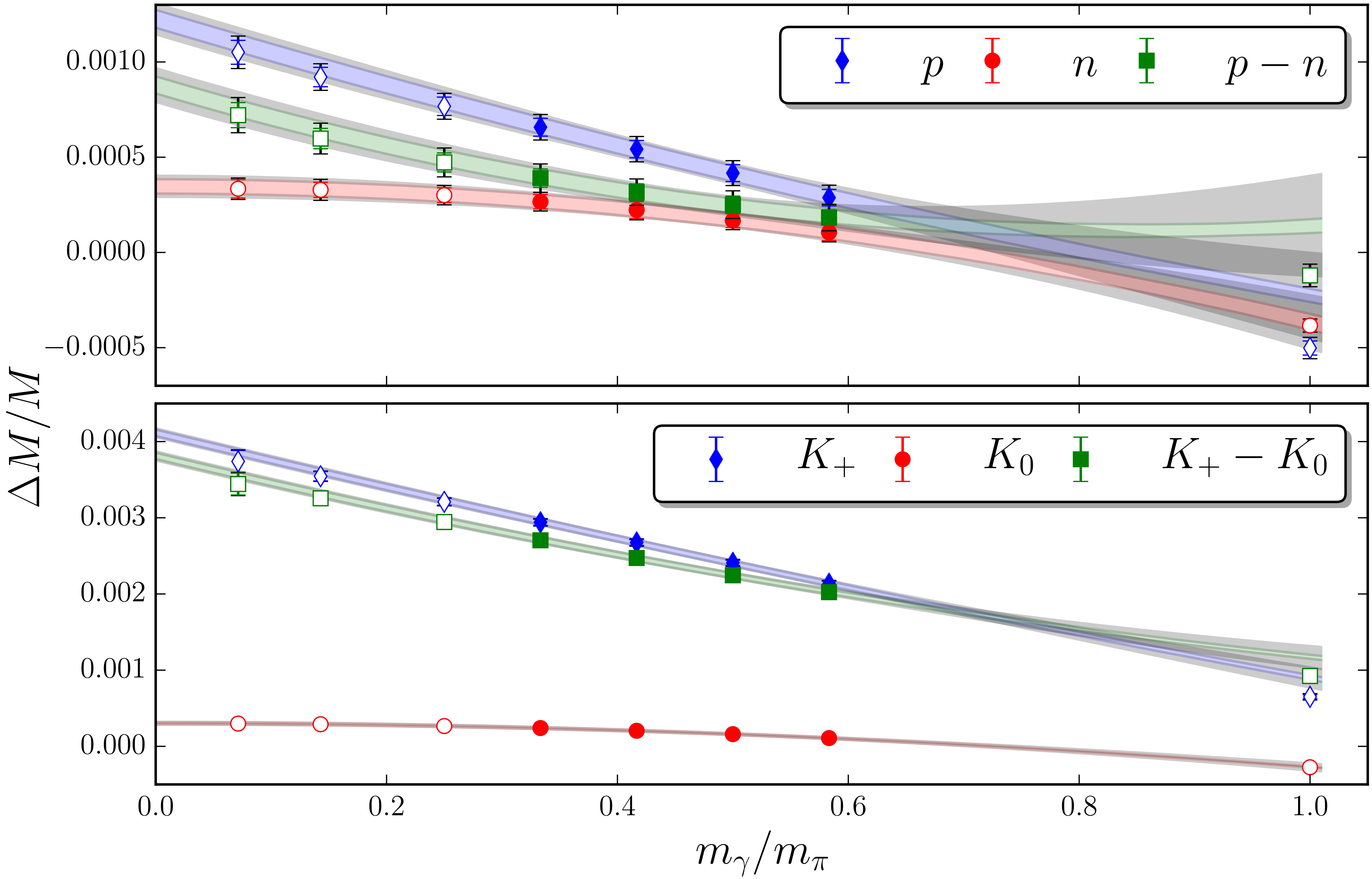
Massive Photons: Numerical Results

$\mathrm{QED_M}$



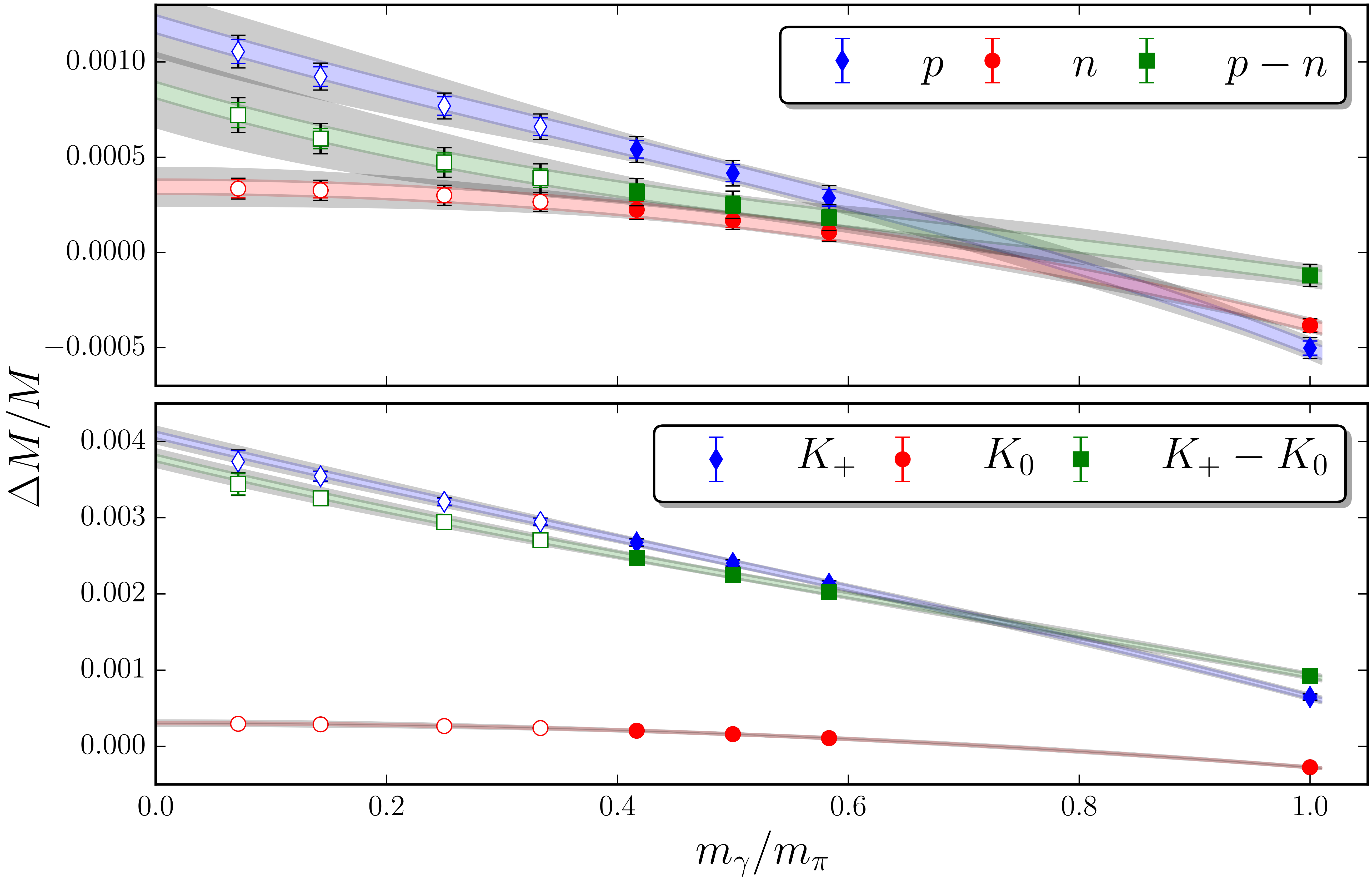
Massive Photons: Numerical Results

QED$_\mathrm{M}$



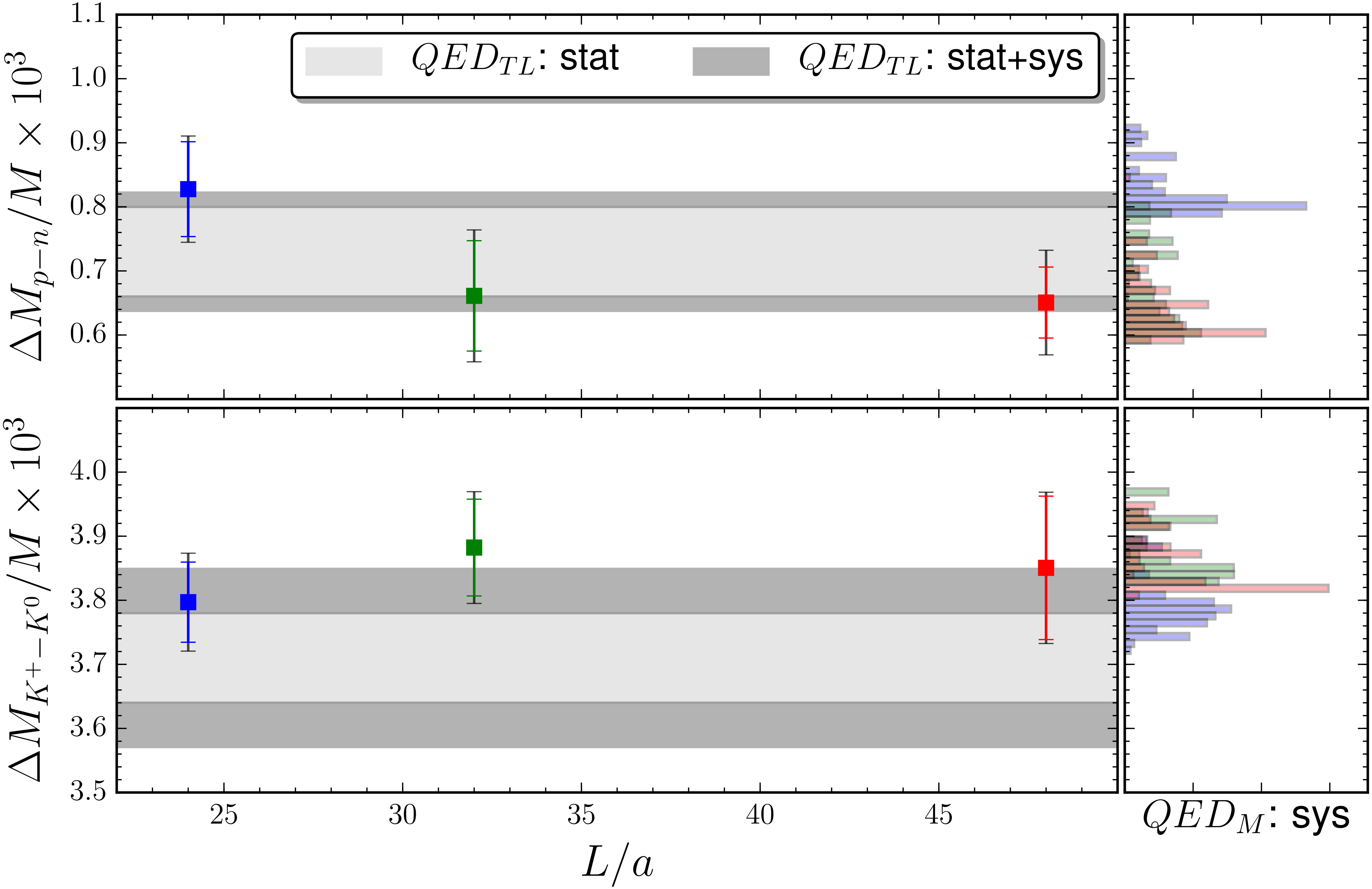
Massive Photons: Numerical Results

QED$_{\mathrm{M}}$



Massive Photons: Numerical Results

QED$_{\mathrm{M}}$



Massive Photons: Cost Comparison

- For this study - the cost arises mostly from the quark-propagators.
  - inversion cost scales linearly in volume
  - L/a=32 calculation cost $515/956 \times (32/24)^3 \simeq 1.3$ x L/a=24
  - L/a=48 calculation cost $342/956 \times (48/24)^3 \times 64/48 \simeq 3.8$ x L/a=24
- The L/a=24 calculations with $m_\gamma/\bar{m}_\pi \in [1/4, 1/2]$ are consistent with all results with comparable precision. This calculation used 4 values of the photon mass, and so the inversion cost is less than using all 3 volumes with $\mathrm{QED_{TL}}$ and comparable to the cost of just doing the L/a=48 $\mathrm{QED_{TL}}$ calculation
- The HMC ensemble generation time scales worse than linearly in the volume, so when this cost is included, our method provides a numerically less expensive calculation than the standard method for comparable results

Massive Photons: Conclusions

- We have introduced a robust alternate IR regulator for including QED effects with LQCD calculations formulated with a local QFT: massive QED, $\mathrm{QED_M}$
  - $\mathrm{QED_M}$ is numerically less expensive than the standard $\mathrm{QED_{\{L,TL\}}}$ schemes as the FV corrections can be controlled with a single volume and multiple values of the photon mass
  - We pushed the method down to $m_\gamma L \simeq 1$ and found we were able to control the FV systematics. Perhaps even smaller values can be utilized. This is important for keeping $m_\gamma < m_\pi$ and having multiple values of the photon mass to control the $m_\gamma \to 0$ extrapolation.
  - $\mathrm{QED_M}$ should offer a significant advantage for calculations involving multiple charged particles in the initial/final state as the photon mass provides a mass gap for radiating soft photons allowing a larger energy range in which standard Lüscher analysis of scattering can be utilized
  - For dynamical $\mathrm{QED_M}$, one may have to worry about additive photon mass corrections - however, $\mathrm{QED_M}$ can be achieved with a Higgs mechanism, so there may be a way to protect the photon mass
  - Or it may be sufficient to add the leading sea-quark effects perturbatively