Kaon Physics

Prospects for Lattice Computations of Rare Kaon Decay Amplitudes

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Outline of Talk

- Long-distance contributions to flavour changing processes

$$\int\!\!\int d^4x\, d^4y\, \langle f\,|\,T[Q_1(x)\,Q_2(y)]\,|\,i\rangle\,,$$

  illustrated through the applications to rare kaon decays

  1. $K \to \pi\ell^+\ell^-$ decays.
  2. $K^+ \to \pi^+\nu\bar{\nu}$ decays.

  (Other applications being studied by the RBC-UKQCD Collaboration* include the K_L-K_S mass difference and ϵ_K.)

- (Status of RBC-UKQCD Collaboration's calculations of $K \to \pi\pi$ decay amplitudes.) *

* RBC=Riken Research Center, Brookhaven National Laboratory, Columbia University; UKQCD = Edinburgh + Southampton.

Most of my understanding of “non-standard" kaon physics comes from my collaborators in the RBC-UKQCD collaboration.

Publications

1. "Prospects for a lattice computation of rare kaon decay amplitudes: 1, $K \to \pi\ell^+\ell^-$ decays"
   N.H.Christ, X.Feng, A.Portelli and C.T.S. Phys.Rev. D. **92** (2015) 094512 [arXiv:1507.03094]
2. "Prospects for a lattice computation of rare kaon decay amplitudes: 2, $K \to \pi\nu\bar{\nu}$ decays"
   N.H.Christ, X.Feng, A.Portelli and C.T.S. arXiv:1605:04442
3. "First exploratory calculation of the long distance contributions to the rare kaon decay $K \to \pi\ell^+\ell^-$"
   N.H.Christ, X.Feng, A.Jüttner, A.Lawson, A.Portelli and C.T.S. (in preparation)
   A preliminary version of the results was presented at Lattice 2015, arXiv:1602.01374.

Some comments from F.Mescia, C,Smith, S.Trine hep-ph/0606081

- Rare kaon decays which are dominated by short-distance FCNC processes, $K \to \pi\nu\bar{\nu}$ in particular, provide a potentially valuable window on new physics at high-energy scales.
- The decays $K_L \to \pi^0 e^+ e^-$ and $K_L \to \pi^0 \mu^+ \mu^-$ are also considered promising because the long-distance effects are reasonably under control using ChPT.
  - They are sensitive to different combinations of short-distance FCNC effects and hence in principle provide additional discrimination to the neutrino modes.
  - A challenge for the lattice community is therefore to calculate the long-distance effects reliably (and to determine the Low Energy Constants of ChPT if appropriate).
- We, the RBC-UKQCD collaboration, are attempting to meet this challenge, but would welcome the help of the wider kaon physics community to do this as effectively as possible.

There are three main contributions to the amplitude:

1. Short distance contributions: F.Mescia, C,Smith, S.Trine hep-ph/0606081

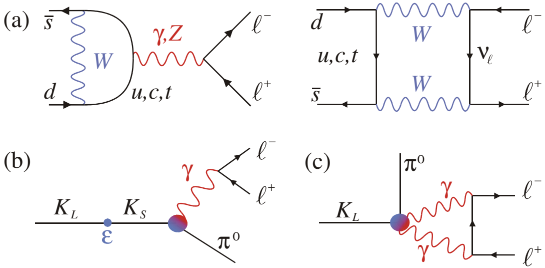
$$H_{\rm eff} = -\frac{G_F\alpha}{\sqrt{2}}\, V_{ts}^* V_{td} \{ y_{7V}(\bar{s}\gamma_\mu d)\,(\bar{\ell}\gamma^\mu \ell) + y_{7A}(\bar{s}\gamma_\mu d)\,(\bar{\ell}\gamma^\mu\gamma_5 \ell)\} + \text{h.c.}$$

   - Direct CP-violating contribution.
   - In BSM theories other effective interactions are possible.

2. Long-distance indirect CP-violating contribution

$$A_{ICPV}(K_L \to \pi^0 \ell^+ \ell^-) = \epsilon A(K_1 \to \pi^0 \ell^+ \ell^-) \simeq \epsilon A(K_S \to \pi^0 \ell^+ \ell^-).$$

3. The two-photon CP-conserving contribution $K_L \to \pi^0(\gamma^*\gamma^* \to \ell^+\ell^-)$.



- The current phenomenological status for the SM predictions is nicely summarised by: V.Cirigliano et al., arXiv1107.6001

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-)_{\mathrm{CPV}} = 10^{-12} \times \left\{ 15.7|a_S|^2 \pm 6.2|a_S| \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right) + 2.4 \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right)^2 \right\}$$

$$\mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-)_{\mathrm{CPV}} = 10^{-12} \times \left\{ 3.7|a_S|^2 \pm 1.6|a_S| \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right) + 1.0 \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right)^2 \right\}$$

  - $\lambda_t = V_{td}V_{ts}^*$ and $\mathrm{Im}\,\lambda_t \simeq 1.35 \times 10^{-4}$.
  - $|a_S|$, the amplitude for $K_S \to \pi^0 \ell^+ \ell^-$ at $q^2 = 0$ as defined below, is expected to be $O(1)$ but the sign of a_S is unknown. $|a_S| = 1.06^{+0.26}_{-0.21}$.
  - For $\ell = e$ the two-photon contribution is negligible.
  - Taking the positive sign (?) the prediction is

$$\begin{aligned} \mathrm{Br}(K_L \to \pi^0 e^+ e^-)_{\mathrm{CPV}} &= (3.1 \pm 0.9) \times 10^{-11} \\ \mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-)_{\mathrm{CPV}} &= (1.4 \pm 0.5) \times 10^{-11} \\ \mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-)_{\mathrm{CPC}} &= (5.2 \pm 1.6) \times 10^{-12}\,. \end{aligned}$$

  - The current experimental limits (KTeV) are:

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) < 2.8 \times 10^{-10} \quad \text{and} \quad \mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-) < 3.8 \times 10^{-10}\,.$$

G.Isidori, G.Martinelli and P.Turchetti, hep-lat/0506026

- We now turn to the CPC decays $K_S \to \pi^0\ell^+\ell^-$ and $K^+ \to \pi^+\ell^+\ell^-$ and consider

$$T_i^\mu = \int d^4x\, e^{-iq\cdot x}\, \langle \pi(p)\,|\, \mathrm{T}\{J^\mu_{\rm em}(x)\, Q_i(0)\,\}\,|\,K(k)\rangle\,,$$

  where Q_i is an operator from the $\Delta S = 1$ effective weak Hamiltonian.
- EM gauge invariance implies that

$$T_i^\mu = \frac{\omega_i(q^2)}{(4\pi)^2}\,\left\{q^2(p+k)^\mu - (m_K^2 - m_\pi^2)\,q^\mu\right\}\,.$$

- Within ChPT the low energy constants a_+ and a_S are defined by

$$a = \frac{1}{\sqrt{2}}\,V_{us}^*V_{ud}\left\{C_1\omega_1(0) + C_2\omega_2(0) + \frac{2N}{\sin^2\theta_W}f_+(0)C_{7V}\right\}$$

  where $Q_{1,2}$ are the two current-current GIM subtracted operators and the C_i are the Wilson coefficients. (C_{7V} is proportional to y_{7V} above).

G.D'Ambrosio, G.Ecker, G.Isidori and J.Portoles, hep-ph/9808289

- Phenomenological values: $a_+ = -0.578 \pm 0.016$ and $|a_S| = 1.06^{+0.26}_{-0.21}$.

What can we achieve in lattice simulations?

- The generic non-local matrix elements which we wish to evaluate is

$$X \equiv \int_{-\infty}^{\infty} dt_x\, d^3x\, \langle \pi(p)\,|\, \mathrm{T}\,[\,J(0)\,H(x)\,]\;|K(k)\rangle$$

$$= i\sum_n \frac{\langle \pi(p)\,|\,J(0)\,|n\rangle\,\langle n\,|H(0)\,|\,K(k)\rangle}{E_K - E_n + i\epsilon} - i\sum_{n_s} \frac{\langle \pi(p)\,|\,H(0)\,|n_s\rangle\,\langle n_s\,|J(0)\,|\,K(k)\rangle}{E_{n_s} - E_\pi + i\epsilon}\,,$$

  - $\{|n\rangle\}$ and $\{|n_s\rangle\}$ represent complete sets of non-strange and strange states.

- In Euclidean space we calculate correlation functions of the form

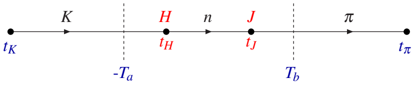
$$C \equiv \int_{-T_a}^{T_b} dt_x \int d^3x\, \langle \phi_\pi(\vec{p}, t_\pi)\, \mathrm{T}\,[\,J(0)\,H(x)\,]\;\phi_K^\dagger(\vec{p}_K, t_K)\,\rangle \equiv \sqrt{Z_K}\,\frac{e^{-E_K|t_K|}}{2m_K}\,X_E\,\sqrt{Z_\pi}\,\frac{e^{-E_\pi t_\pi}}{2E_\pi}\,,$$

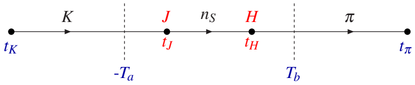
where $X_E = X_{E\_} + X_{E_+}$ and

$$X_{E\_} = -\sum_n \frac{\langle \pi(p)\,|\,J(0)\,|n\rangle\,\langle n\,|H(0)\,|\,K(k)\rangle}{E_K - E_n}\left(1 - e^{(E_K - E_n)T_a}\right) \quad \text{and}$$

$$X_{E_+} = \sum_{n_s} \frac{\langle \pi(p)\,|\,H(0)\,|n_s\rangle\,\langle n_s\,|J(0)\,|\,K(k)\rangle}{E_{n_s} - E_\pi}\left(1 - e^{-(E_{n_s} - E_\pi)T_b}\right)\,.$$

4-pt Euclidean Correlation Functions





- In Euclidean space we calculate correlation functions of the form

$$C \equiv \int_{-T_a}^{T_b} dt_x \int d^3x \, \langle \phi_\pi(\vec{p}, t_\pi) \, \mathrm{T}\,[\,J(0)\,H(x)\,]\ \phi_K^\dagger(\vec{p}_K, t_K)\,\rangle \equiv \sqrt{Z_K}\,\frac{e^{-E_K|t_K|}}{2m_K}\,X_E\,\sqrt{Z_\pi}\,\frac{e^{-E_\pi t_\pi}}{2E_\pi}\,,$$

where $X_E = X_{E_-} + X_{E_+}$ and

$$X_{E_-} = -\sum_n \frac{\langle \pi(p)\,|\,J(0)\,|n\rangle\,\langle n\,|H(0)\,|\,K\rangle}{E_K - E_n}\left(1 - e^{(E_K - E_n)T_a}\right) \quad \text{and}$$

$$X_{E_+} = \sum_{n_s} \frac{\langle \pi(p)\,|\,H(0)\,|n_s\rangle\,\langle n_s\,|J(0)\,|\,K\rangle}{E_{n_s} - E_\pi}\left(1 - e^{-(E_{n_s} - E_\pi)T_b}\right)\,.$$

- In practice we may need to modify the above formulae to recognise the discrete nature of the lattice.
- For $E_K > E_n$ there are unphysical exponentially growing terms which need to be subtracted! This is a common feature in calculations of long-distance effects in Euclidean space. This requires the consideration of π, $\pi\pi$ and $\pi\pi\pi$ intermediate states.

Removal of single pion intermediate state

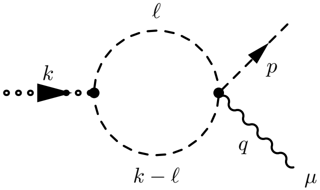
- For illustration, I consider the kaon to be at rest.
  - $X_{E_-} = -\sum_n \frac{\langle \pi(p) \,|\, J(0) \,|n\rangle \,\langle n \,|H(0) \,|\, K\rangle}{E_K - E_n} \left(1 - e^{(E_K - E_n)T_a}\right)$

- We use two methods to remove the contribution from the single pion state.

1. We determine the matrix elements $\langle \pi|H|K\rangle$ and $\langle \pi|J|\pi\rangle$ and the energies from two and three-point correlations functions and then perform the subtraction directly.
2. We add a term $c_S\,\bar{s}d$ to the effective Hamiltonian, with c_S chosen for each momentum so that

$$\langle \pi \,|H - c_S\,\bar{s}d|K\rangle = 0\,.$$

   - The theoretical demonstration that the addition of a term proportional to $\bar{s}d$ does not change the physical amplitude can be found in our paper arXiv:1507.03094.

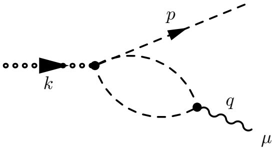


- In the continuum, space-time symmetries protect us from two-pion intermediate states:

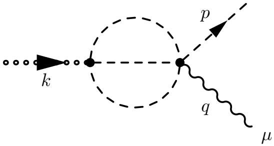
$$\langle\pi(p_1)|J_\mu|\pi(p_2)\pi(p_3)\rangle = \epsilon_{\mu\nu\rho\sigma}p_1^\nu p_2^\rho p_3^\sigma F(s,t,u)$$

- After integrating over the momenta of the two intermediate pions, the only independent vectors are k, p and ϵ_γ and so the indices of the Levi-Civita tensor cannot be saturated.
- This still leaves lattice artefacts two-pion contributions ($\propto a^2$) amplified by the growing exponential factors. While we expect these to be very small (as is the case for Δm_K), this will have to be confirmed numerically.
- Recently we have also determined the finite-volume corrections for the two-pion contribution to $\Delta m_K = m_{K_L} - m_{K_S}$. N.H.Christ, X.Feng, G.Martinelli & CTS, arXiv:1504.01170

The three pion contribution



(a)

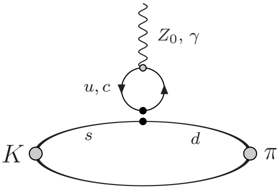


(b)

- The finite-volume effects which vanish as powers of the volume are absent from diagram (a) for $q^2 < 4m_\pi^2$.
- The three-pion on-shell intermediate state contribution is heavily phase-space suppressed and is expected to be negligible (but in principle is also calculable as with method 1 for the single pion contribution).
- The suppression of finite-volume effects which only vanish as powers of the volume due to 2 or 3 particle on-shell intermediate states follows in a similar way.
- (It is only recently that the finite-volume corrections for three particle states have become understood theoretically, but the theory has not been applied in numerical calculations.) M.T.Hansen and S.R.Sharpe, arXiv:1504.04248

$$T_i^\mu = \int d^4x\, e^{-iq\cdot x}\, \langle \pi(p)\,|\,\mathrm{T}\{J^\mu(x)\, Q_i(0)\,\}\,|\,K(k)\rangle\,,$$

- Each of the two local Q_i operators can be normalized in the standard way and for J we imagine taking the conserved vector current.
- We must treat additional divergences as $x \to 0$.



- Quadratic divergence is absent by gauge invariance $\Rightarrow$ Logarithmic divergence.
  - Checked explicitly for Wilson and Clover at one-loop order.

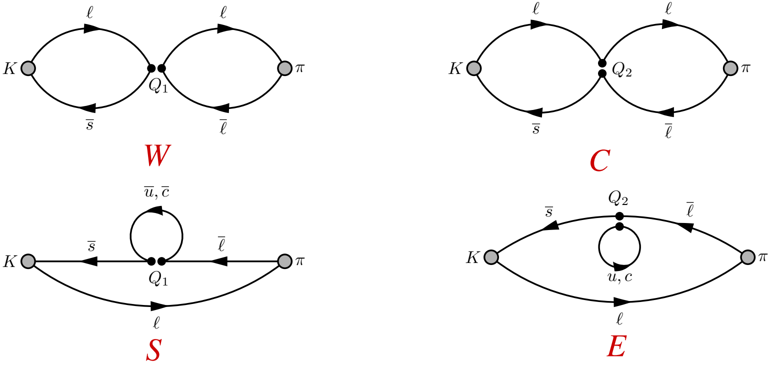
    G.Isidori, G.Martinelli and P.Turchetti, hep-lat/0506026
  - Absence of power divergences does not require GIM.
  - Logarithmic divergence cancelled by GIM.

Short Distance Effects - Postscript

- In the calculation described below we have followed the IMT approach, but the conserved vector current with DWF is a 5-D operator which adds considerably to the cost.
- This is not possible for $K \to \nu\bar{\nu}$ decays because the axial current is present so that the GIM mechanism does not result in the absence of logarithmic divergences. This is discussed in some detail below.

Many diagrams to evaluate!

- For example for K^+ decays we need to evaluate the diagrams obtained by inserting the current at all possible locations in the three point function (and adding the disconnected diagrams):



- W=Wing, C=Connected, S=Saucer, E=Eye.
- For K_S decays there is an additional topology with a gluonic intermediate state.

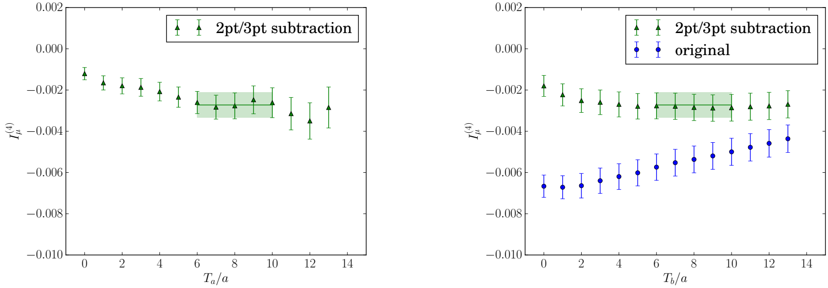
Exploratory numerical study

N.Christ, X.Feng, A.Jüttner, A.Lawson, A.Portelli and CTS (in preparation)

- The numerical study is performed on the $24^3 \times 64$ DWF+Iwasaki RBC-UKQCD ensembles with $am_l = 0.01$ ($m_\pi \simeq 420$ MeV), $am_s = 0.04$, $a^{-1} \simeq 1.73$ fm.
- 128 configurations were used with $\vec{k} = \vec{0}$ and $\vec{p} =$(1,0,0), (1,1,0) and (1,1,1) in units of $2\pi/L$.
- With this kinematics we are in the unphysical region, $q^2 < 0$.
- The charm quark is also lighter than physical $m_c^{\overline{\text{MS}}}(2\,\text{GeV}) \simeq 520$ MeV.
- The calculation is performed using the conserved vector current (5-dimensional), J_{em}.
- Disconnected diagrams not included.
- All results are preliminary.

Method 1 for $\vec{p} = (1,0,0)$

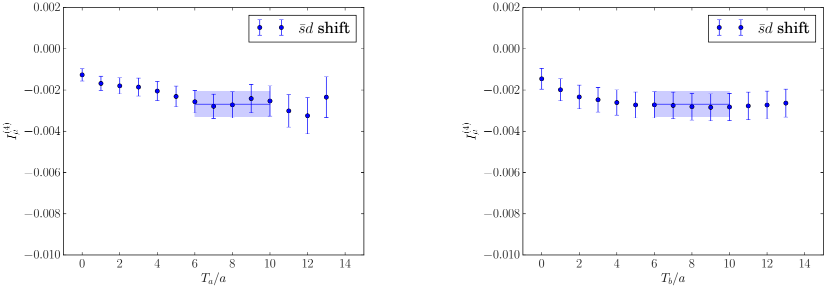
Preliminary



$\int_{t_J-T_A}^{t_J+8} \tilde{\Gamma}_0^{(4)}\, dt_H$ $\int_{t_J-6}^{t_J+T_B} \tilde{\Gamma}_0^{(4)}\, dt_H$

$$A_0(q^2) = -0.0027(6).$$

Preliminary



$$\int_{t_J - T_A}^{T_J+8} \left(\tilde{\Gamma}_0^{(4)} - c_s \tilde{\Gamma}_0^{\bar{s}d(4)}\right) dt_H \qquad \int_{T_J-6}^{t_J+T_B} \left(\tilde{\Gamma}_0^{(4)} - c_s \tilde{\Gamma}_0^{\bar{s}d(4)}\right) dt_H$$

$$A_0(q^2) = -0.0027(6).$$

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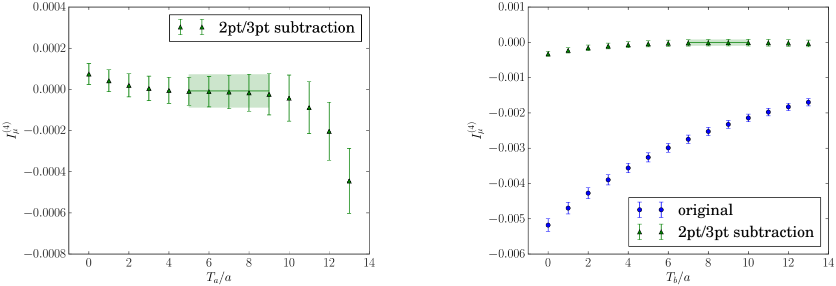
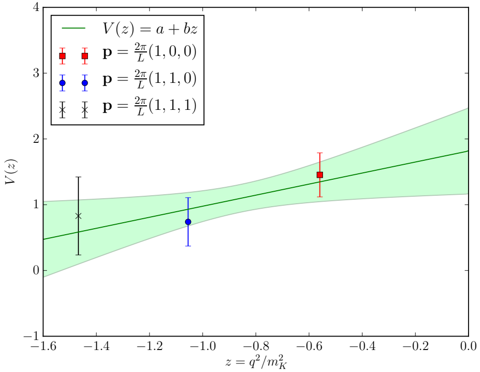
Numerical check that the matrix element with H replaced by $\bar{s}d$ is consistent with zero.

Preliminary



$$A_0^{\bar{s}d}(q^2) = -0.00007(8).$$

Working Plot



- $V(z)$ here is simply $\omega_+(q^2/m_K^2)$.

2. $K \to \pi\nu\bar{\nu}$ Decays

N.H.Christ, X.Feng, A.Portelli and CTS, arXiv:1605:04442

- NA62 and KOTO are beginning their experimental programme to study these decays. I repeat that these FCNC processes provide ideal probes for the observation of new physics effects.
- The dominant contributions from the top quark $\Rightarrow$ they are also very sensitive to V_{ts} and V_{td}.
- Experimental results and bounds:

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\text{exp}} = 1.73^{+1.15}_{-1.05} \times 10^{-10}$$

A.Artamonov et al. (E949), arXiv:0808.2459

$$\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu}) \le 2.6 \times 10^{-8} \text{ at 90\% confidence level},$$

J.Ahn et al. (E291a), arXiv:0911.4789

- Sample recent theoretical predictions:

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\text{SM}} = (9.11 \pm 0.72) \times 10^{-11}$$
$$\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})_{\text{SM}} = (3.00 \pm 0.30) \times 10^{-11},$$

A.Buras, D.Buttazzo, J.Girrbach-Noe, R.Knejgens, arXiv:1503.02693

- To what extent can lattice calculations reduce the theoretical uncertainty?

Short and Long-Distance Contributions

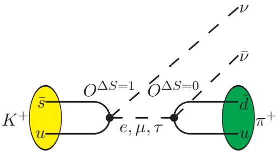
- To what extent can lattice calculations reduce the theoretical uncertainty?
- $K \to \pi\nu\bar{\nu}$ decays are SD dominated and the hadronic effects can be determined from CC semileptonic decays such as $K^+ \to \pi^0 e^+ \nu$.
  - Lattice calculations of the $K_{\ell 3}$ form factors are well advanced,

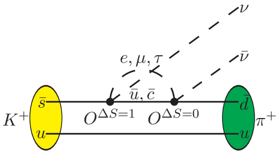
    P.A.Boyle et al. (RBC-UKQCD), arXiv:1504.01692
- LD contributions, i.e. contributions from distances greater than $1/m_c$ are negligible for K_L decays and are expected to be $O(5\%)$ for K^+ decays.
  - K_L decays are therefore one of the cleanest places to search for the effects of new physics.
  - The aim of our study is to compute the LD effects in K^+ decays.
    These provide a significant, if probably still subdominant, contribution to the theoretical uncertainty (which is dominated by the uncertainties in CKM matrix elements).
  - A phenomenological estimate of the long distance effects, estimated these to enhance the branching fraction by 6% with an uncertainty of 3%.

    G.Isidori, F.Mescia and C.Smith, hep-ph/0503107
- Lattice QCD can provide a first-principles determination of the LD contribution with controlled errors.
  - Given the NA62 experiment, it is timely to perform a lattice QCD calculation of these effects.

WW-Diagrams

- For this doubly weak decay there are a number of novel diagrams to evaluate:

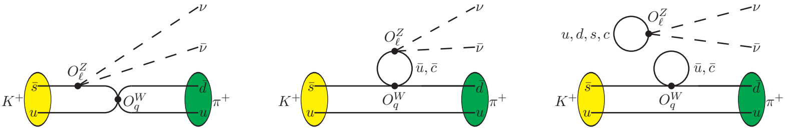




WW-diagrams

$$\mathcal{H}_{\rm eff}^{\rm LO} = -i\frac{G_F}{\sqrt{2}}\sum_{q,\ell}\left(V_{qs}^*O_{q\ell}^{\Delta S=1} + V_{qd}O_{q\ell}^{\Delta S=0}\right) - i\frac{G_F}{\sqrt{2}}\sum_q \lambda_q O_q^W - i\frac{G_F}{\sqrt{2}}\sum_\ell O_\ell^Z\,,$$

$$\begin{aligned} O_{q\ell}^{\Delta S=1} &= C_{\Delta S=1}^{\overline{\rm MS}}(\mu)\,\left[(\bar{s}q)_{V-A}\,(\bar{\nu}_\ell\ell)_{V-A}\right]^{\overline{\rm MS}}(\mu),\\ O_{q\ell}^{\Delta S=0} &= C_{\Delta S=0}^{\overline{\rm MS}}(\mu)\,\left[(\bar{\ell}\nu_\ell)_{V-A}\,(\bar{q}d)_{V-A}\right]^{\overline{\rm MS}}(\mu). \end{aligned}$$



Z-exchange diagrams

$$\mathcal{H}_{\text{eff}}^{\text{LO}} = -i\frac{G_F}{\sqrt{2}}\sum_{q,\ell}\left(V_{qs}^* O_{q\ell}^{\Delta S=1} + V_{qd}O_{q\ell}^{\Delta S=0}\right) - i\frac{G_F}{\sqrt{2}}\sum_q \lambda_q O_q^W - i\frac{G_F}{\sqrt{2}}\sum_\ell O_\ell^Z\,,$$

$$\begin{aligned} O_q^W &= C_1^{\overline{\text{MS}}}(\mu)\,Q_{1,q}^{\overline{\text{MS}}}(\mu) + C_2^{\overline{\text{MS}}}(\mu)\,Q_{2,q}^{\overline{\text{MS}}}(\mu), \\ O_\ell^Z &= C_Z^{\overline{\text{MS}}}(\mu)\,\left[J_\mu^Z\,\bar\nu_\ell\gamma^\mu(1-\gamma_5)\nu_\ell\right]^{\overline{\text{MS}}}(\mu) \end{aligned}$$

- The issues encountered in $K^+ \to \pi^+\ell^+\ell^-$ decays (additional ultra-violet divergences, subtraction or suppression of growing unphysical exponential terms and FV effects which fall as powers of the volume) must also be dealt with here.
- Theoretical paper ~~almost~~ complete. N.H.Christ, X.Feng, A.Portelli, CTS, arXiv:1605.04442
- An exploratory study of $K^+ \to \pi^+\nu\bar{\nu}$ decays is also underway and the parameters and early results were presented at Lattice 2015 by Xu Feng.
  X.Feng, https://indico2.riken.jp/indico/confSpeakerIndex.py?confId=1805
- Here I will focus on the treatment of UV divergences.
  - For $K \to \pi\nu\bar{\nu}$ decays the contributions from the axial current and the breaking of chiral symmetry by mass terms $\Rightarrow$ the logarithmic divergences, which are proportional to m_q^2, are not cancelled by the GIM mechanism.

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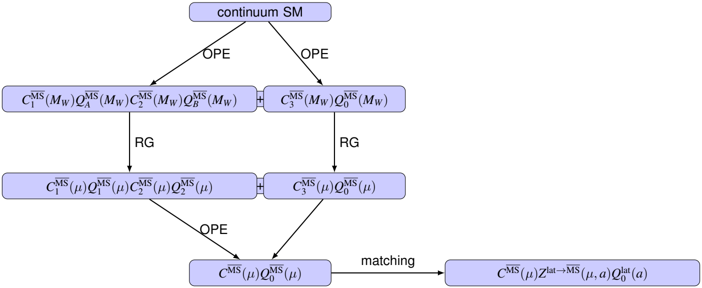

- **Step 1** At $\mu = M_W$, the W and Z bosons are integrated out and the second-order weak interaction is written as a combination of a bilocal operator $\int d^4x\, T[Q_A(x)Q_B(0)]^{\overline{\rm MS}}$ and a local operator $Q_0^{\overline{\rm MS}}$.

  The presence of the local operator serves two purposes:

  1. to represent phenomena which appear local below the scale of M_W such as those containing the top quark;
  2. to act as a counter-term removing the UV divergence when $x \simeq 0$.

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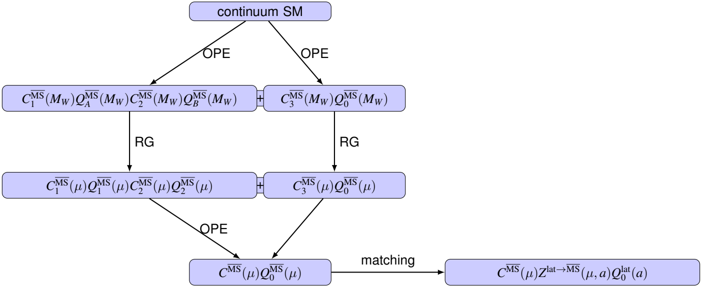
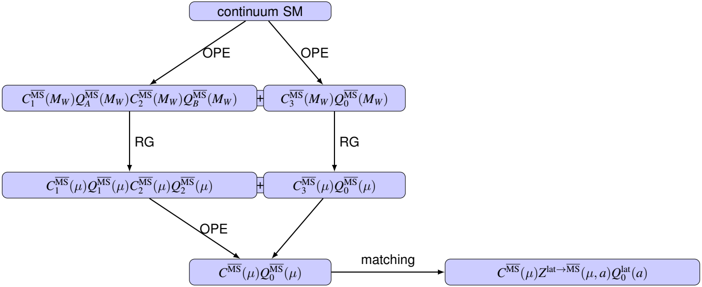


- **Step 2** The renormalisation group equations are used to evolve the Wilson coefficients to a lower scale μ.
  - The RQE are an extension of those governing local operators;

    G.Buchalla, A.J.Buras and M.E.Lautenbacher, hep-ph/9512380
  - The evolution includes a mixing of the singular part of the bilocal operator $\int d^4x\, T[Q_A(x)Q_B(0)]^{\overline{\rm MS}}$ into the local operator $Q_0^{\overline{\rm MS}}$.

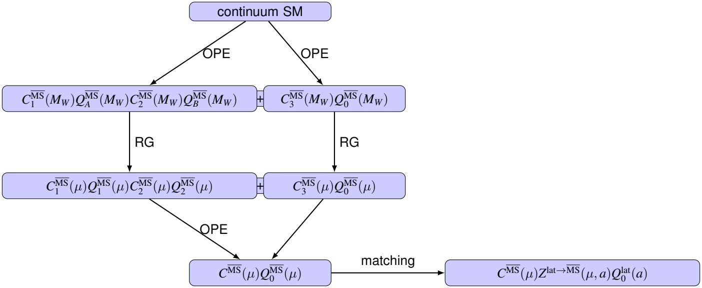
Schematic of a Perturbative Calculation



- **Step 3** At $\mu = O(m_c)$ we perform a second OPE integrating out the charm quark.

$$\int d^4x\ \langle T[Q_A(x)Q_B(0)]^{\overline{\mathrm{MS}}}(\mu)\rangle = r_{AB}^{\overline{\mathrm{MS}}}(\mu)\langle Q_0^{\overline{\mathrm{MS}}}(x=0,\mu)\rangle\,.$$

Schematic of a Perturbative Calculation



- **Step 4** At this stage we need the $K^+ \to \pi^+ \nu\bar\nu$ matrix element of the local operator $Q_0^{\overline{\rm MS}}(\mu)$ which can be obtained in the standard way from a lattice calculation + NPR into RI-SMOM + perturbative matching into $\overline{\rm MS}$.

- We propose an alternative approach in which the charm quark is not integrated out. This allows us to avoid:
  1. perturbation theory at scales of $O(m_c)$;
  2. neglecting higher order terms in the OPE which typically are suppressed only by μ^2/m_c^2.
- Instead we propose to calculate the matrix elements of the bilocal operator $\int d^4x\, T[Q_A(x)Q_B(0)]^{\overline{\rm MS}}(\mu)$ and the local operator $Q_0^{\overline{\rm MS}}(\mu)$ and to combine them to obtain the physical amplitude.
- We introduce the shorthand notation
$$\{Q_A^S Q_B^S\}^{S'}(y) \equiv \int d^4x\, T\{Q_A^S(x)Q_B^S(y)\}^{S'}\,.$$
  - S indicates the scheme used to define the local operators.
  - S' labels the method used to treat the singularity when $x=y$.
  - WLOG we will take $y=0$.
- We will need to evaluate Green functions of bilocal operators:
$$\langle\{Q_A^{\rm RI}(\mu_0)Q_B^{\rm RI}(\mu_0)\}_a^{\rm lat}\rangle = \int d^4x\,\langle T[Q_A^{\rm RI}(x,\mu_0)Q_B^{\rm RI}(0,\mu_0)]_a^{\rm lat}\rangle\,.$$
  - Remaining a-dependence is due to the short-distance divergence as $x\to 0$.

Alternative Approach (cont.)

- The renormalised bilocal operator is defined by writing

$$\{Q_A^{\mathrm{RI}}(\mu_0)Q_B^{\mathrm{RI}}(\mu_0)\}_{\mu_0}^{\mathrm{RI}} = \{Q_A^{\mathrm{RI}}(\mu_0)Q_B^{\mathrm{RI}}(\mu_0)\}_a^{\mathrm{lat}} - X_{AB}(\mu_0,a)Q_0^{\mathrm{RI}}(\mu_0)\,,$$

where the subtraction constant X is determined by imposing a condition such as

$$\langle\{Q_A^{\mathrm{RI}}(\mu_0)Q_B^{\mathrm{RI}}(\mu_0)\}_{\mu_0}^{\mathrm{RI}}\rangle_{p_i^2=\mu_0^2} = \langle\{Q_A^{\mathrm{RI}}(\mu_0)Q_B^{\mathrm{RI}}(\mu_0)\}_a^{\mathrm{lat}}\rangle_{p_i^2=\mu_0^2} - X_{AB}(\mu_0,a)\langle Q_0^{\mathrm{RI}}(\mu_0)\rangle_{p_i^2=\mu_0^2} = 0\,.$$

- Finally we write the $\overline{\mathrm{MS}}$ bilocal operator as

$$\{Q_A^{\overline{\mathrm{MS}}}Q_B^{\overline{\mathrm{MS}}}\}_\mu^{\overline{\mathrm{MS}}} = Z_{Q_A}^{\mathrm{RI}\to\overline{\mathrm{MS}}}(\mu,\mu_0)Z_{Q_B}^{\mathrm{RI}\to\overline{\mathrm{MS}}}(\mu,\mu_0)\{Q_A^{\mathrm{RI}}Q_B^{\mathrm{RI}}\}_{\mu_0}^{\mathrm{RI}} + Y_{AB}(\mu,\mu_0)Q_0^{\mathrm{RI}}(\mu_0)\,,$$

where the coefficient Y_{AB} is determined by

$$\langle\{Q_A^{\overline{\mathrm{MS}}}Q_B^{\overline{\mathrm{MS}}}\}_\mu^{\overline{\mathrm{MS}}}\rangle_{p_i^2=\mu_0^2} = \frac{Z_q^{\mathrm{RI}}(\mu_0)}{Z_q^{\overline{\mathrm{MS}}}(\mu)}Y_{AB}(\mu,\mu_0)\langle Q_0\rangle_{p_i^2=\mu_0^2}^{(0)}\,.$$

Summary and Conclusions on Prospects for Rare Kaon Decays

- For $K^+ \to \pi^+ \ell^+ \ell^-$ or $K_S \to \pi^0 \ell^+ \ell^-$ decays we now have a “complete" theoretical framework with which to perform lattice computations of the amplitudes.

  N.H.Christ, X.Feng, A.Portelli and C.T.Sachrajda, arXiv:1507.03094

  - Exploratory numerical simulations are underway and the preliminary results are very encouraging.
  - To use this framework in a simulation with physical quark masses would require a major project.
  - This would undoubtedly happen if there was a strong prospect of the corresponding experimental programme and will probably happen as part of the $K^+ \to \pi^+ \nu \bar{\nu}$ project.
- For the evaluation of the LD contributions to $K^+ \to \pi^+ \nu \bar{\nu}$ decays we are very close to being at the same stage, with a theoretical paper to be released in the next few weeks.
  - The exploratory numerical results are surprisingly (to me) encouraging.

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