Heavy Flavor Physics: BSM phenomenology

Aida X. El-Khadra
(University of Illinois)

Symposium on Effective Field Theories and Lattice Gauge Theory

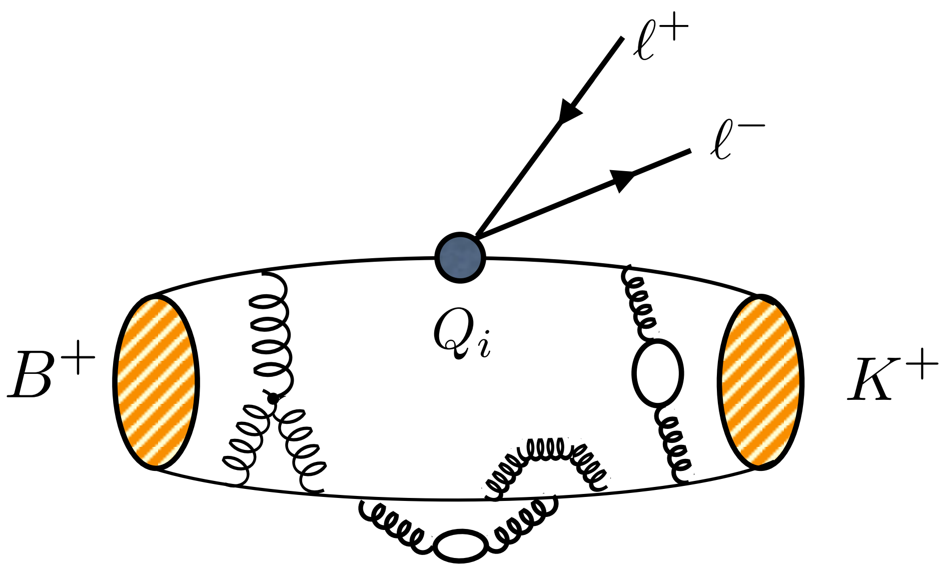
May 18-21, 2016 | TUM Institute for Advanced Study

Outline

- Motivation and introduction
  - for BSM focus (mostly) on loop processes
- LQCD results for
  - semileptonic B meson form factors
  - neutral B meson mixing matrix elements
  - summary of recent progress
- Phenomenology
  - SM pre/post-dictions
  - CKM unitarity & BSM implications
  - Lepton Flavor Universality
- Summary

Motivation

example: $B^+ \to K^+ \ell^+ \ell^-$



Experiment vs. SM theory:

(experiment) = (known) x (**CKM factor**) x (had. matrix element)

$$\frac{d\Gamma(B \to \pi \ell \nu)}{dq^2}, \frac{d\Gamma(B \to K \ell^+ \ell^-)}{dq^2}, \ldots$$

$$\frac{d\Gamma(B \to D \ell \nu)}{d\omega}, \frac{d\Gamma(B \to D \tau \nu)}{d\omega}, \ldots$$

$$\Delta m_{d(s)}$$

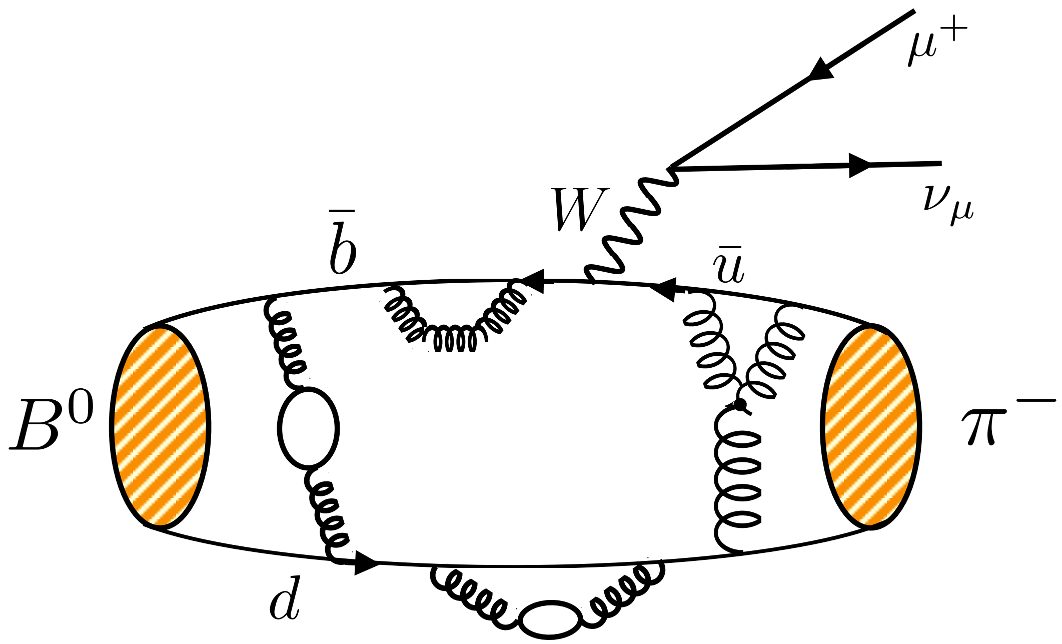
$$\vdots$$

Lattice QCD

parameterize the ME in terms of form factors, decay constants, bag parameters, ...

Semileptonic B-meson decay to light hadrons

Example: $B \to \pi \ell \nu$

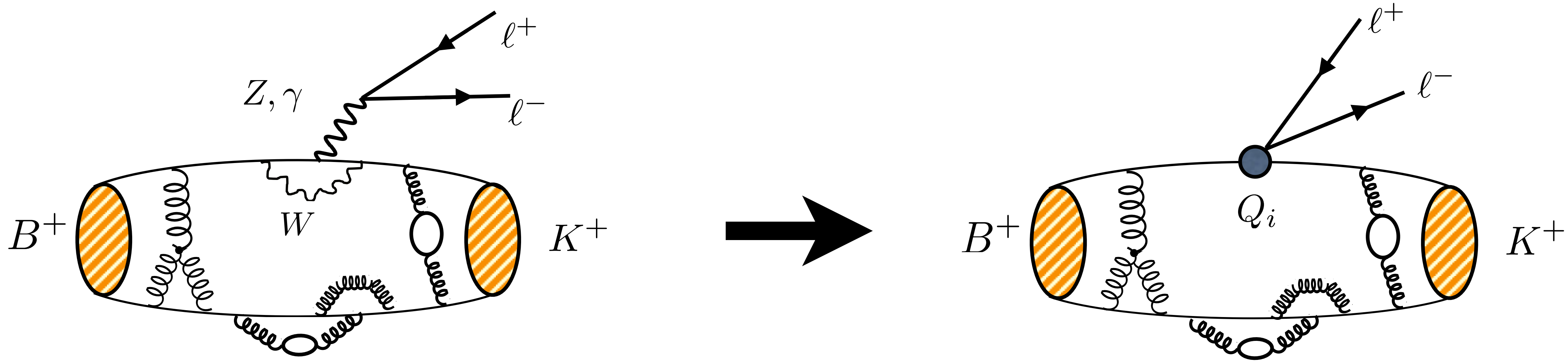


$$\frac{d\Gamma(B\to\pi\ell\nu)}{dq^2} = (\text{known}) \times |V_{ub}|^2 \times |f_+(q^2)|^2$$

- ★ shape for semileptonic B decays:

  use z-expansion for model-independent parameterization of q^2 dependence
- ★ calculate the complete set of form factors, $f_+(q^2), f_0(q^2)$ and $f_T(q^2)$ with LQCD.

Rare semileptonic B decay



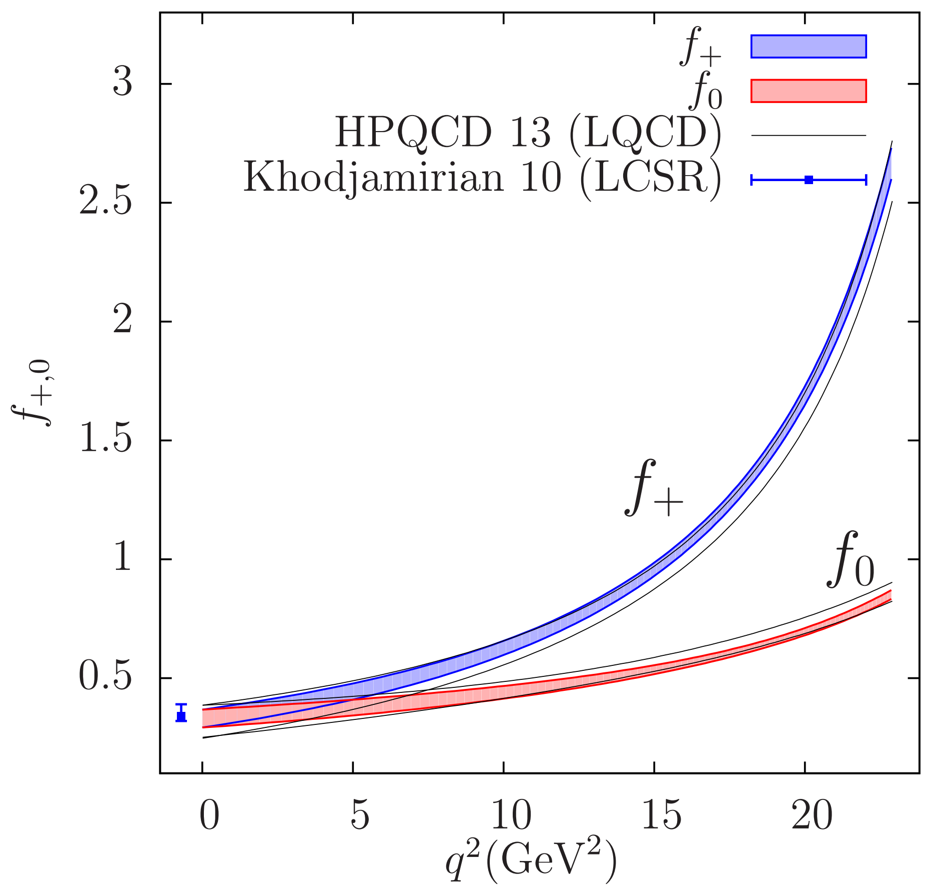
$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tq}^* V_{tb} \sum_i C_i(\mu) Q_i + \ldots$$

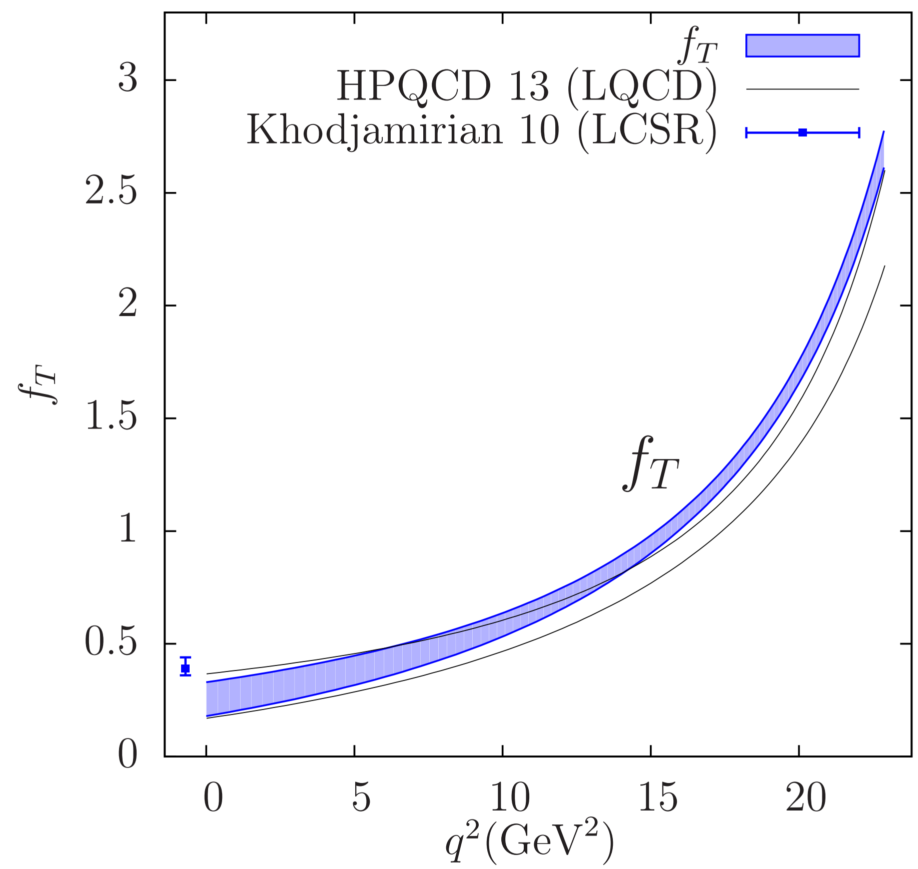
Parameterize the amplitude in terms of the three form factors $f_{+,0,T}(q^2)$:

$$A(B \to P\,\ell\ell) \sim C_7^{\rm eff} f_T + (C_9^{\rm eff} + C_{10}) f_+ + \text{nonfactorizable terms}$$

more on these later

Form factors for $B \to K\,\ell\ell$



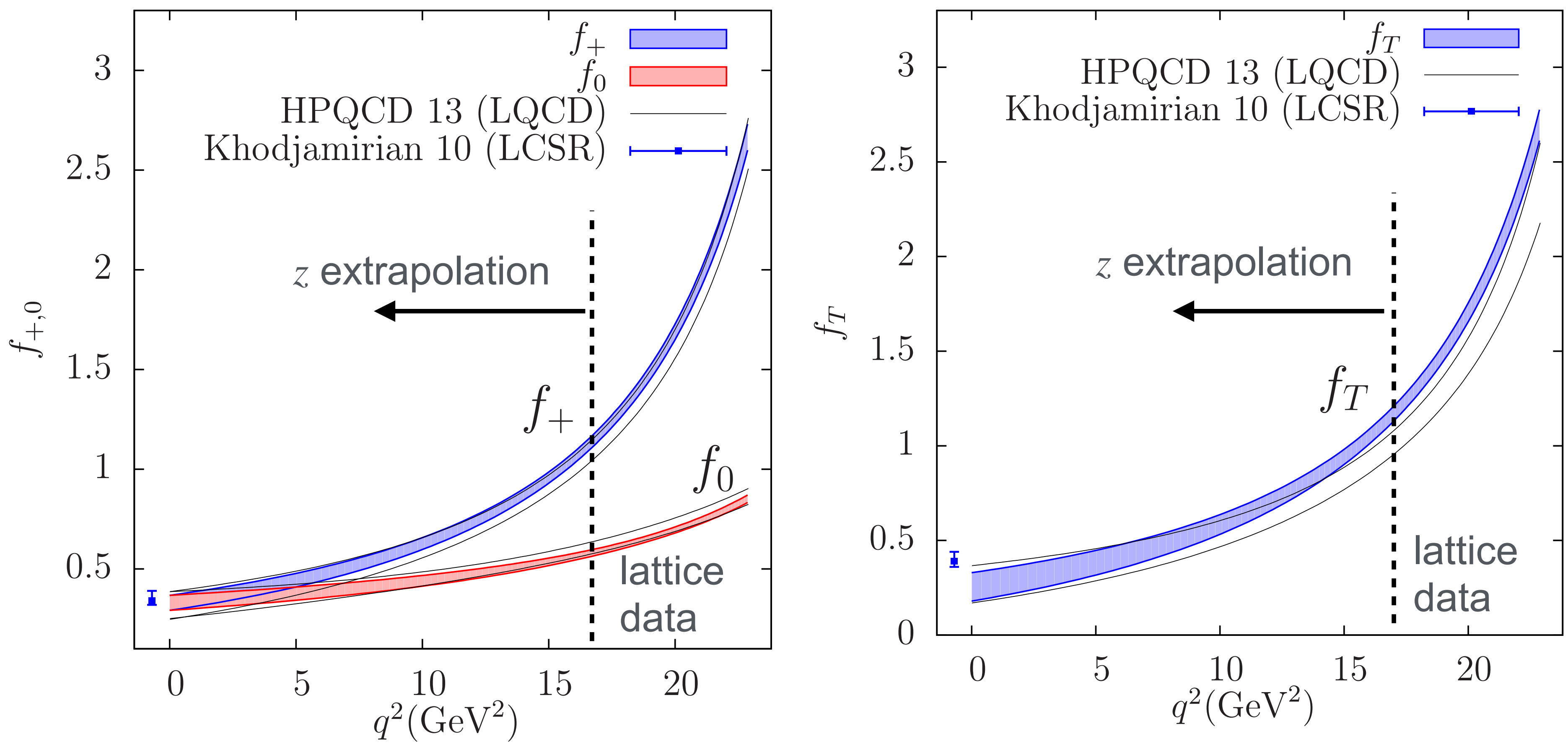


HPQCD (arXiv:1306.0434, 1306.2384, PRL 2013)

FNAL/MILC (arXiv:1509.06235, PRD 2016)

- Two LQCD calculations (on overlapping ensemble sets, different valence actions): HPQCD (NRQCD b + HISQ), FNAL/MILC (Fermilab b + asqtad)
- consistent results for all three form factors
- also consistent with LCSR (Khodjamarian et al, arXiv:1006.4945, JHEP 2010)
- Note: First LQCD calculation of $\Lambda_b \to \Lambda\,\ell^+\ell^-$ form factors (10 total) (Detmold and Meinel, arXiv:1602.01399, PRD 2016)

Form factors for $B \to K \ell\ell$



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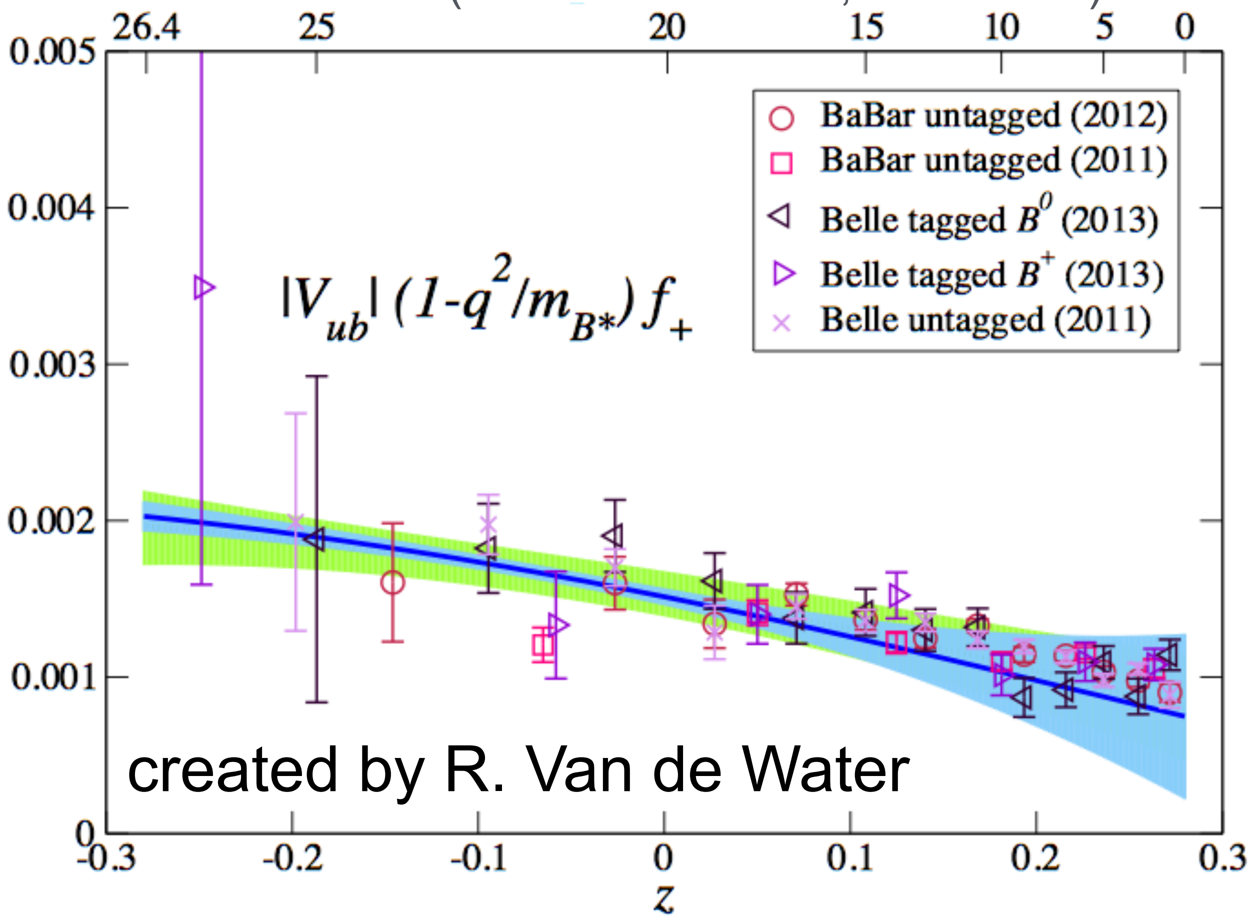

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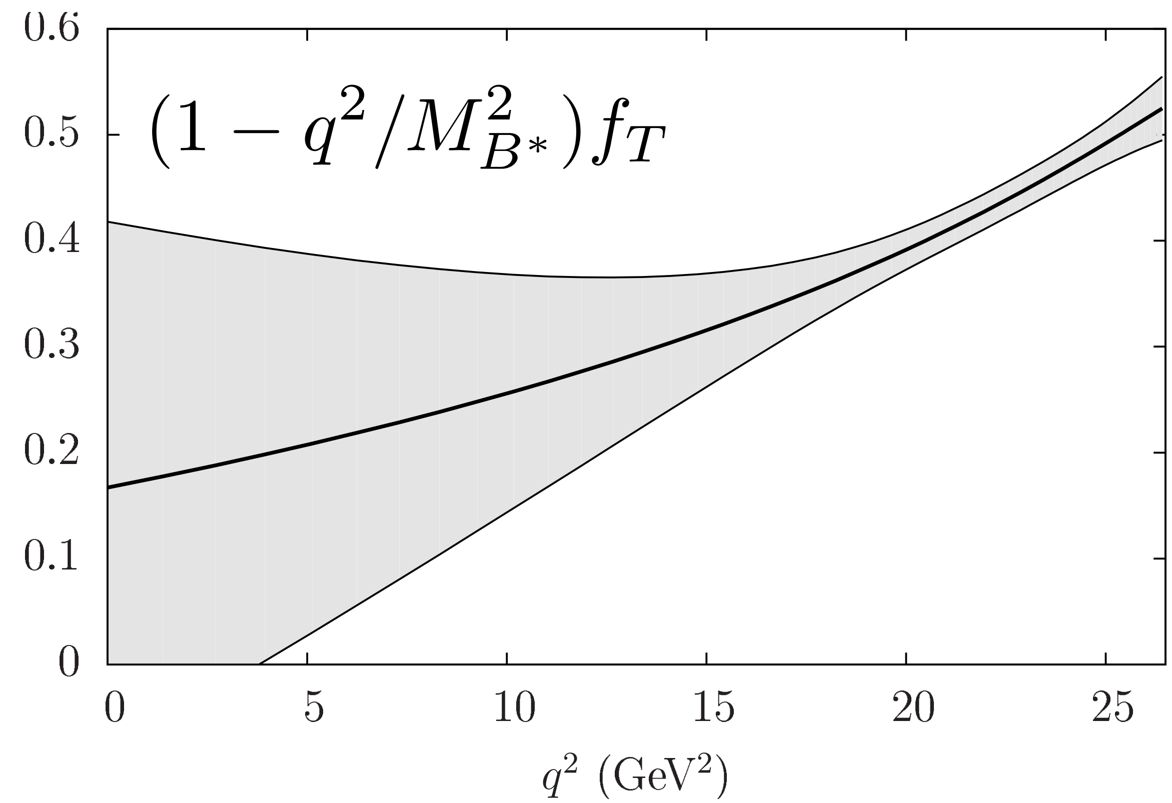

- Two independent calculations (RBC, FNAL/MILC): consistent results for f_0, f_+
- Shape of f_+ agrees with experiment and uncertainties are commensurate
- Fit lattice form factors together with experimental data to determine $|V_{ub}|$ **and** improved form factors (f_+, f_0)
- **First** calculation of f_T (FNAL/MILC)

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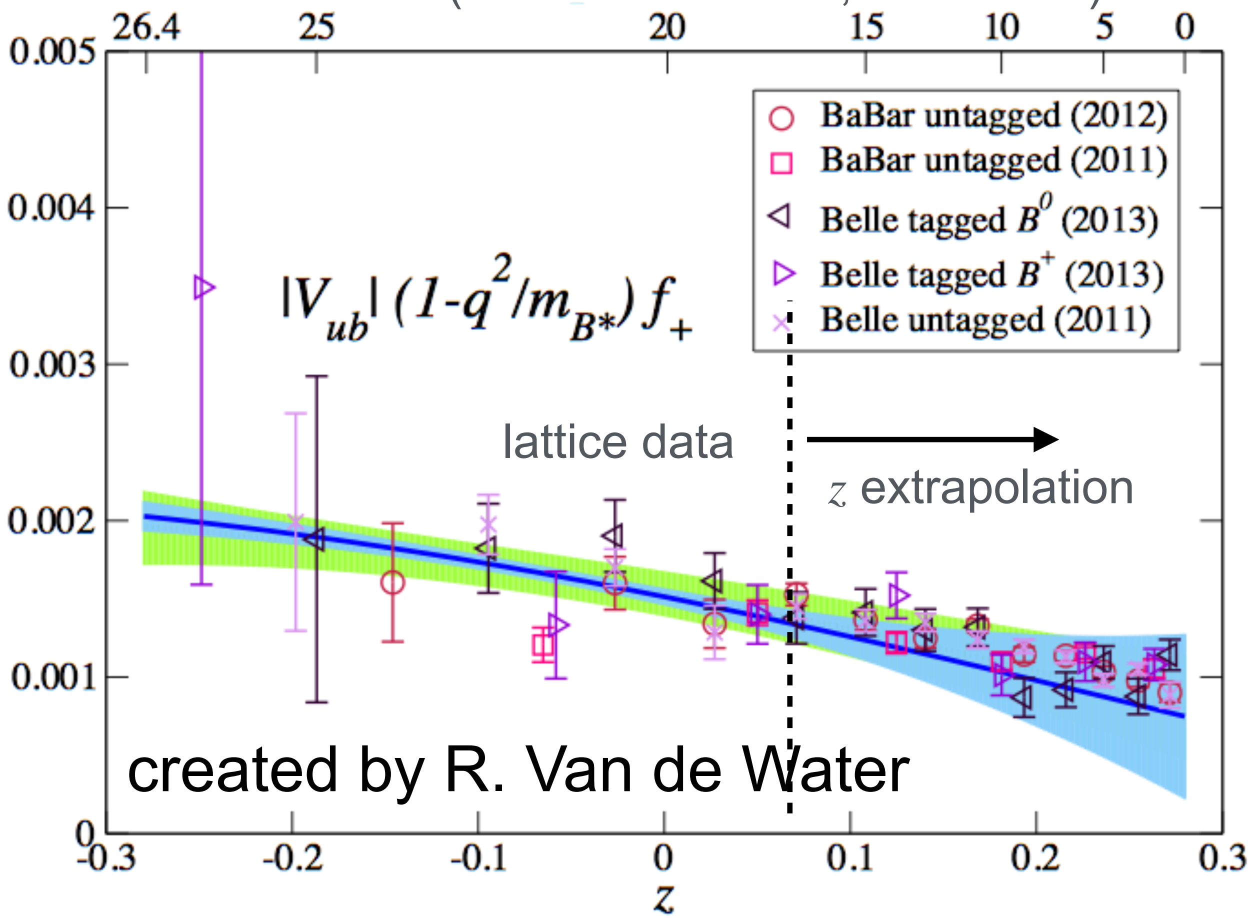

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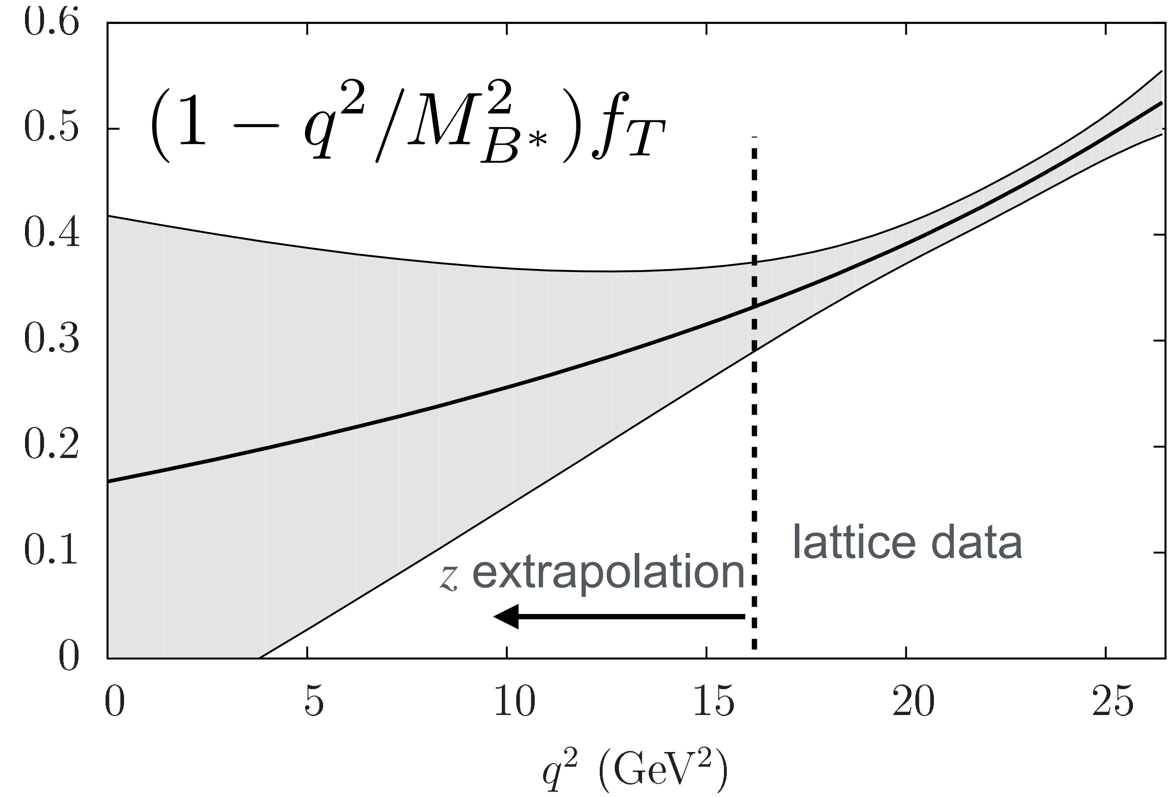

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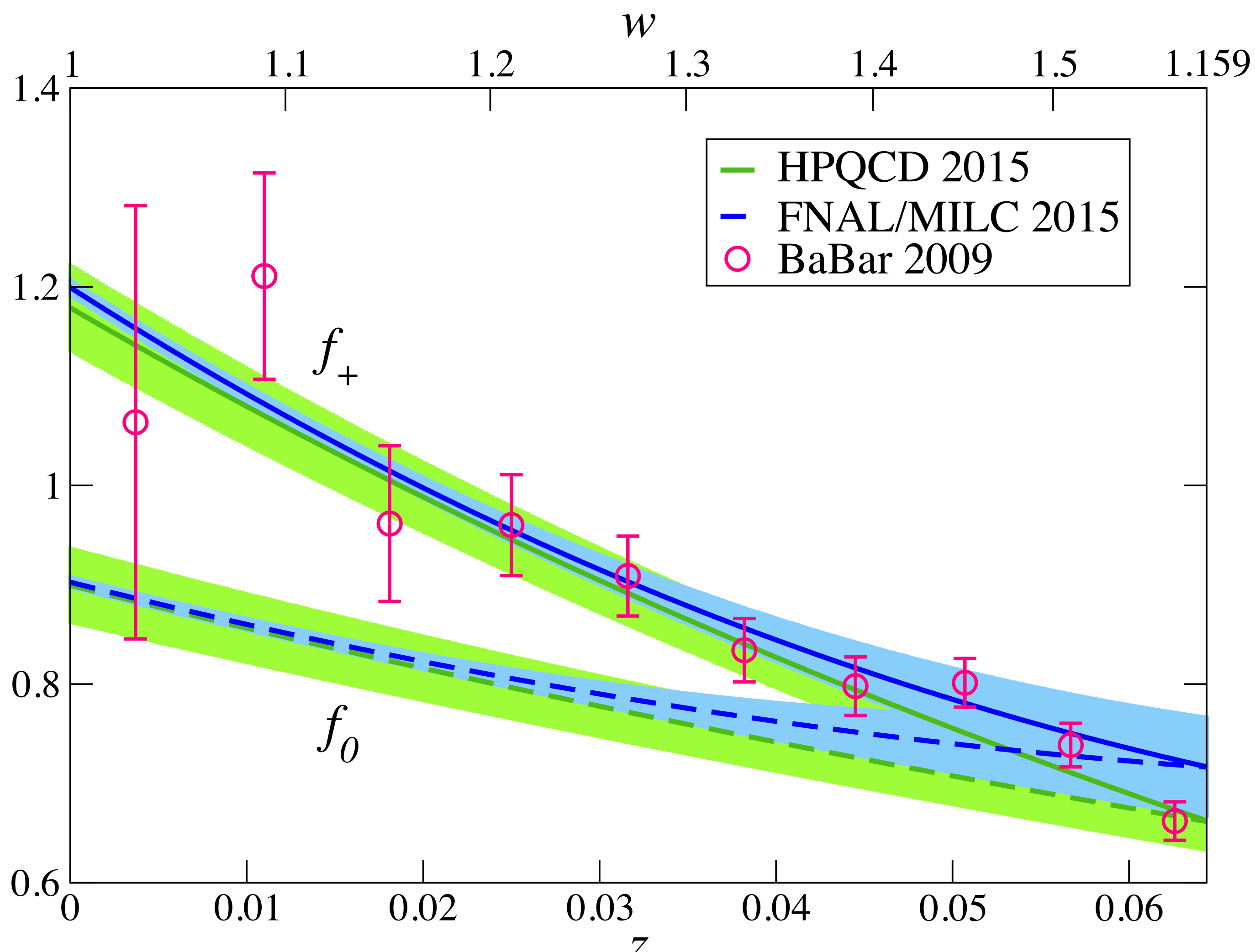
Form factors for $B \to D\,\ell\nu,\ \ell = e, \mu, \tau$



created by R. Van de Water

HPQCD (arXiv:1505.03925, PRD 2015)
FNAL/MILC (arXiv:1503.07237, PRD 2015)

- Two LQCD calculations (FNAL/MILC, HPQCD)
- Combined fit to LQCD form factors and BaBar data.
- LQCD form factor uncertainties (~1.2%) smaller than experiment.

- Form factors can be used to calculate the CKM free ratio:

$$R(D) \equiv \frac{\mathcal{B}(B \to D\tau\nu_\tau)}{\mathcal{B}(B \to D\ell\nu)}$$

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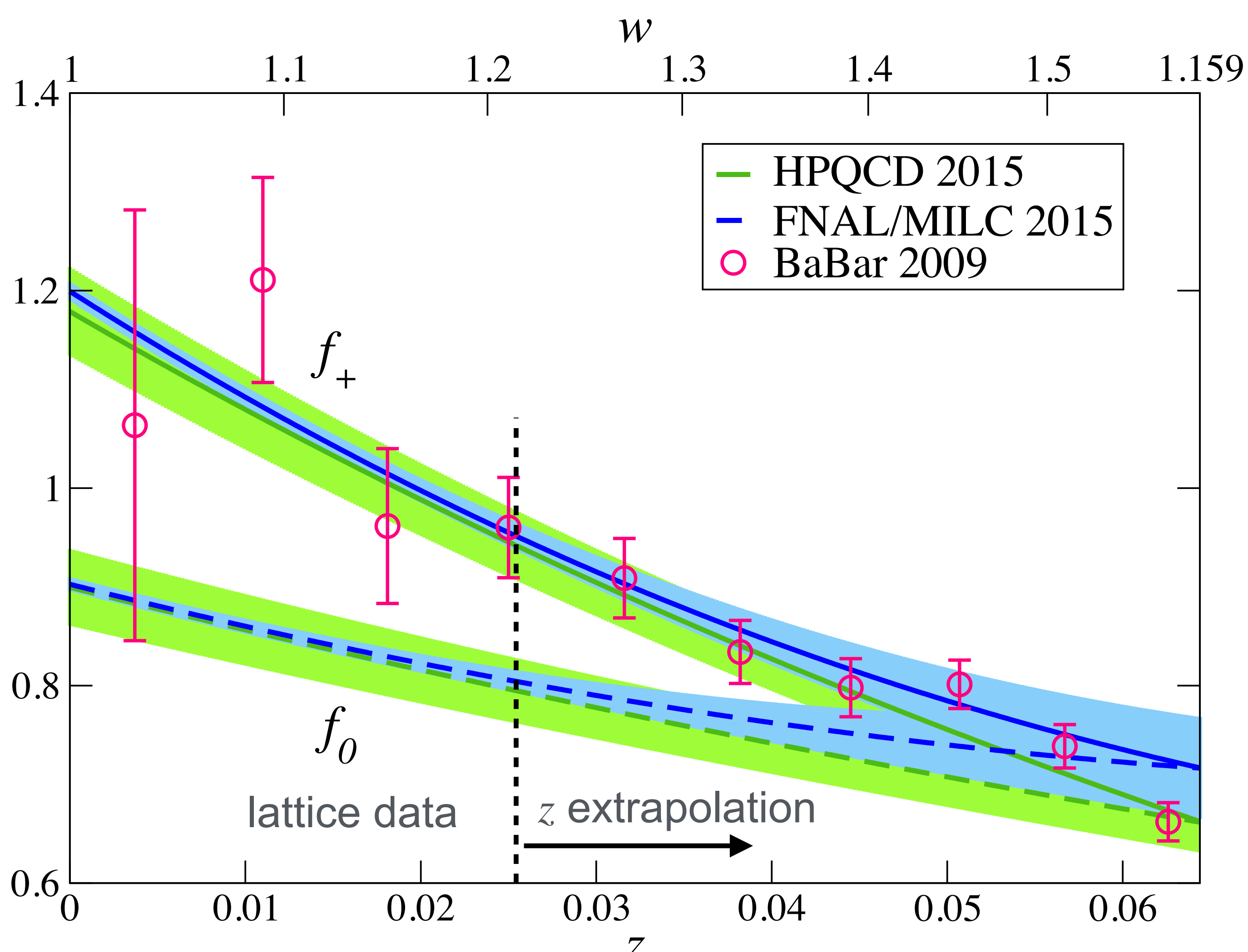

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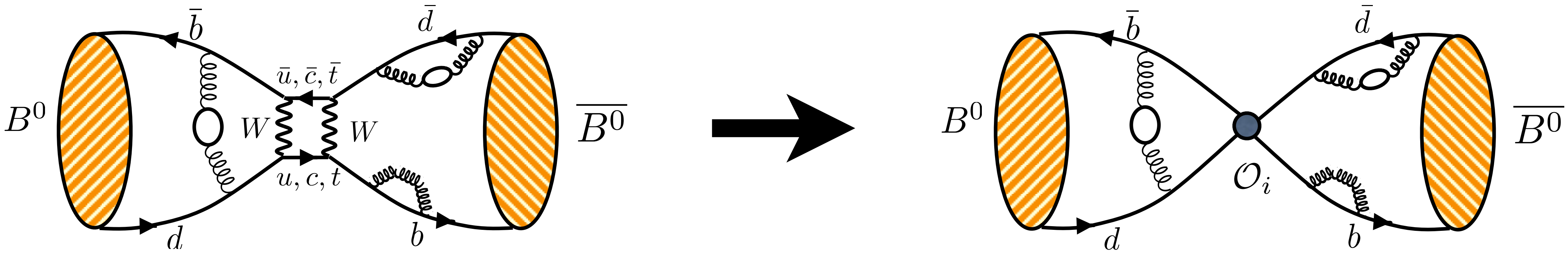
Neutral B mixing

Standard Model



SM: $$\Delta M_q = (\text{known}) \times |V_{tq}^* V_{tb}|^2 \times \langle \bar{B}_q^0 | \mathcal{O}_1 | B_q^0 \rangle$$

also:

$$\frac{\Delta M_s}{\Delta M_d} = \frac{m_{B_s}}{m_{B\,d}} \times \left| \frac{V_{ts}}{V_{td}} \right|^2 \times \xi^2$$ with $$\xi \equiv \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

$$\Delta\Gamma_q = \left[G_1 \, \langle \bar{B}_q^0 | \mathcal{O}_1 | B_q^0 \rangle + G_3 \, \langle \bar{B}_q^0 | \mathcal{O}_3 | B_q^0 \rangle \right] \cos\phi_q + O(1/m_b)$$

Neutral B mixing

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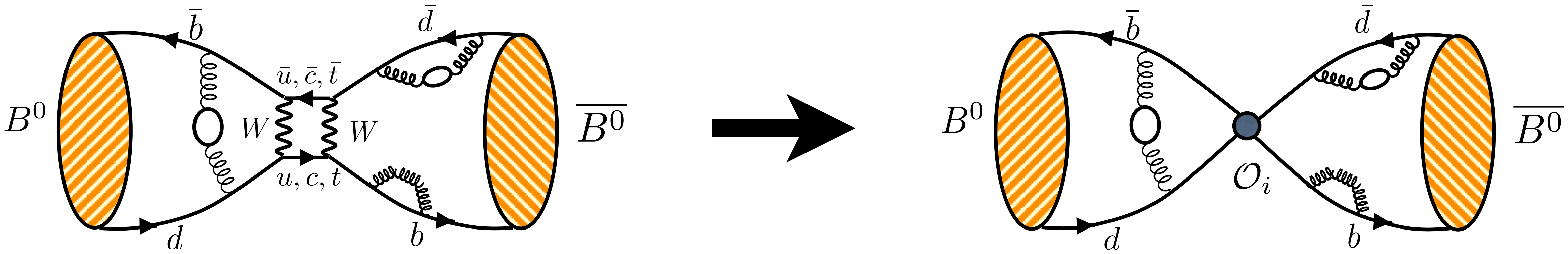


In general :

$$\mathcal{H}_{\text{eff}} = \sum_{i=1}^{5} c_i(\mu) \mathcal{O}_i(\mu)$$

SM:

$$\mathcal{O}_1 = (\bar{b}^\alpha \gamma_\mu L q^\alpha)\ (\bar{b}^\beta \gamma_\mu L q^\beta)$$

$$\mathcal{O}_2 = (\bar{b}^\alpha L q^\alpha)\ (\bar{b}^\beta L q^\beta)$$

$$\mathcal{O}_3 = (\bar{b}^\alpha L q^\beta)\ (\bar{b}^\beta L q^\alpha)$$

BSM:

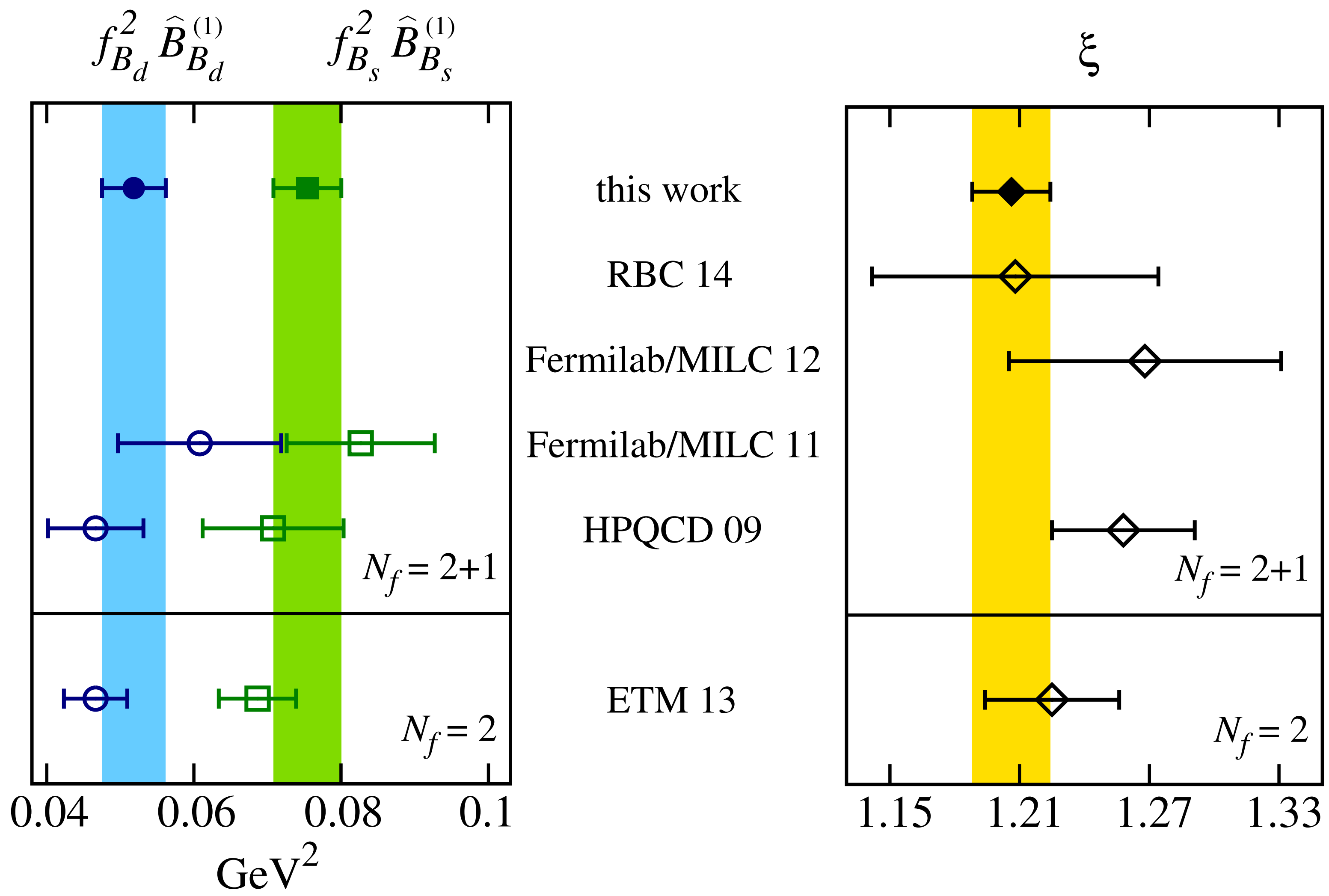
$$\mathcal{O}_4 = (\bar{b}^\alpha L q^\alpha)\ (\bar{b}^\beta R q^\beta)$$

$$\mathcal{O}_5 = (\bar{b}^\alpha L q^\beta)\ (\bar{b}^\beta R q^\alpha)$$

Recent and ongoing LQCD calculations of K, D, and B mixing quantities now include results for hadronic matrix elements of all five operators.

Neutral B mixing

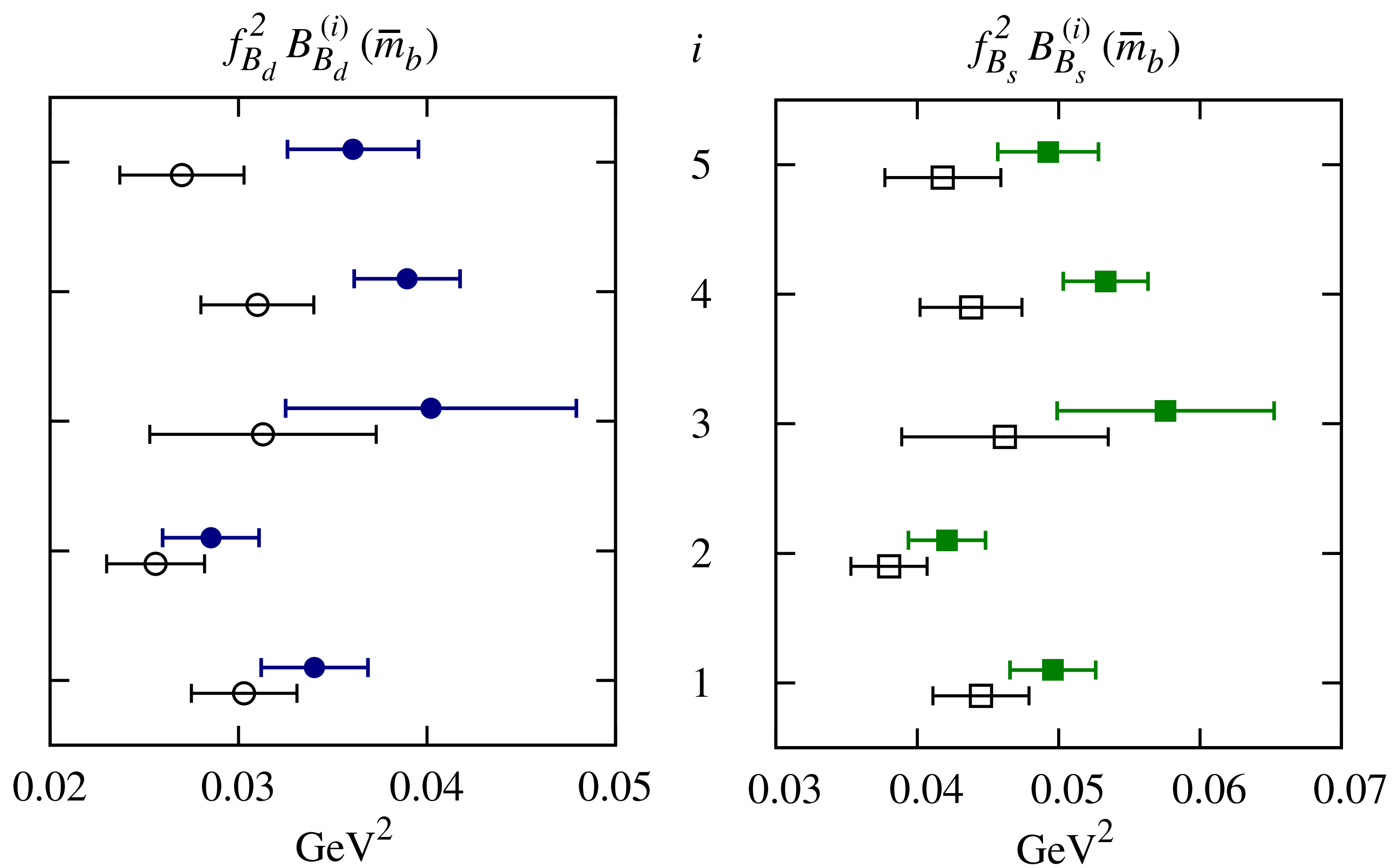
FNAL/MILC (arXiv:1602.03560)



- new LQCD calculation by FNAL/MILC (Fermilab b + asqtad)
- significant reduction of errors, especially for ξ
- first three flavor LQCD result for all five matrix elements

Neutral B mixing

ETM (n_f=2, arXiv:1308.1851, JHEP 2014) vs. FNAL/MILC (n_f=3, arXiv:1602.03560)



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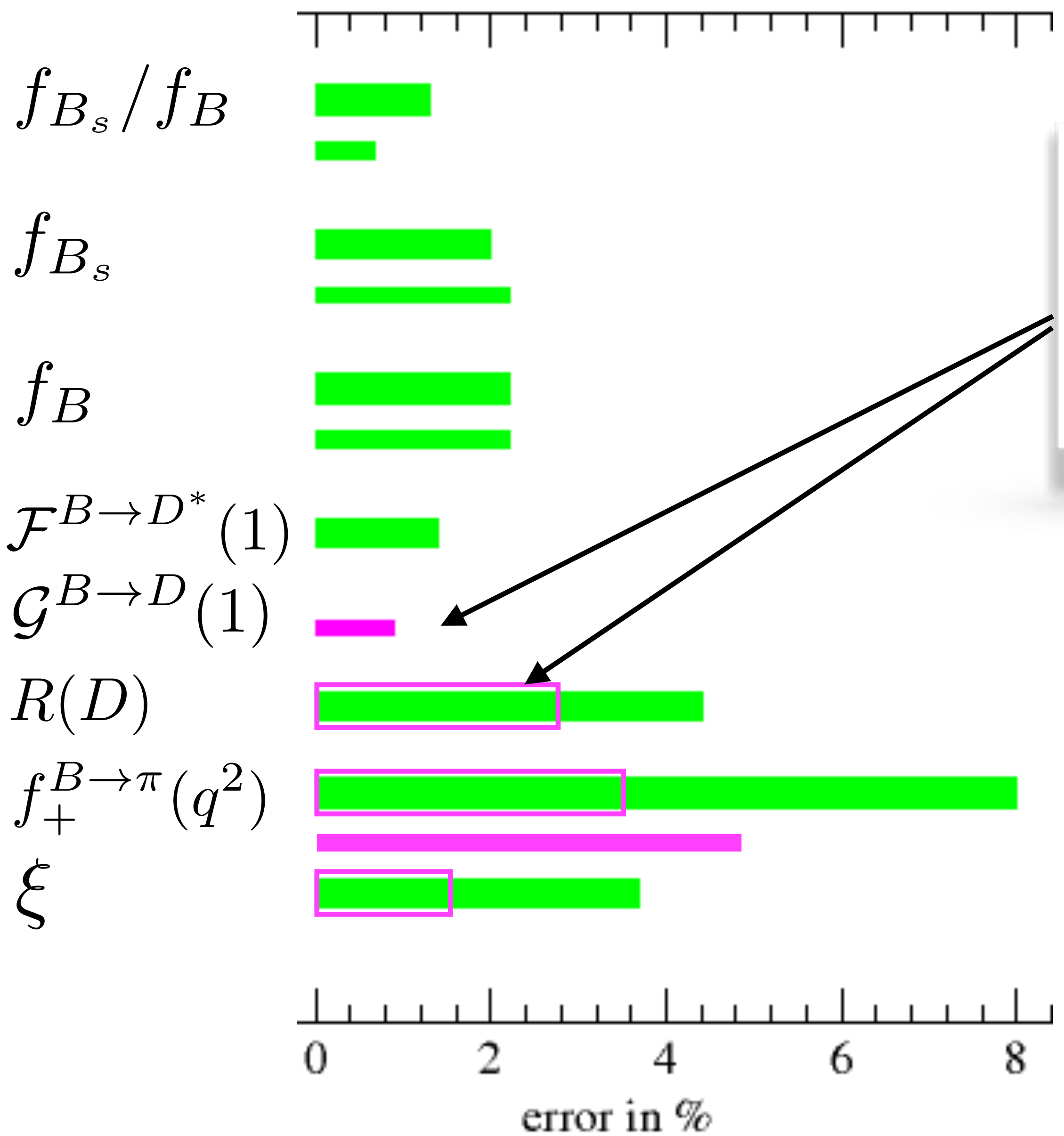
Summary of recent progress

errors (in %) **(preliminary) FLAG-3 averages** + **new results**



Semileptonic decays

form factors for $B \to D\,\ell\nu$ at nonzero recoil by
FNAL/MILC (Bailey et al,arXiv:1503.07237, PRD 2015)
HPQCD (Na et al,arXiv:1505.03925, PRD 2015)

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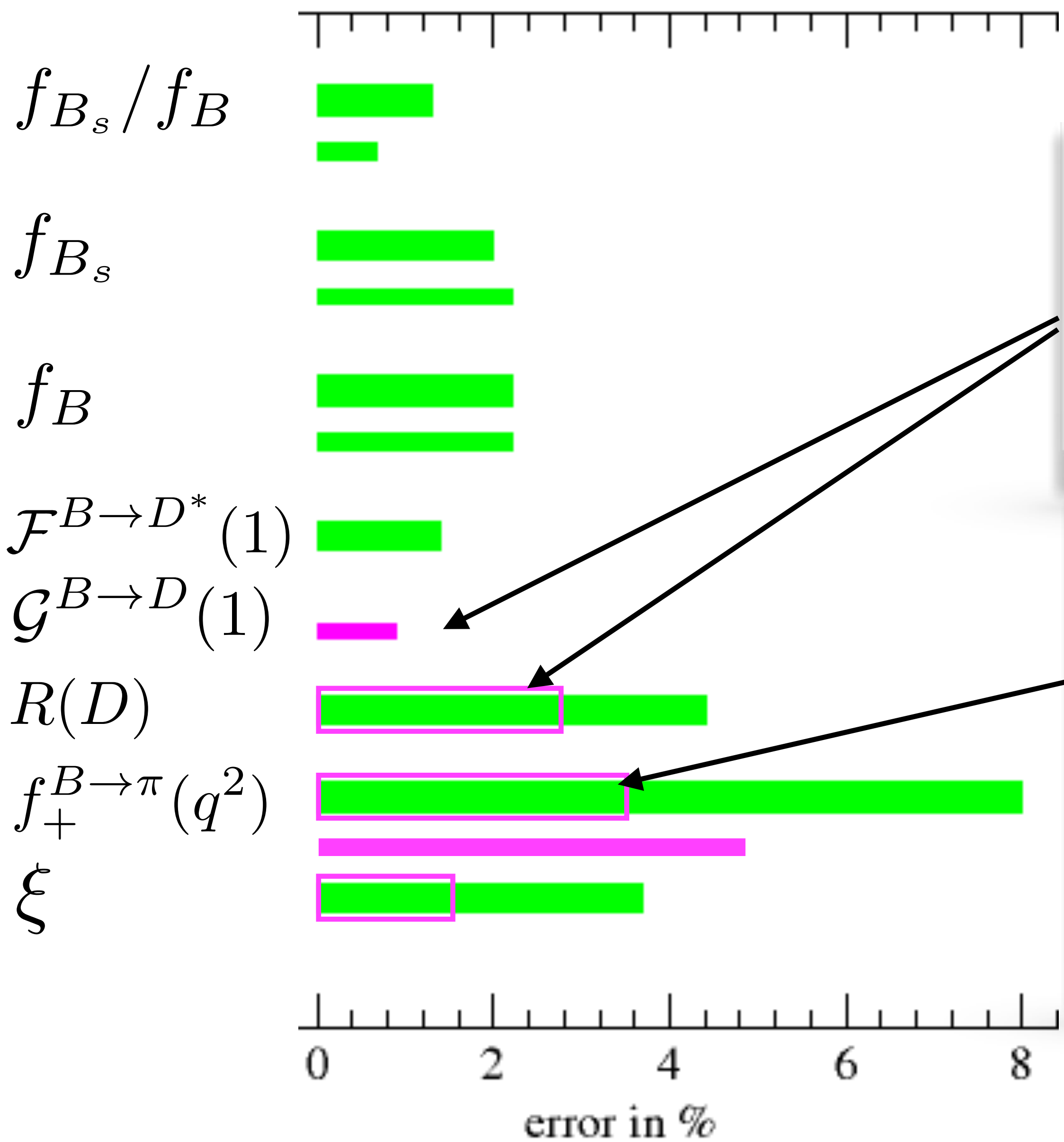

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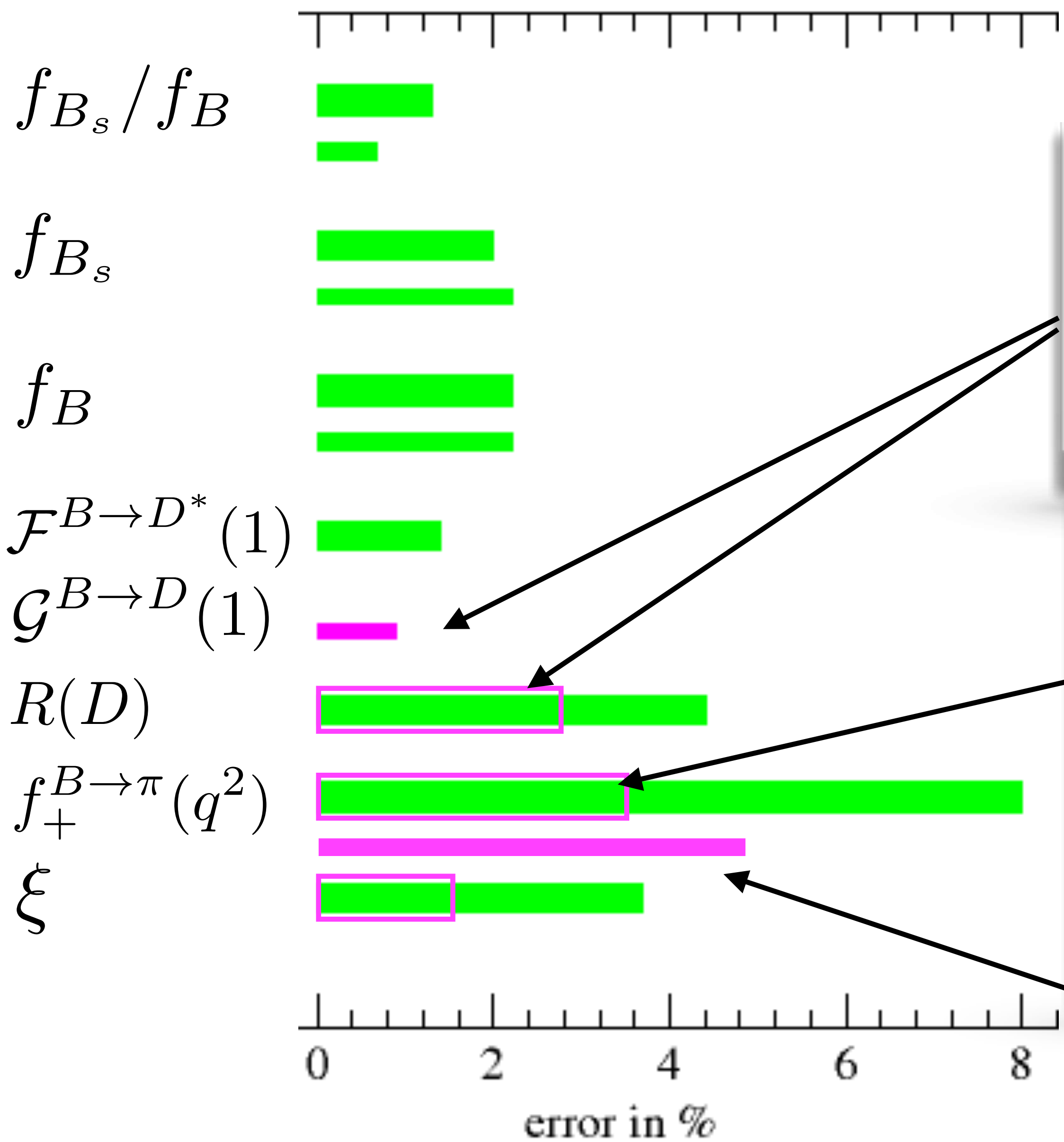

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$\Lambda_b \to p/\Lambda_b \to \Lambda_c$ and $\Lambda_b \to \Lambda\,\ell\ell$
(Detmold et al, arXiv:1503.01421, PRD 2015; arXiv:1602.01399, PRD 2016)

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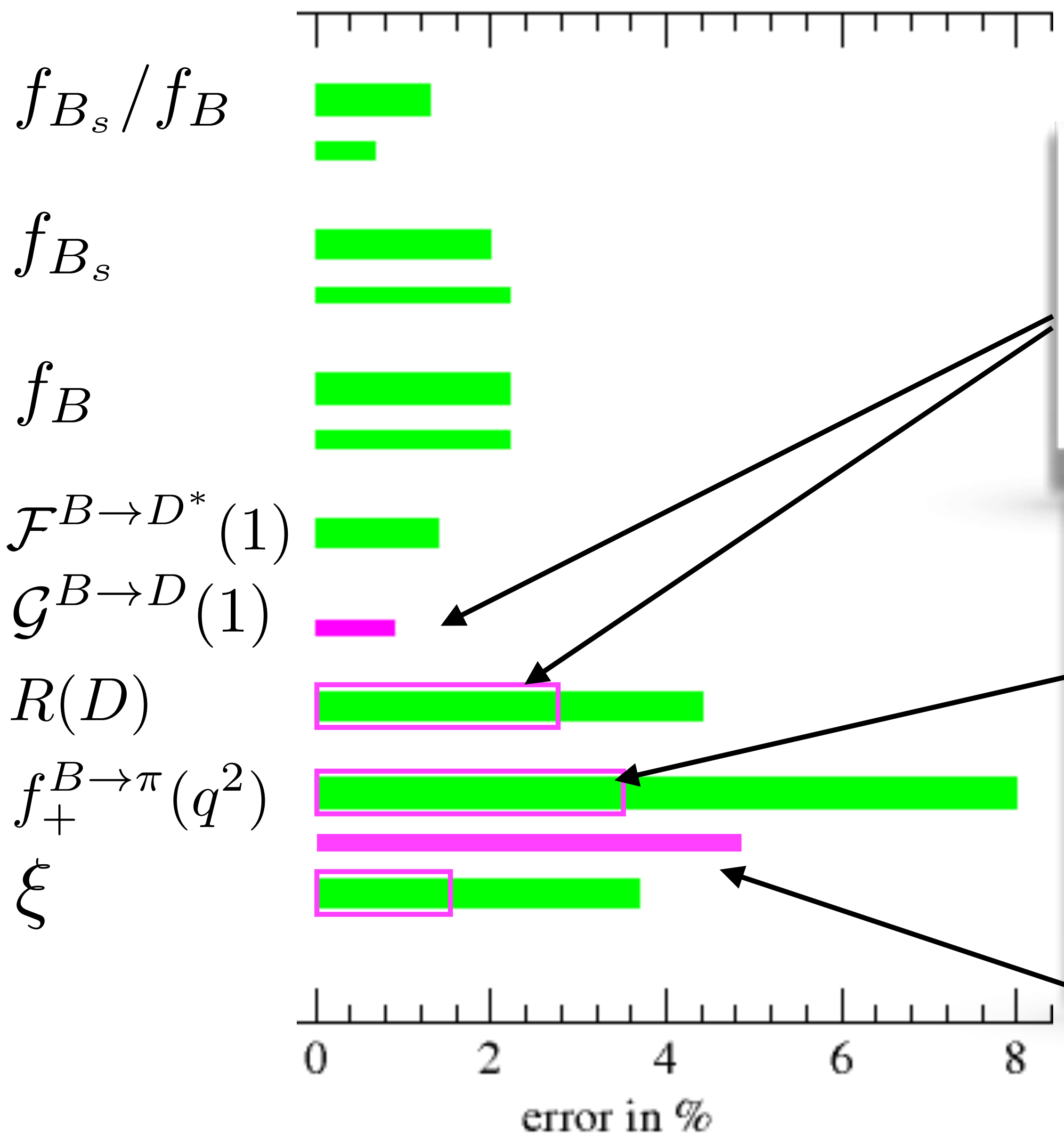

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Theory uncertainties are commensurate with experimental errors

Summary of recent progress

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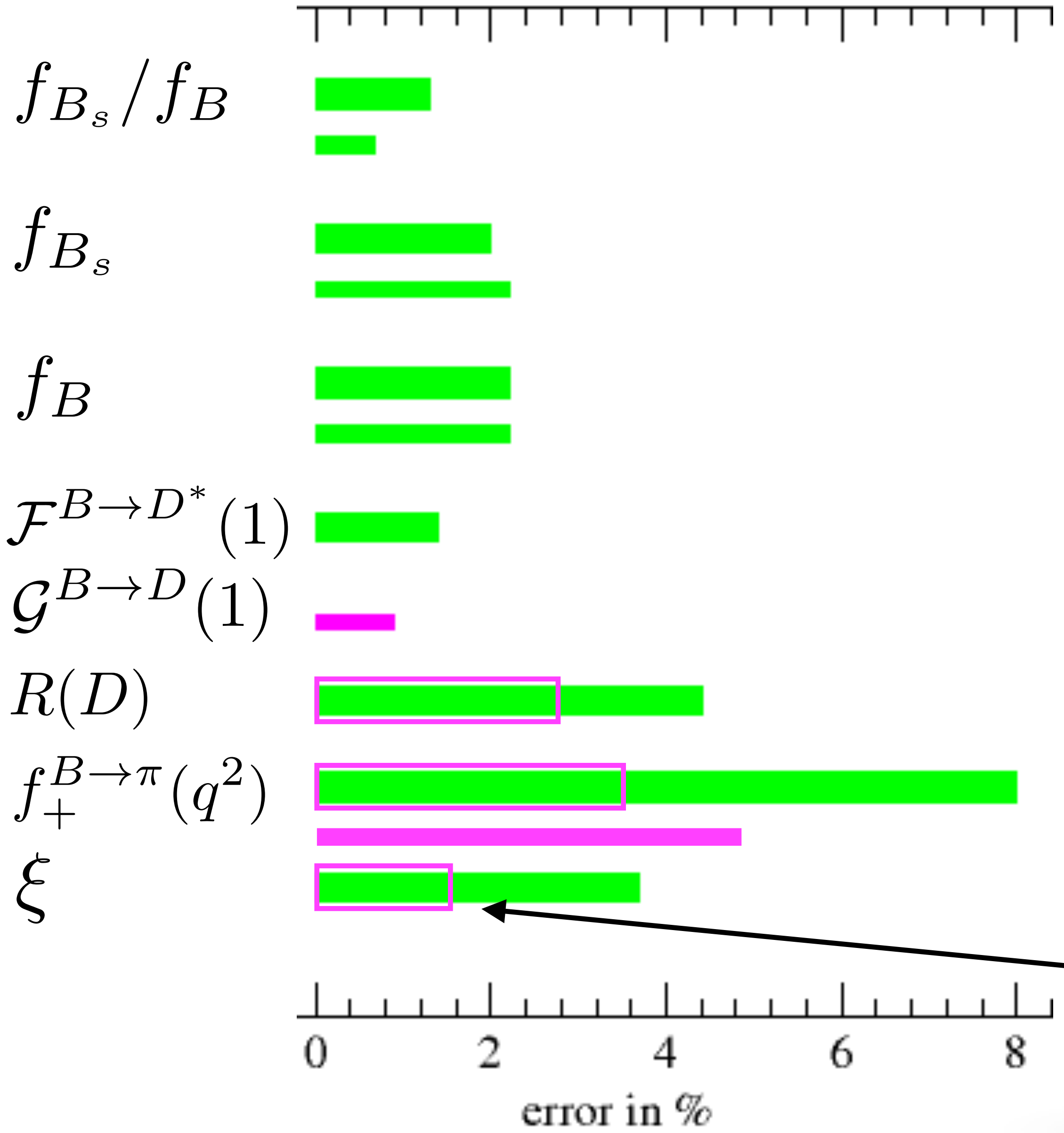

B meson mixing

First calculation of all five MEs with n_f=3 by FNAL/MILC (Bazavov et al,arXiv:1602.03560)

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Rare semileptonic B decay

Nonfactorizable contributions:

$$\langle P\,\ell\ell|Q_i(y)|B\rangle \sim \bar{u}_\ell \gamma^\mu v_\ell \int d^4x\, e^{iq(x-y)} \langle P|T J^\mu_{\rm em}(x) Q_i(y)|B\rangle$$

- at low recoil (high q^2) use OPE (Grinstein&Pirjol, hep-ph/0404250, PRD 2004; others):
  expand in $1/q^2 \sim 1/m_b^2$

$$\langle P\,\ell\ell|Q_i|B\rangle \sim f_{+,0,T}$$ + quark-hadron duality violations

(Beylich et al, arXiv:1101.5118, EPJC 2011)

- at high recoil (low q^2) use SCET:
  $E_P \sim m_b/2$, expand in Λ/M_B

$$\langle P\,\ell\ell|Q_i|B\rangle \sim f_{+,0,T} + \phi_B \star T \star \phi_P$$

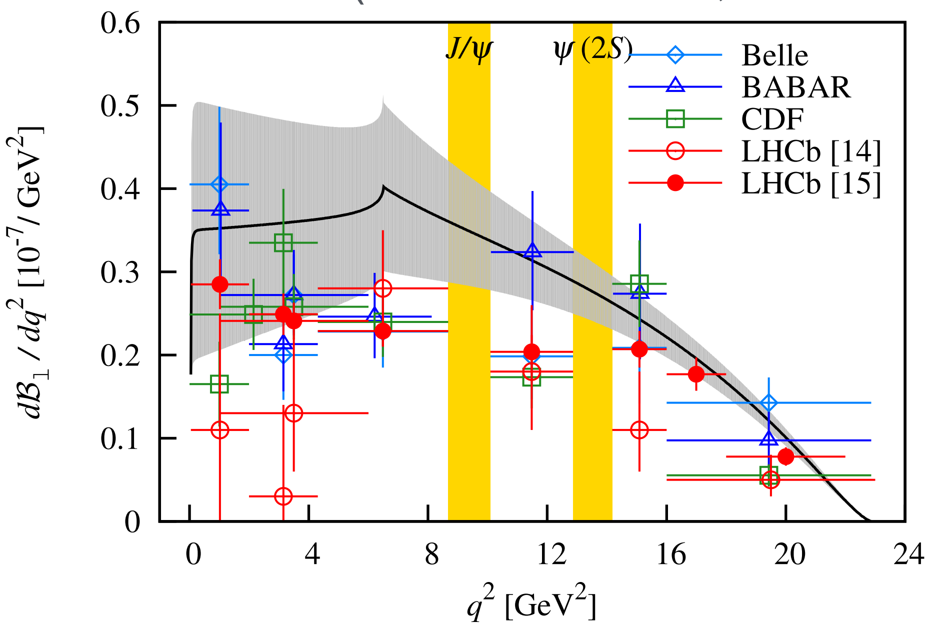
hard ~ $m_b{}^2$

hard collinear ~ $\Lambda\, m_b{}^2$

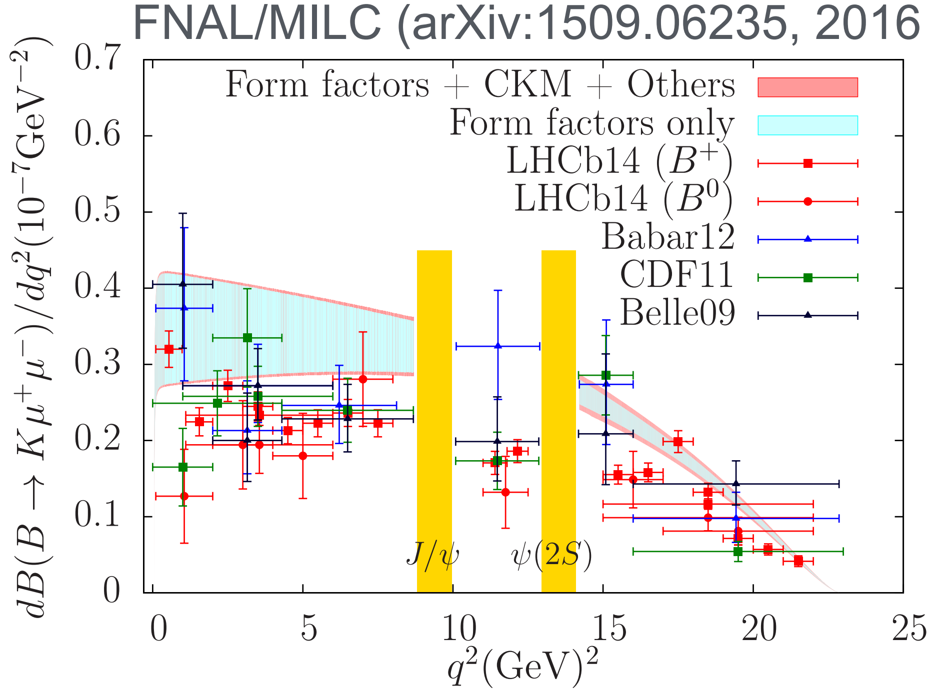
Phenomenology for $B \to K, \pi\, \ell^+\ell^-$

Experiment vs. Theory

HPQCD (arXiv:1306.0434, 2013 PRL)



FNAL/MILC (arXiv:1509.06235, 2016 PRD)



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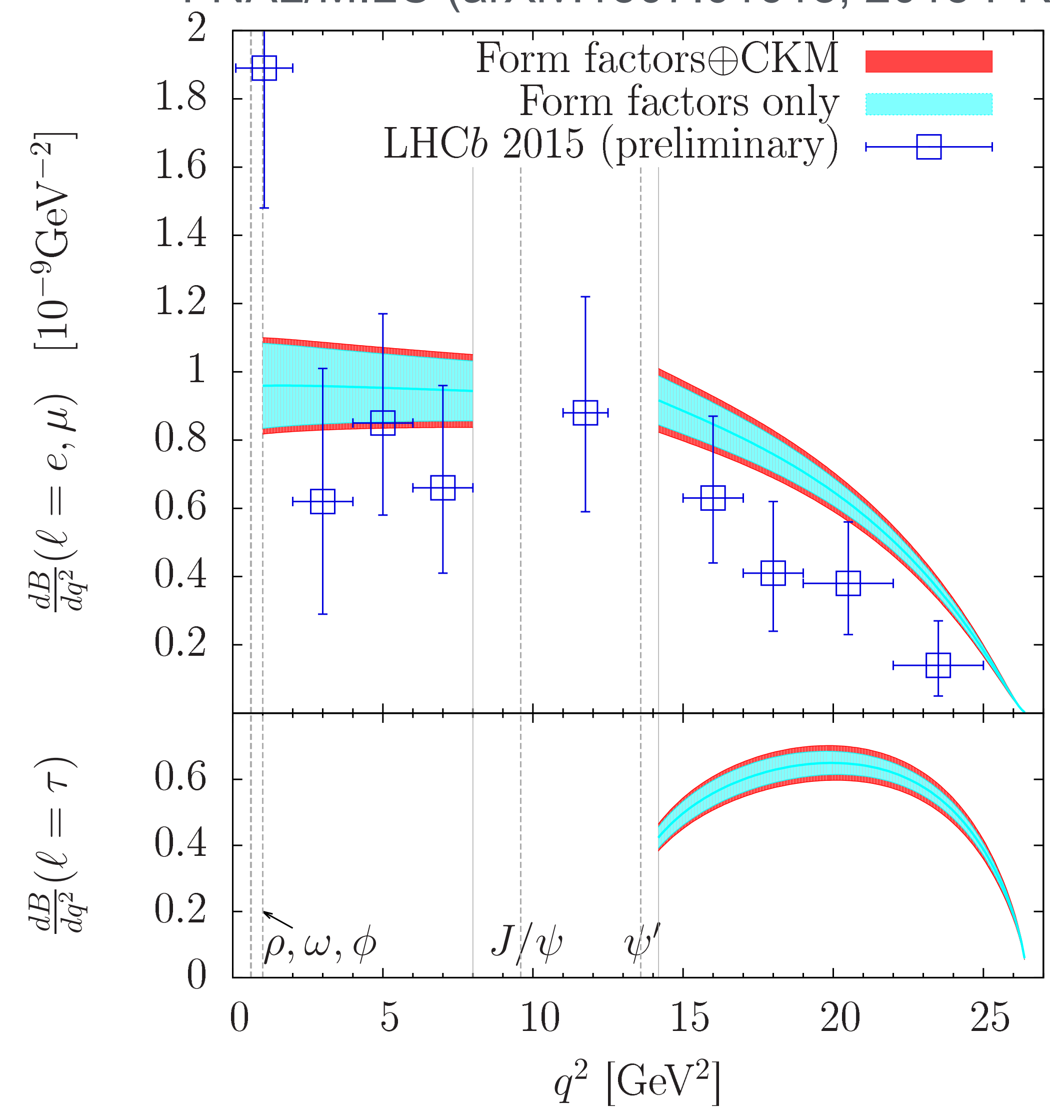

Phenomenology for $B \to K, \pi\, \ell^+ \ell^-$

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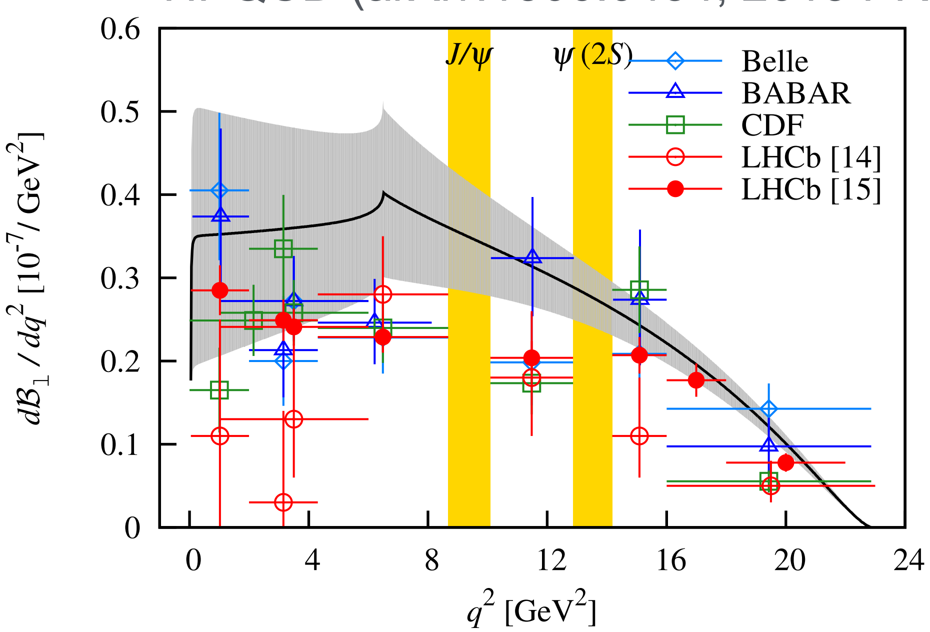

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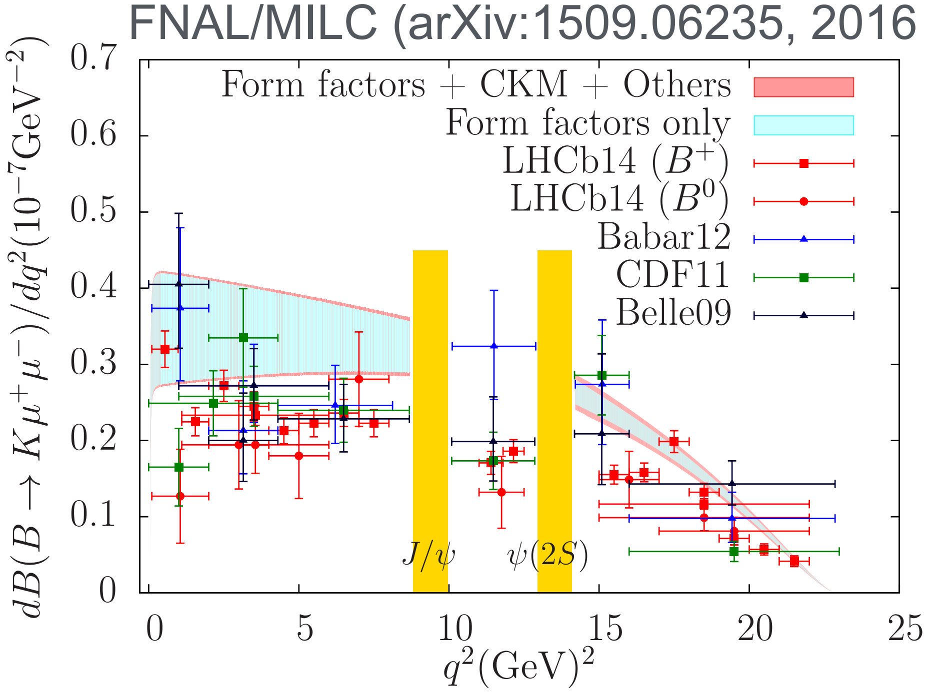

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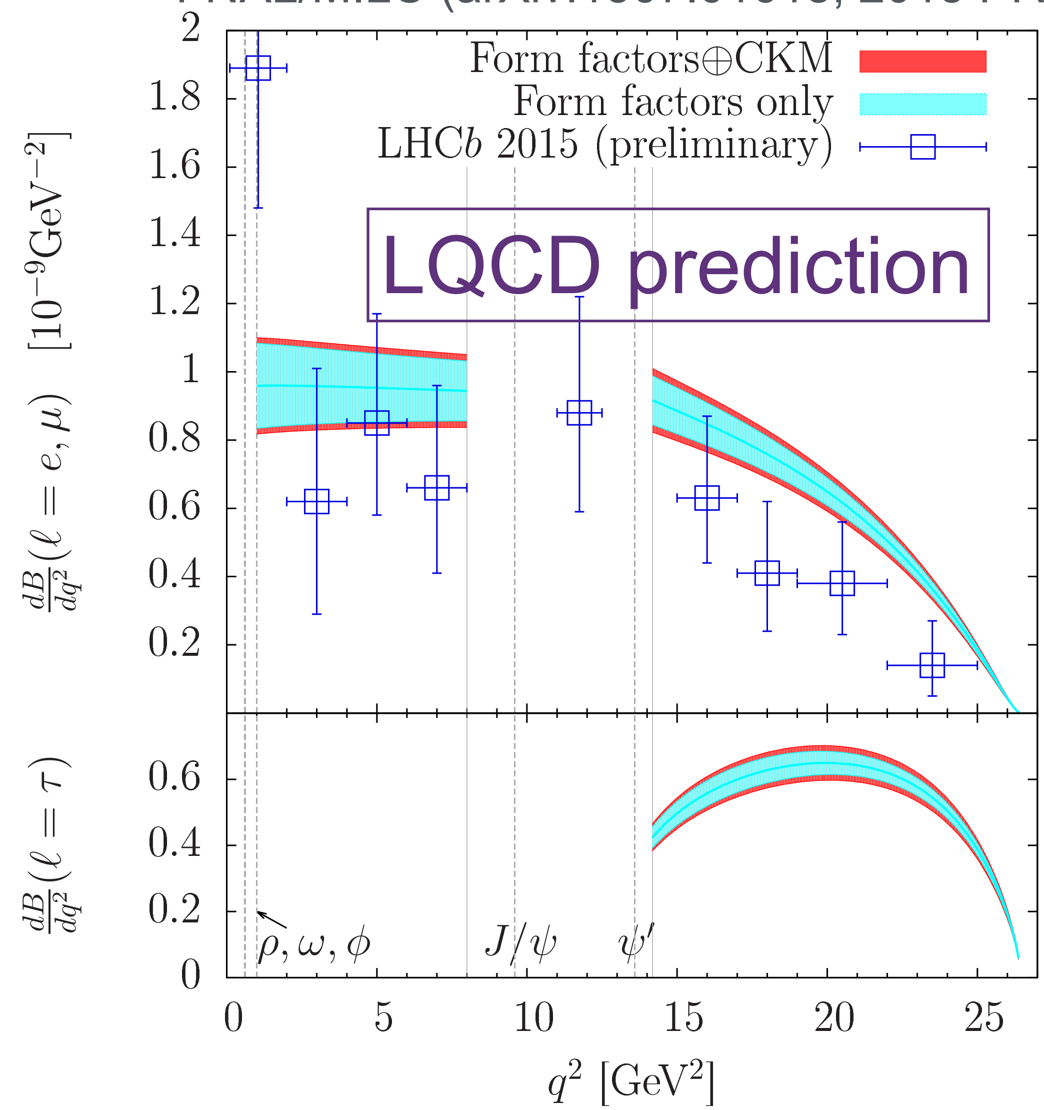


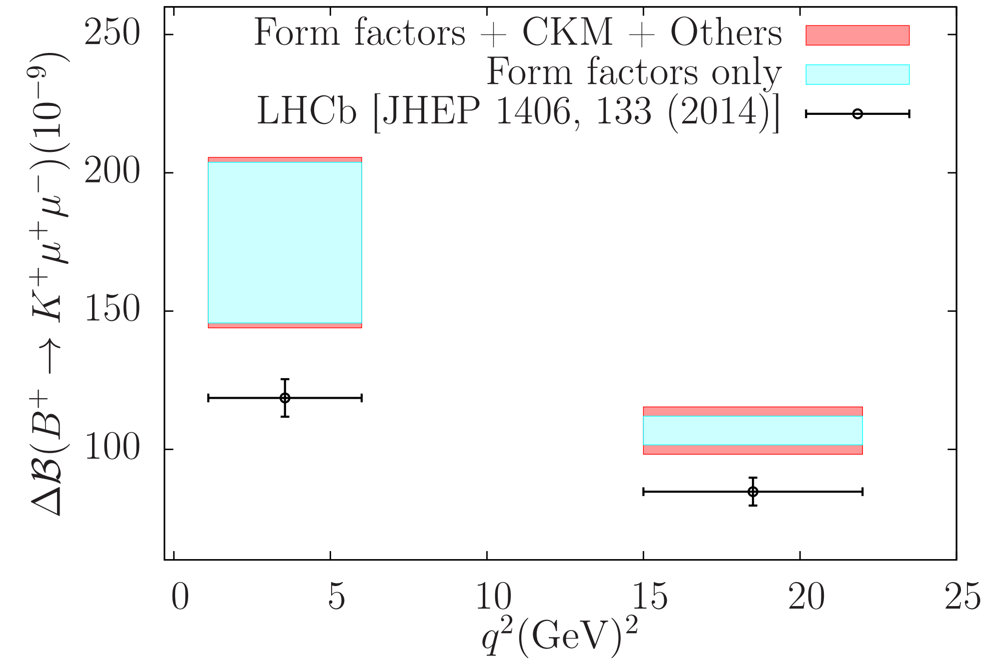
Phenomenology for $B \to K, \pi\, \ell^+ \ell^-$

Experiment vs. theory

- LHCb data + FNAL/MILC form factors (arXiv:1509.00414, JHEP 2015;1403.8044, JHEP 2014)
- focus on large bins above and below charmonium resonances
- theory errors commensurate with experiment
- yields $\sim$1-2σ tensions
- $\Rightarrow$ determine $|V_{td}/V_{ts}|, |V_{td}|, |V_{ts}|$

  or constrain Wilson coefficients

D. Du et al (arXiv:1510.02349, PRD 2016)



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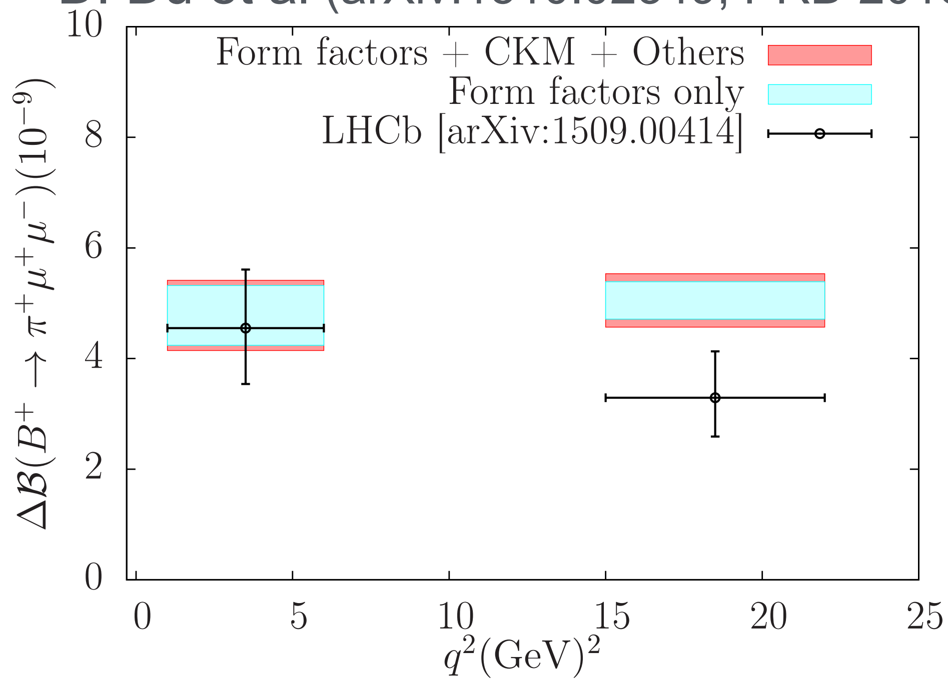
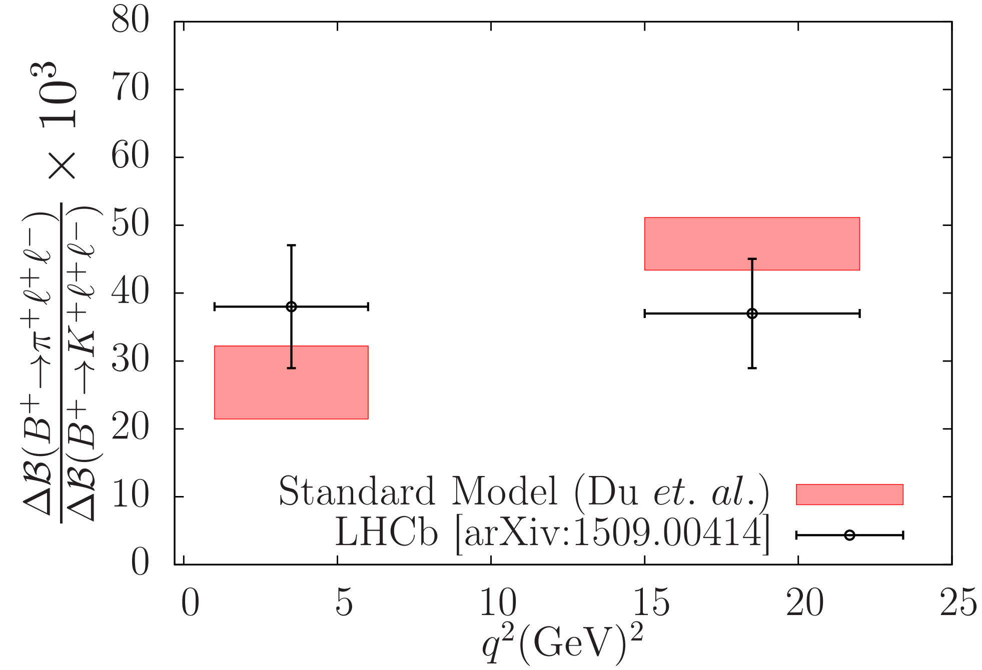
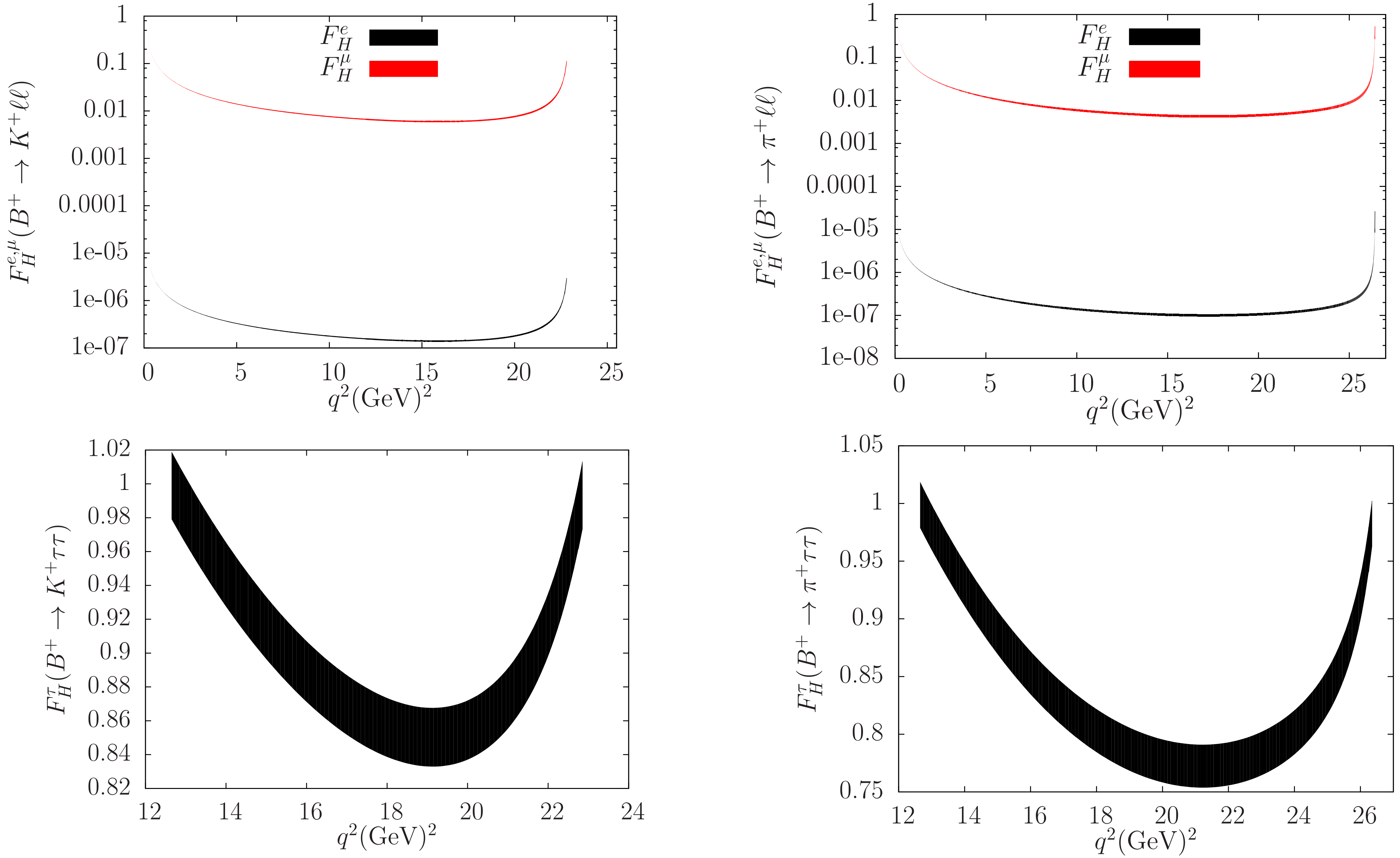


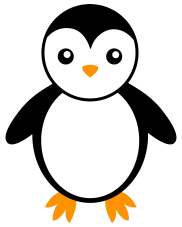


Phenomenology for $B \to K, \pi\, \ell^+ \ell^-$

in the SM: $A_{\rm FB} = 0 \quad F_H^\ell \sim m_\ell^2 / M_B^2$

D. Du et al (arXiv:1510.02349, PRD 2016)





Phenomenology for $\Lambda_b \to \Lambda\, \ell^+ \ell^-$

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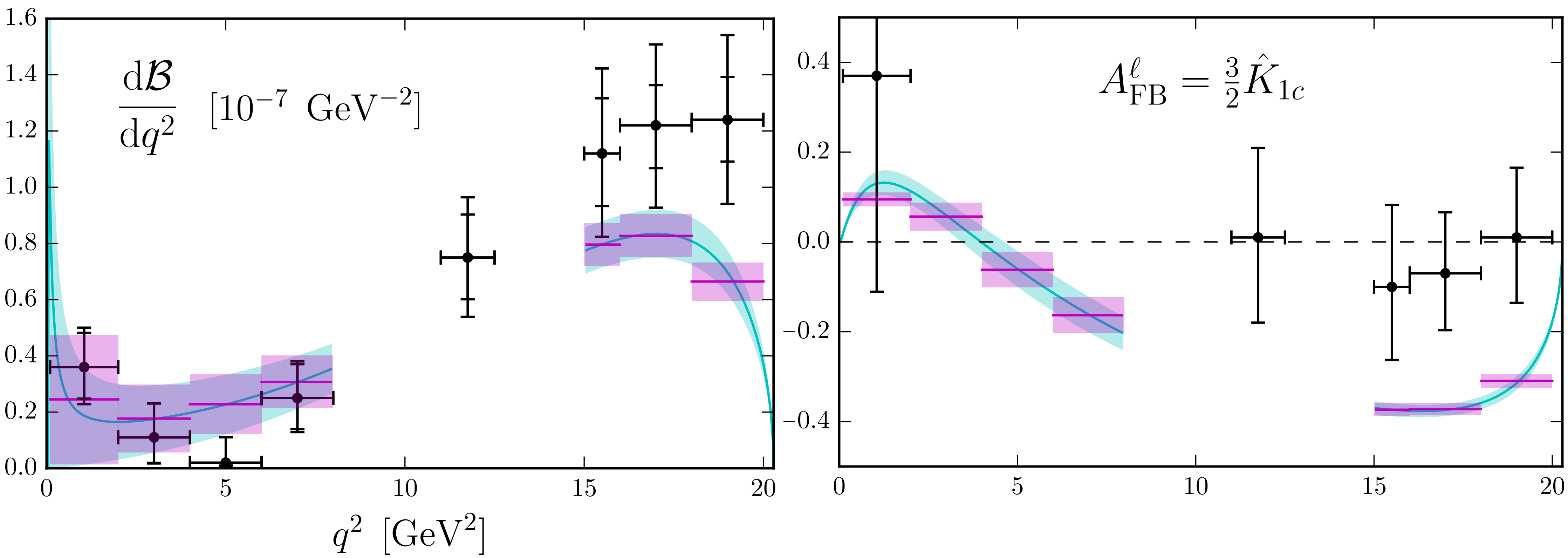
  (arXiv:1503.07138, JHEP 2015)
- focus on regions above and below charmonium resonances
- exp. data lie above SM theory $\sim$1-3σ tensions

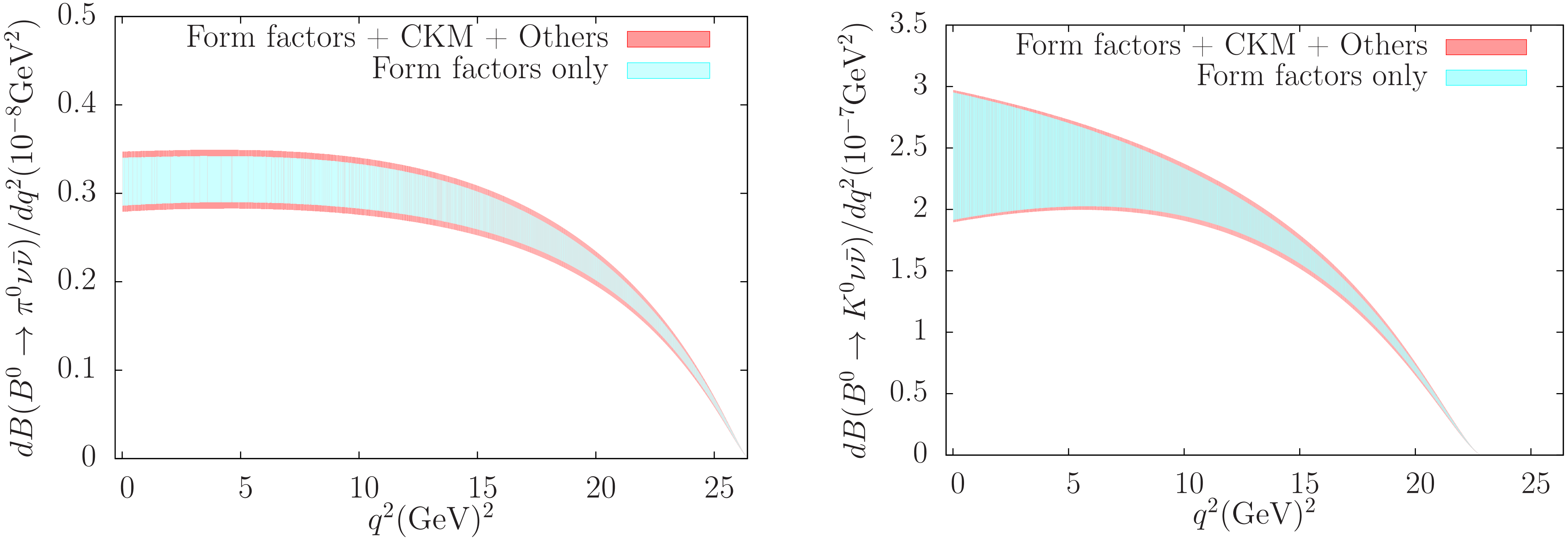
Detmold & Meinel (arXiv:1602.01399, PRD 2016)



Phenomenology for $B^0 \to K^0, \pi^0\, \nu\bar{\nu}$

theoretically clean

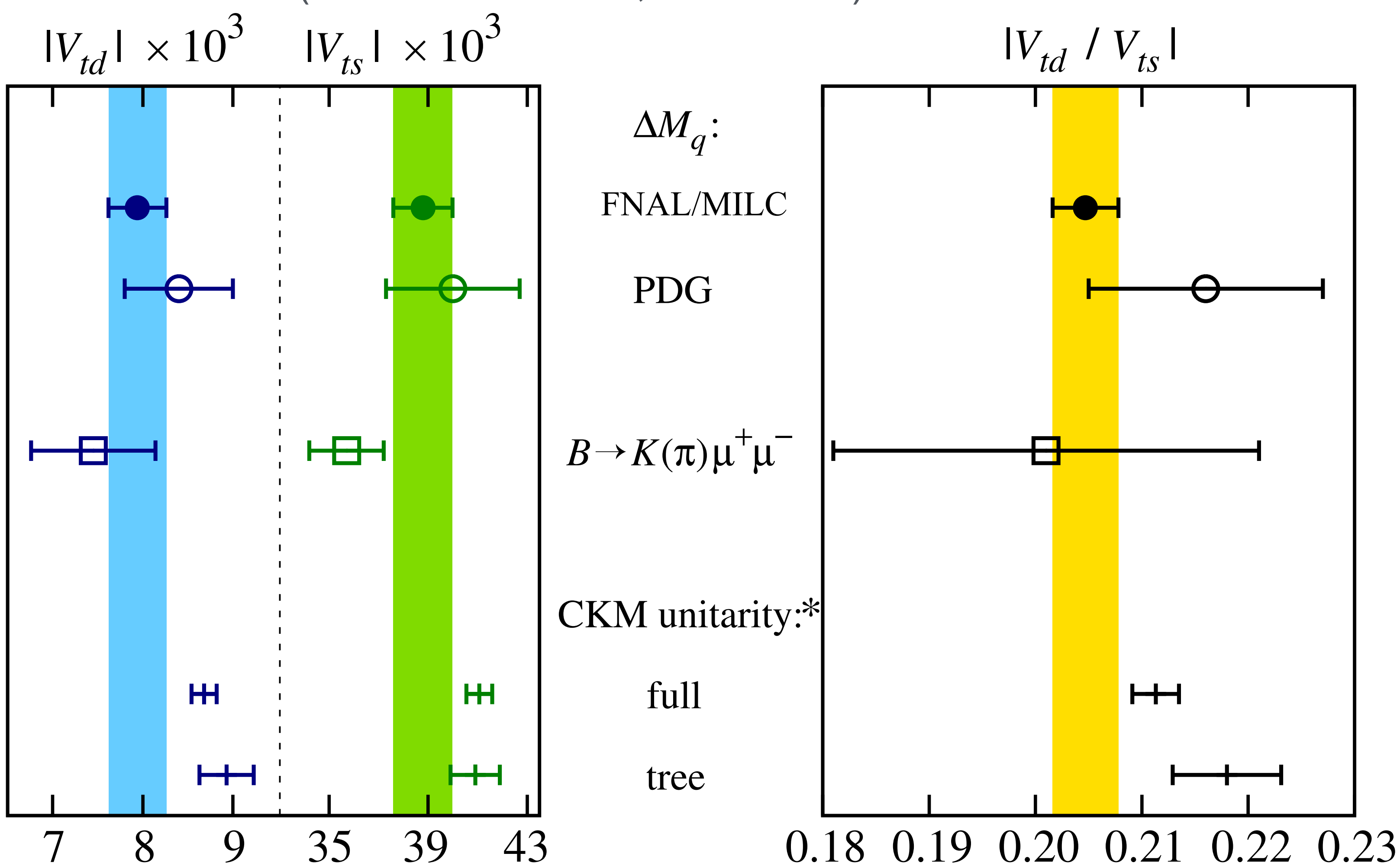
D. Du et al (arXiv:1510.02349, PRD 2016)



- Motivation and introduction
  - for BSM focus (mostly) on loop processes
- LQCD results for
  - semileptonic B meson form factors
  - neutral B meson mixing matrix elements
  - summary of recent progress
- Phenomenology
  - SM pre/post-dictions
  - CKM unitarity & BSM implications
  - Lepton Flavor Universality
- Conclusions & Outlook

B Mixing and FCNC decays

D. Du et al (arXiv:1510.02349, PRD 2016)



Blanke & Buras:
(arXiv:1602.04020, EPJC 2016)

tension between $\Delta M_{s,d}$ & ϵ_K inconsistent with CMFV (Constrained Minimal Flavor Violation)

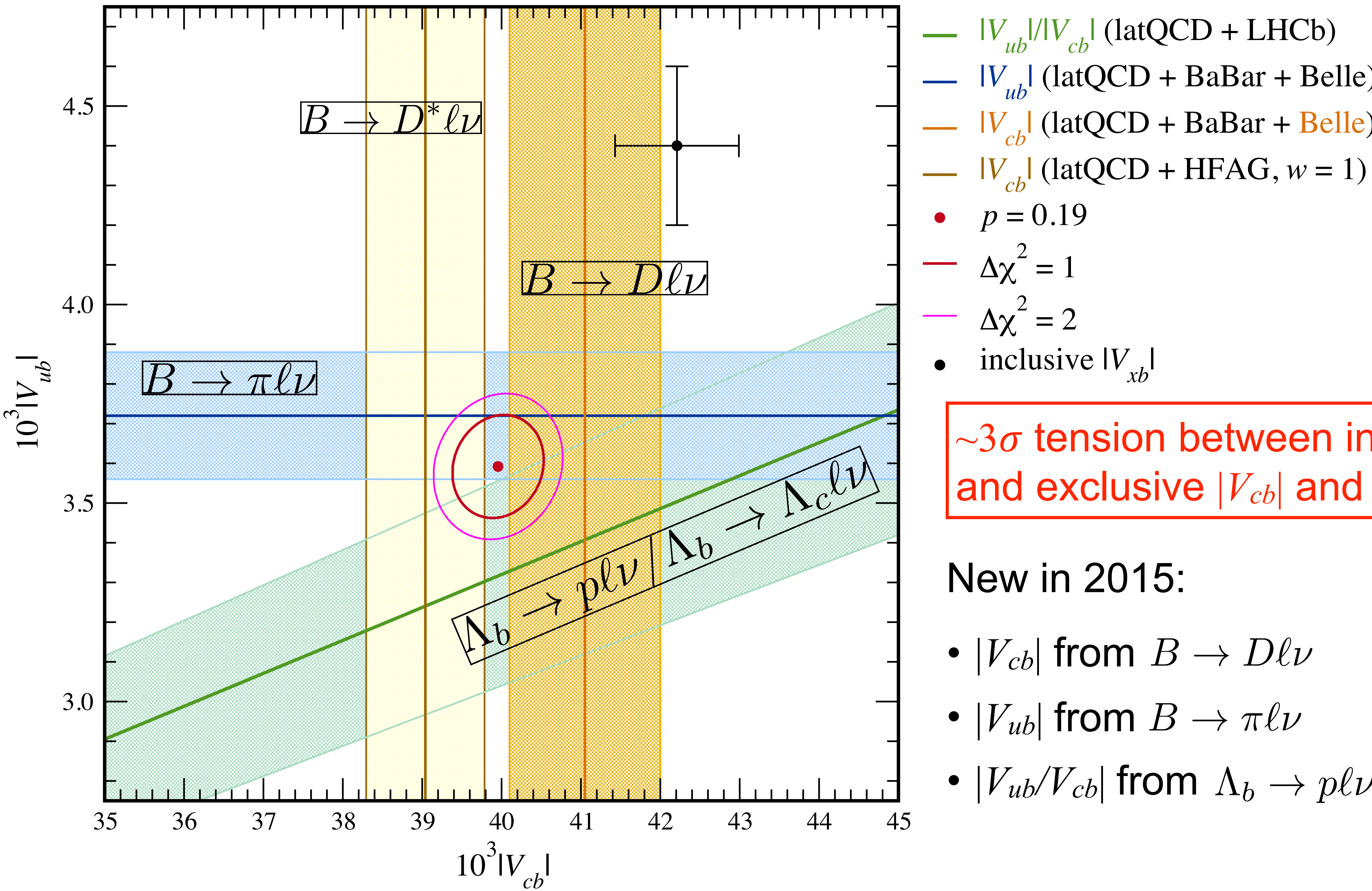
Buras & De Fazio
(arXiv:1604.02334):
implications for "331" models

$\sim 2\sigma$ tensions between loop processes and CKM unitarity.

*from CKMfitter 2015 (hep-ph/0406186, http://ckmfitter.in2p3.fr)

Exclusive vs. inclusive $|V_{cb}|$ and $|V_{ub}|$

A. Kronfeld (priv. communication)



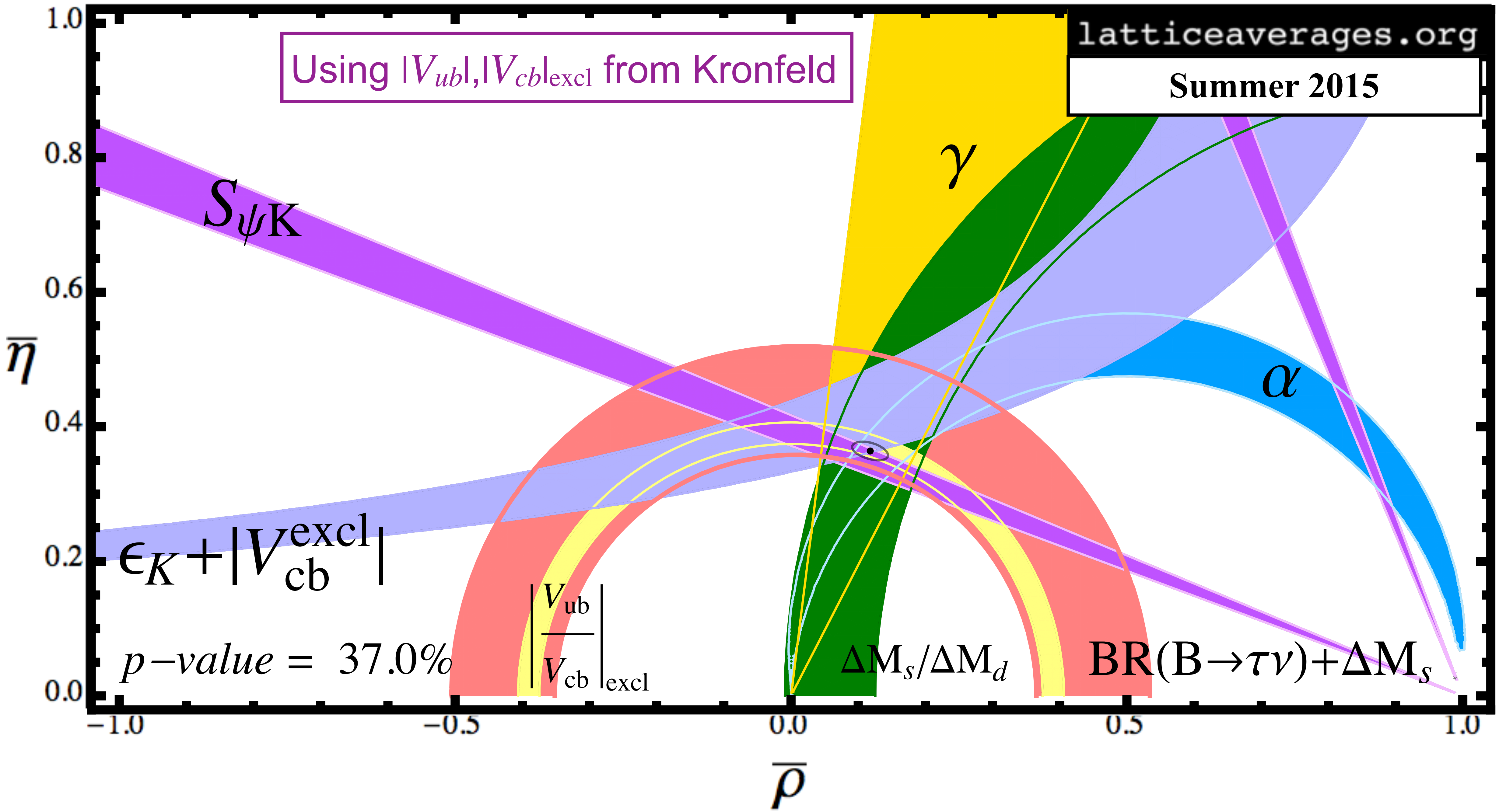
$\sim 3\sigma$ tension between inclusive and exclusive $|V_{cb}|$ and $|V_{ub}|$

New in 2015:

- $|V_{cb}|$ from $B \to D\ell\nu$
- $|V_{ub}|$ from $B \to \pi\ell\nu$
- $|V_{ub}/V_{cb}|$ from $\Lambda_b \to p\ell\nu/\Lambda_b \to \Lambda_c\ell\nu$

UT analysis

Laiho, Lunghi & Van de Water (Phys.Rev.D81:034503,2010), E. Lunghi, private comm.



UT analysis

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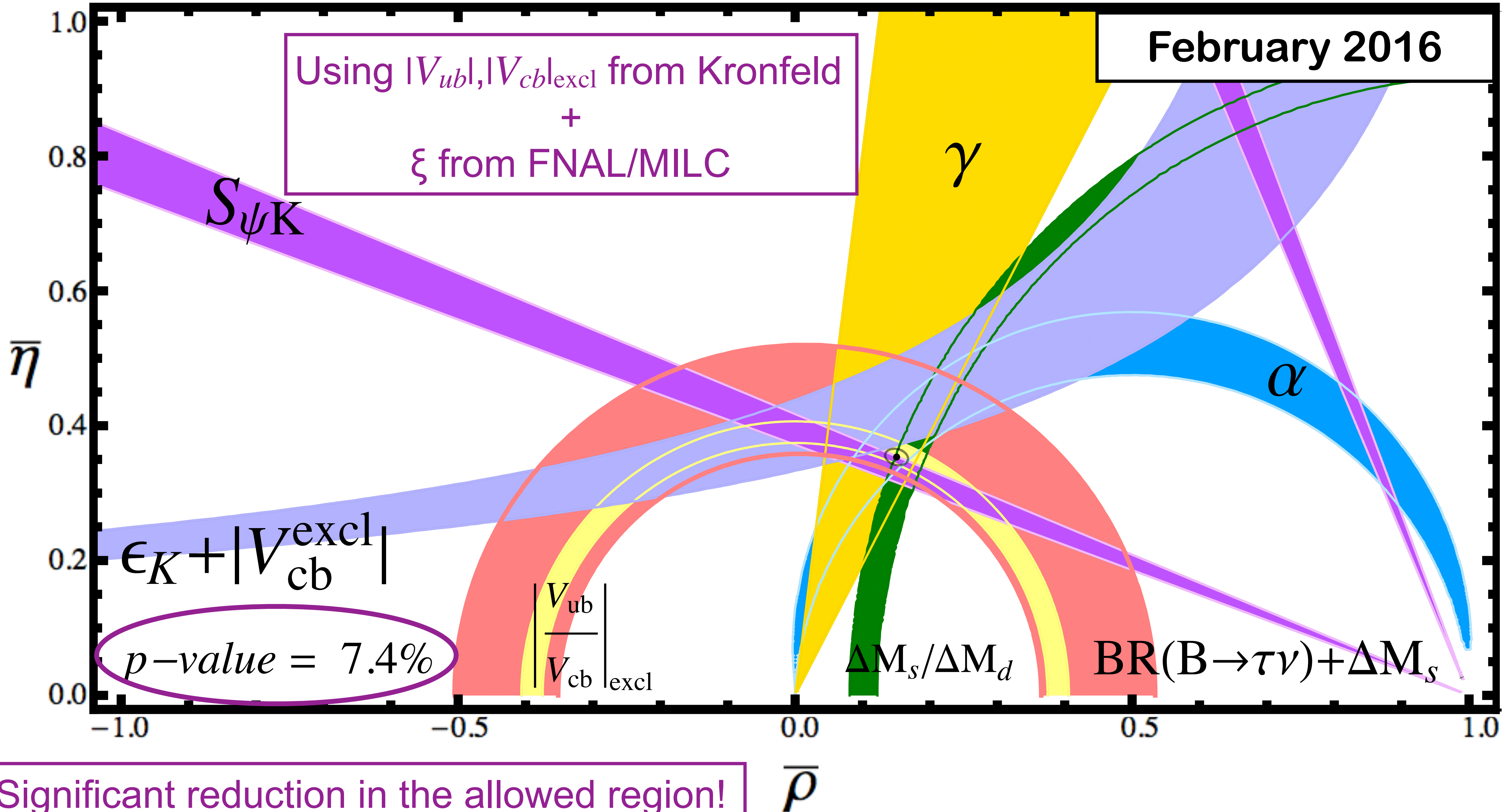


Significant reduction in the allowed region!

BSM phenomenology for $B \to K, \pi\, \ell^+\ell^-$

Constraints on Wilson coefficients (C_9, C_{10})

- New physics contributions modify the Wilson coefficients:

$$C_i \to C_i + C_i^{\mathrm{NP}}$$

at the high scale, $\mu_0 = 120\ \mathrm{GeV}$

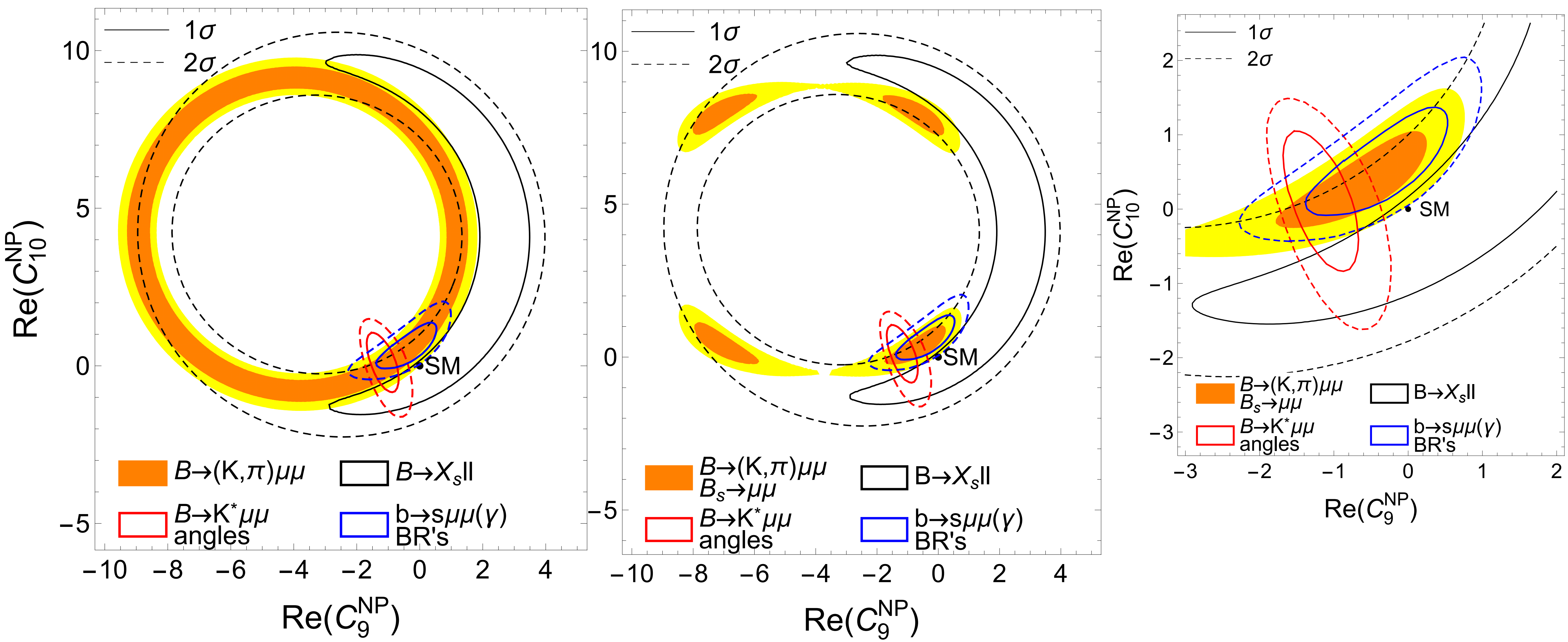
- take $C_{7,8}^{\mathrm{NP}} = 0$ using constraints from $B \to X_s\gamma$
- assume MFV so that $C_i(b \to s\,\ell\ell) = C_i(b \to d\,\ell\ell)$
- assume $C_{9,10}^{\mathrm{NP}}$ are real (no new CP violating phases)
- take measured $\Delta\mathcal{B}(B \to K, \pi\, \mu^+\mu^-)$ in $\Delta q^2 = 1-6, 15-22\ \mathrm{GeV}^2$
- and FNAL/MILC form factors

- add $B_s \to \mu^+\mu^-$ constraint with lattice f_{Bs}

BSM phenomenology for $B \to K, \pi\, \ell^+ \ell^-$

Constraints on Wilson coefficients (C_9, C_{10})

D. Du et al (arXiv:1510.02349, PRD 2016)

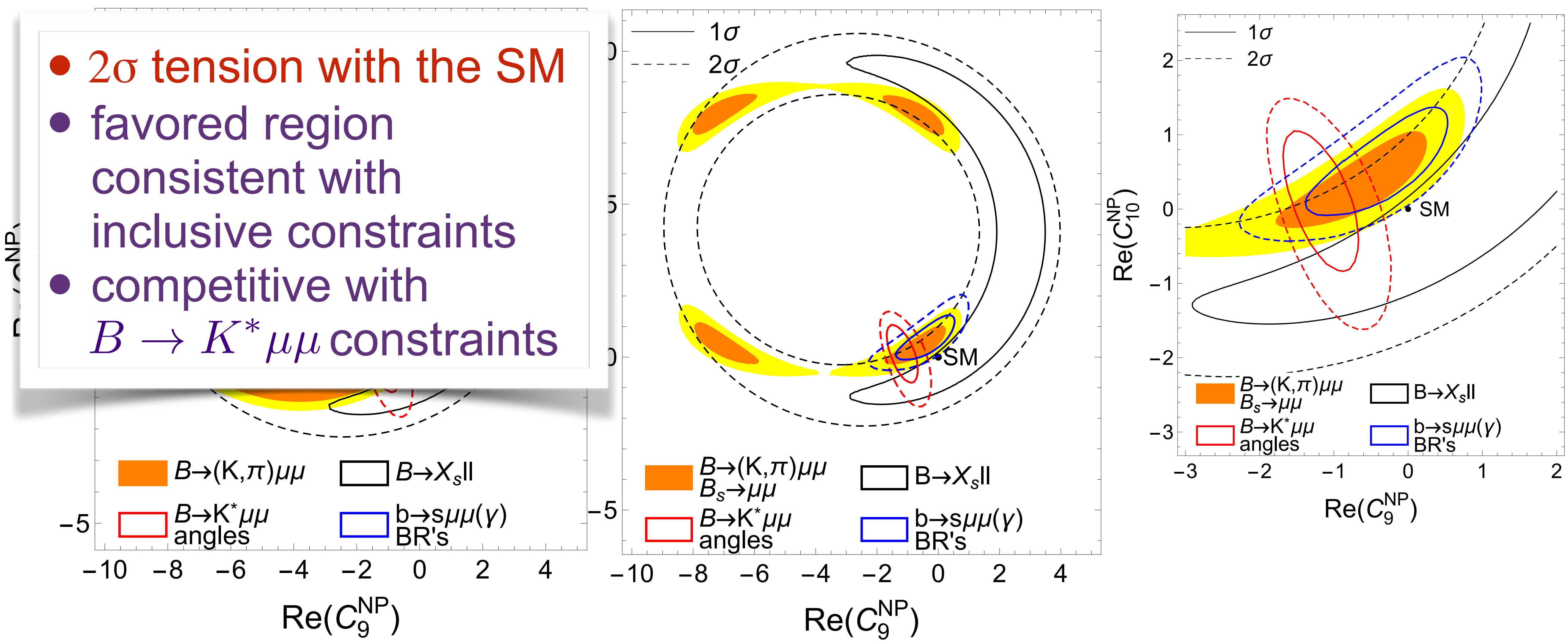


BSM phenomenology for $B \to K, \pi\ \ell^+\ell^-$

Constraints on Wilson coefficients (C_9, C_{10})

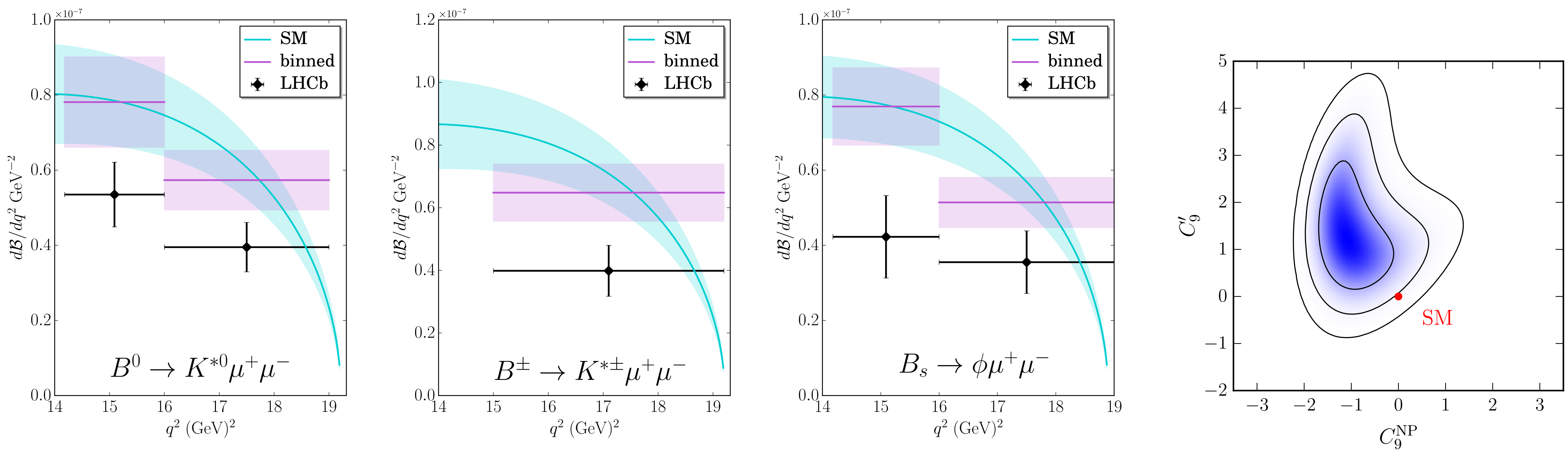
D. Du et al (arXiv:1510.02349, PRD 2016)

- 2σ tension with the SM
- favored region consistent with inclusive constraints
- competitive with $B \to K^* \mu\mu$ constraints



BSM phenomenology for $B \to K^* \mu^+ \mu^-$

Horgan et al (arXiv:1310.3887, PRL 2014; arXiv:1310.3722, PRD 2014, arXiv: 1501.00367)



caveat: K^*, ϕ treated as stable (narrow width approximation)

Theoretical framework for weak decays to resonances, $B \to K\pi\,\ell\ell$ etc… being developed

(Briceño et al, arXiv:1406.5965, PRD 2015; Agadjanov et al, arXiv:1605.03386)

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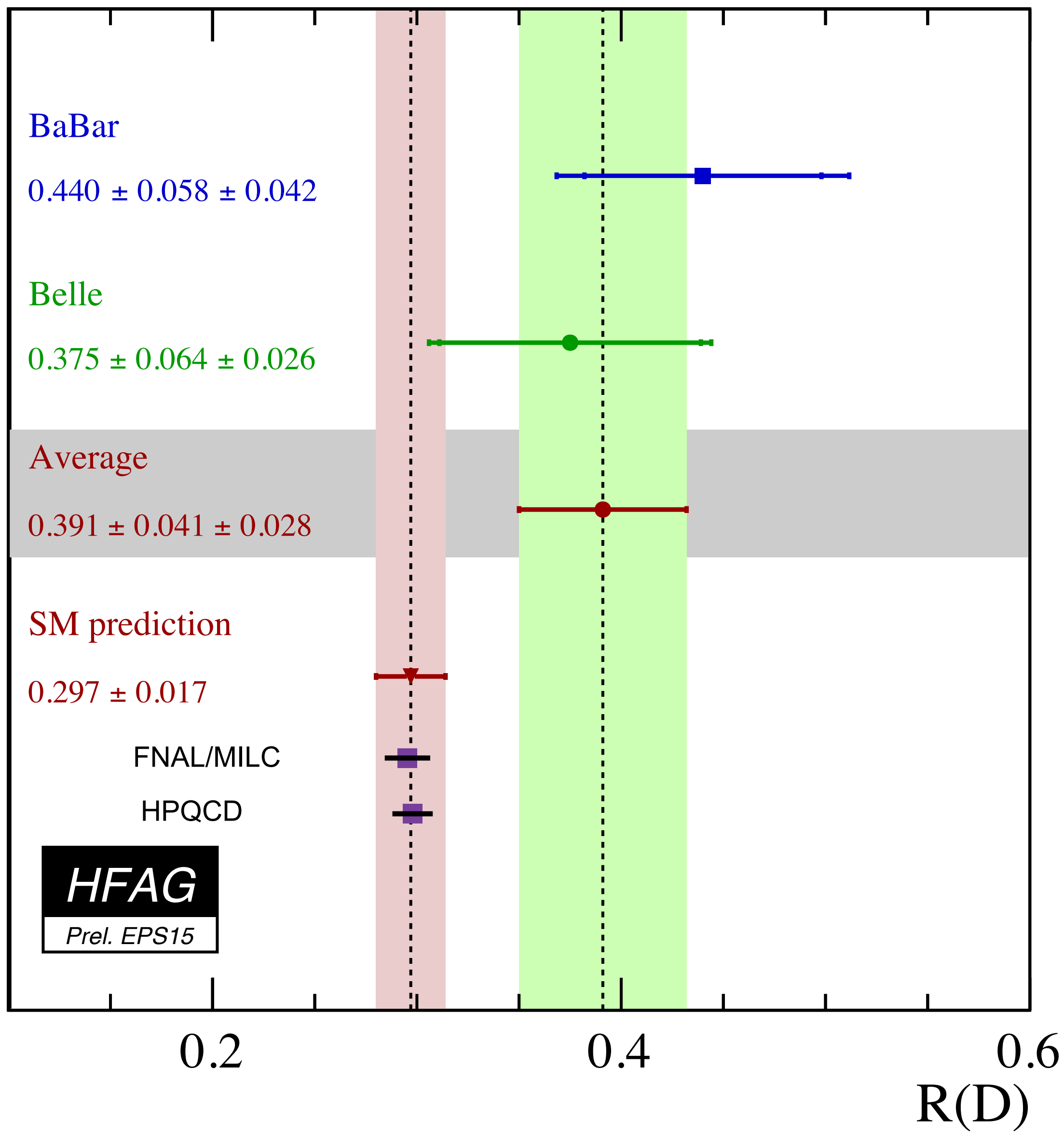
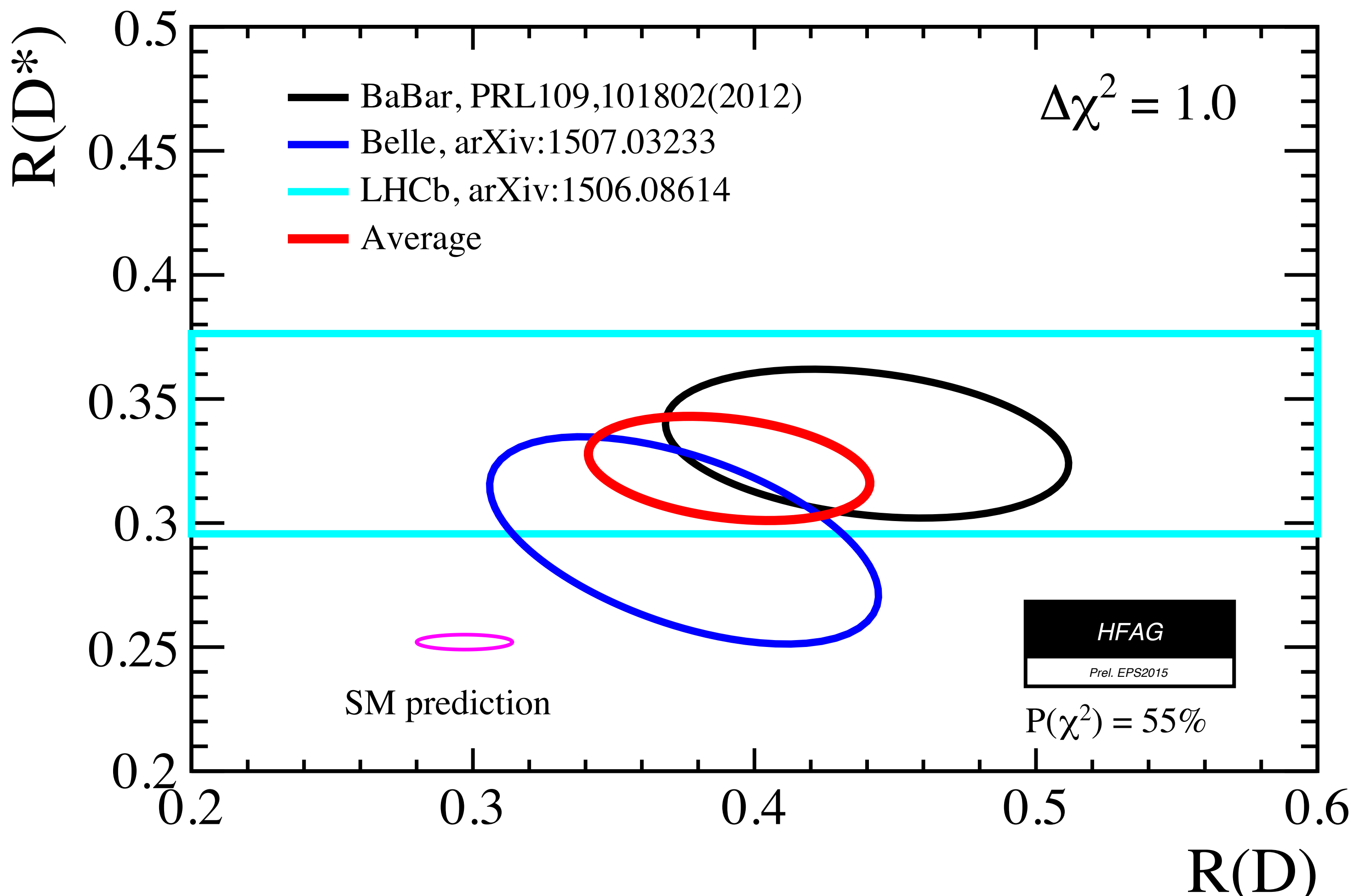
BSM phenomenology: LFU τ/ℓ

$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\nu)}$$

HFAG average for EPS 2015



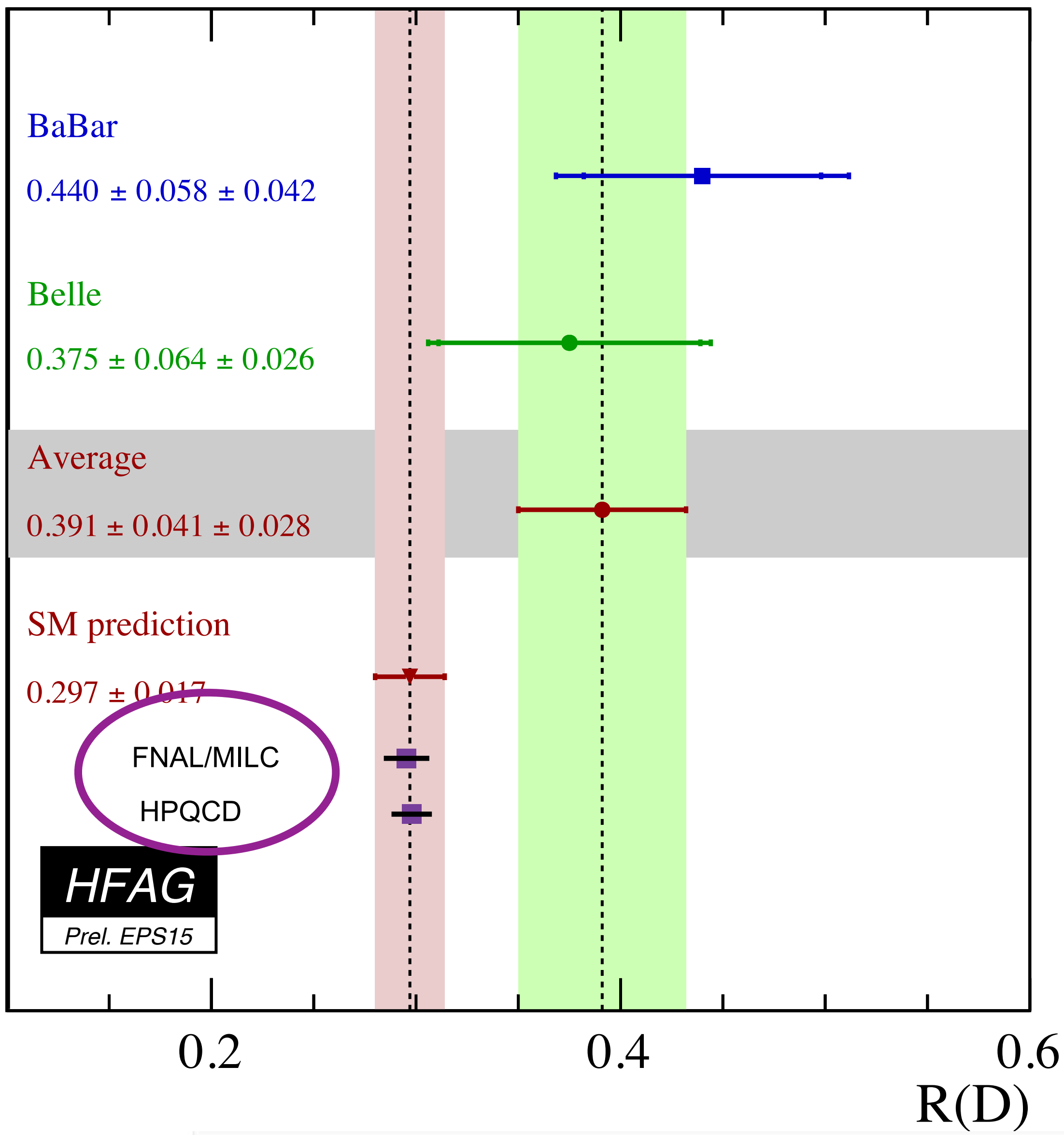


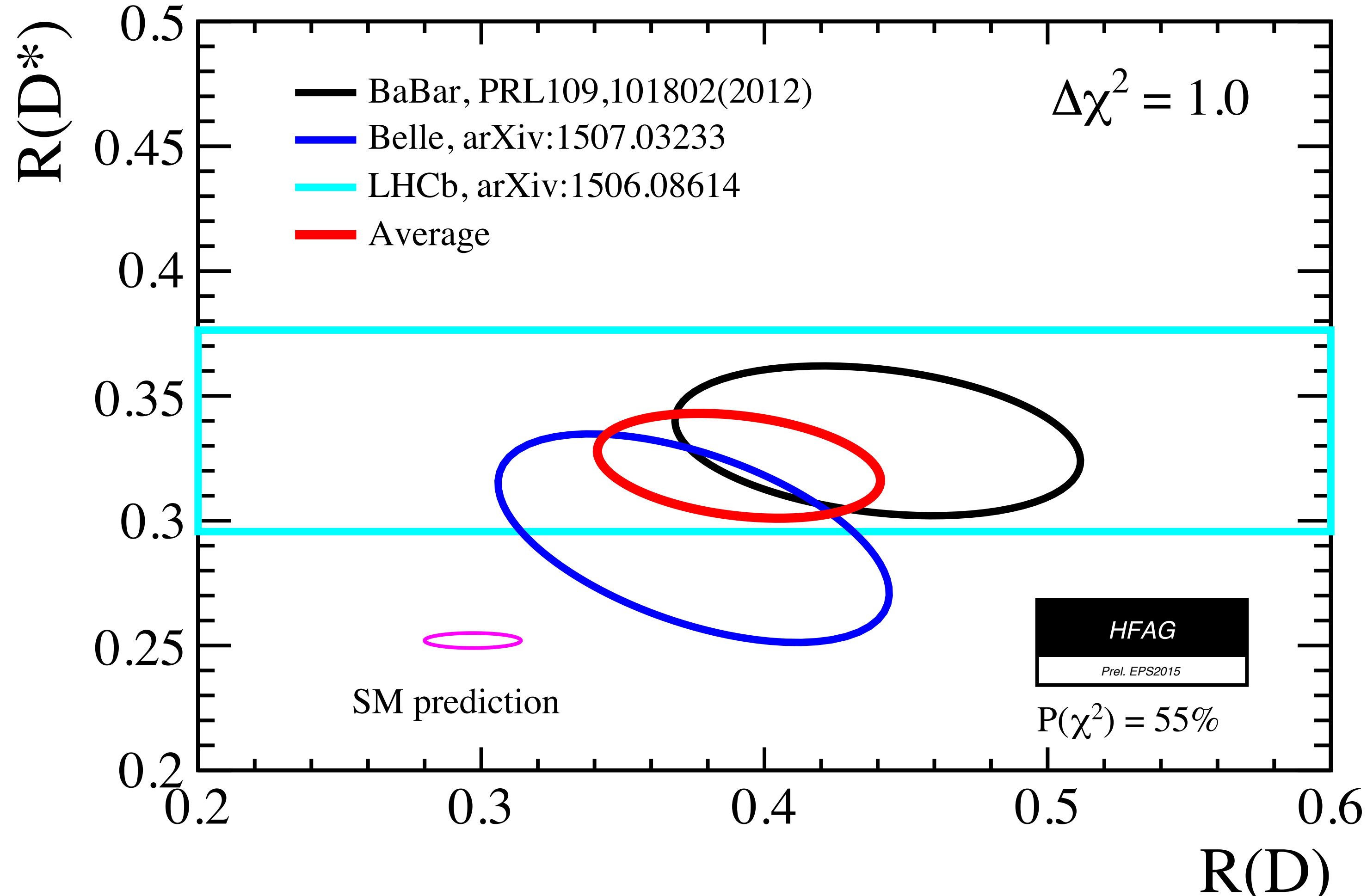
HFAG average: combined 3.9σ excess

BSM phenomenology: LFU τ/ℓ

$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\nu)}$$

HFAG average for EPS 2015

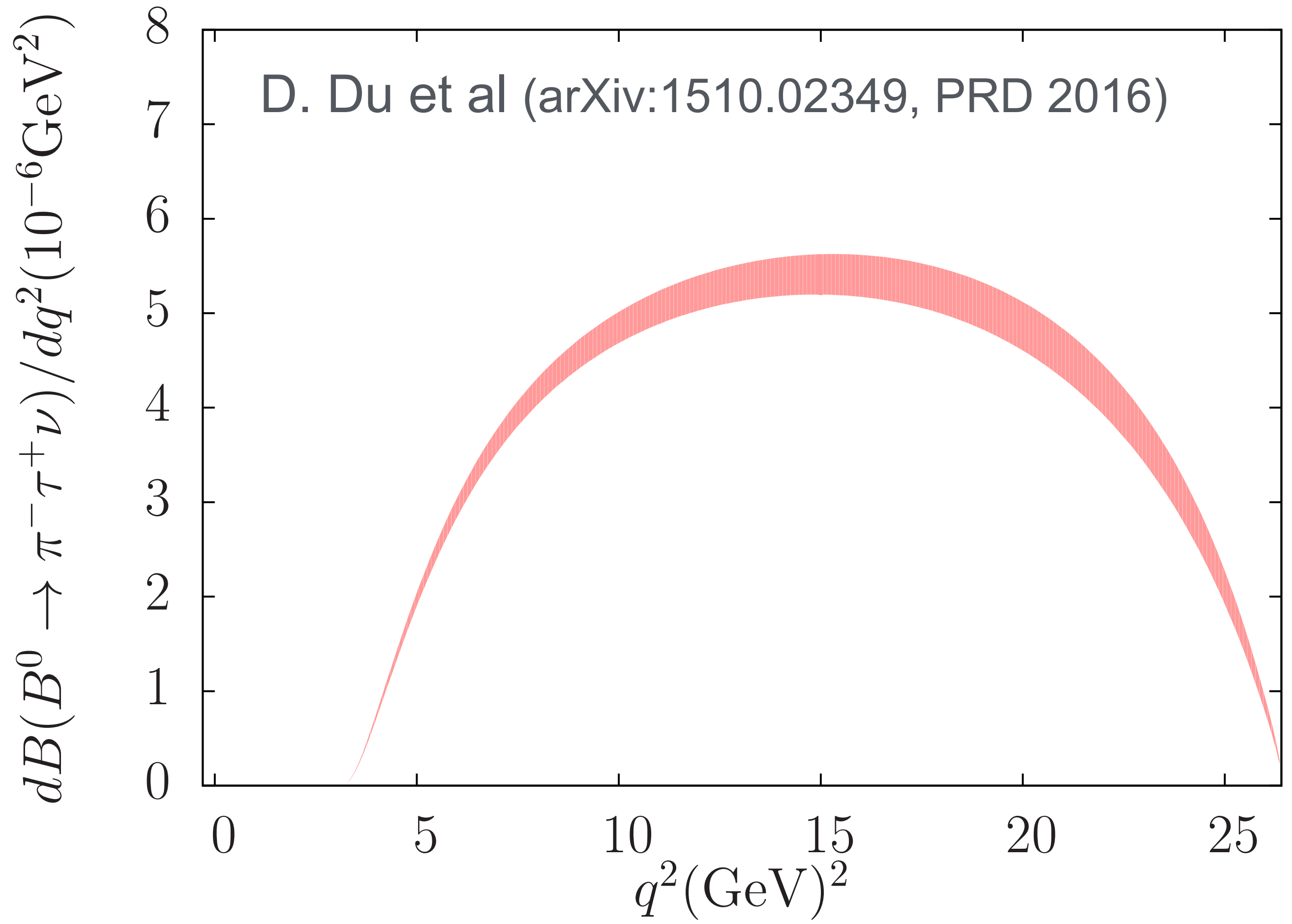




HFAG average: combined 3.9σ excess

still need LQCD form factors for $B \to D^*$ at nonzero recoil for $\mathrm{R}(D^*)$

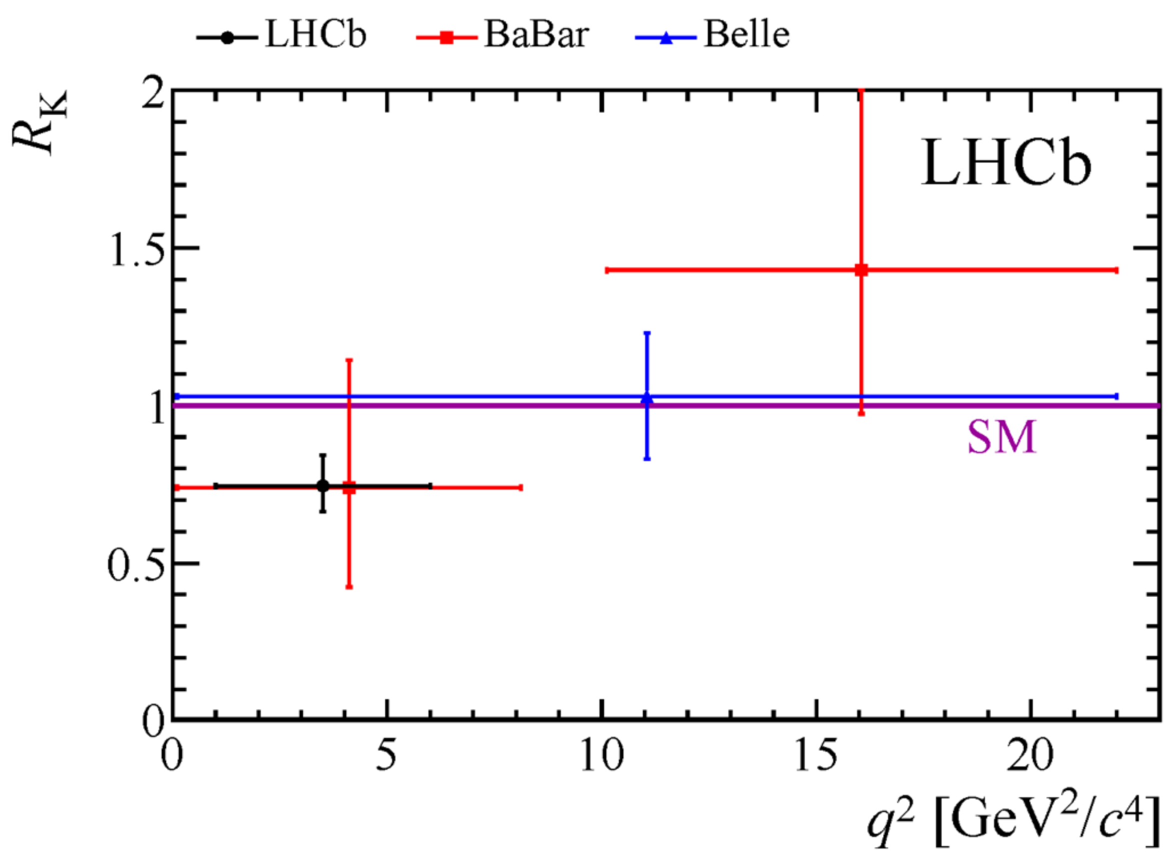
BSM phenomenology: LFU τ/ℓ



SM prediction for $R(\pi) = \dfrac{\mathcal{B}(B \to \pi\,\tau\nu_\tau)}{\mathcal{B}(B \to \pi\,\ell\nu)} = 0.641(17)$

BSM phenomenology: LFU μ/e

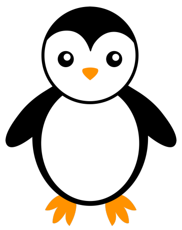
Lepton universality test: $B \to K\mu^+\mu^- / B \to Ke^+e^-$



LHCb (arXiv:1406.6482, PRL 2014):

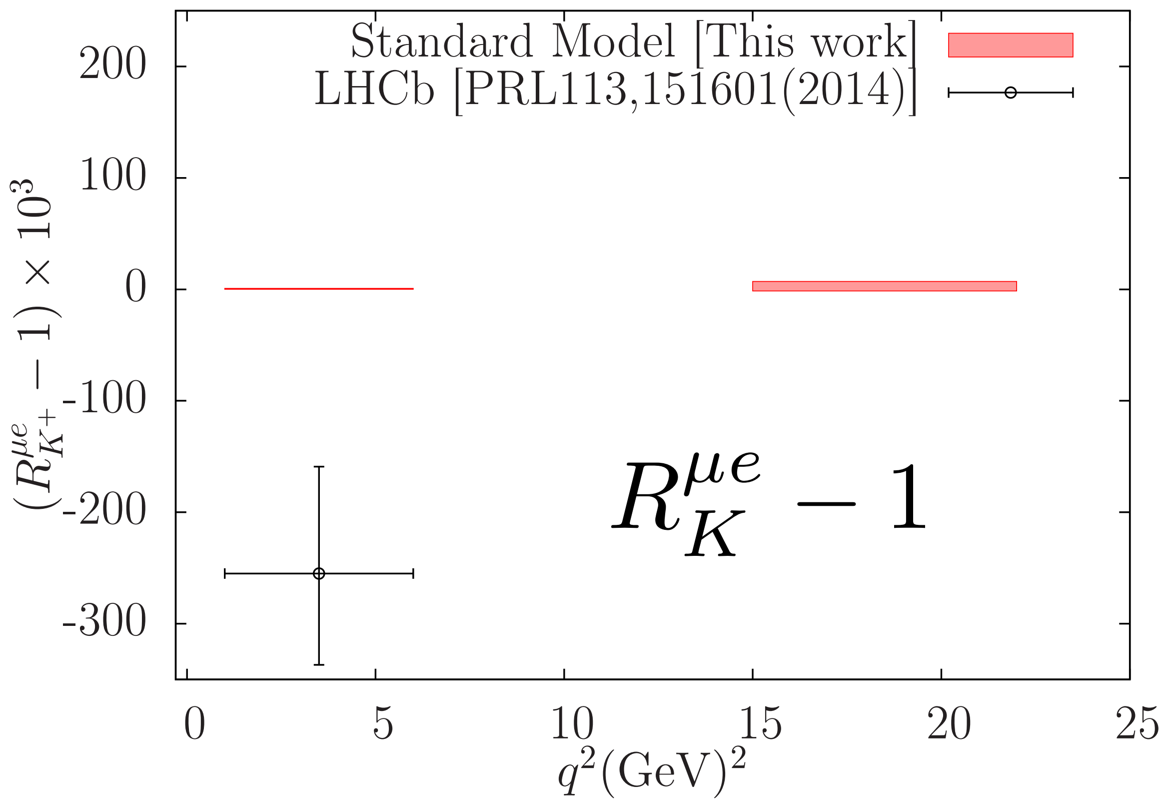
$$\boxed{R_K = 0.745\,\left({}^{90}_{74}\right)(36)}$$

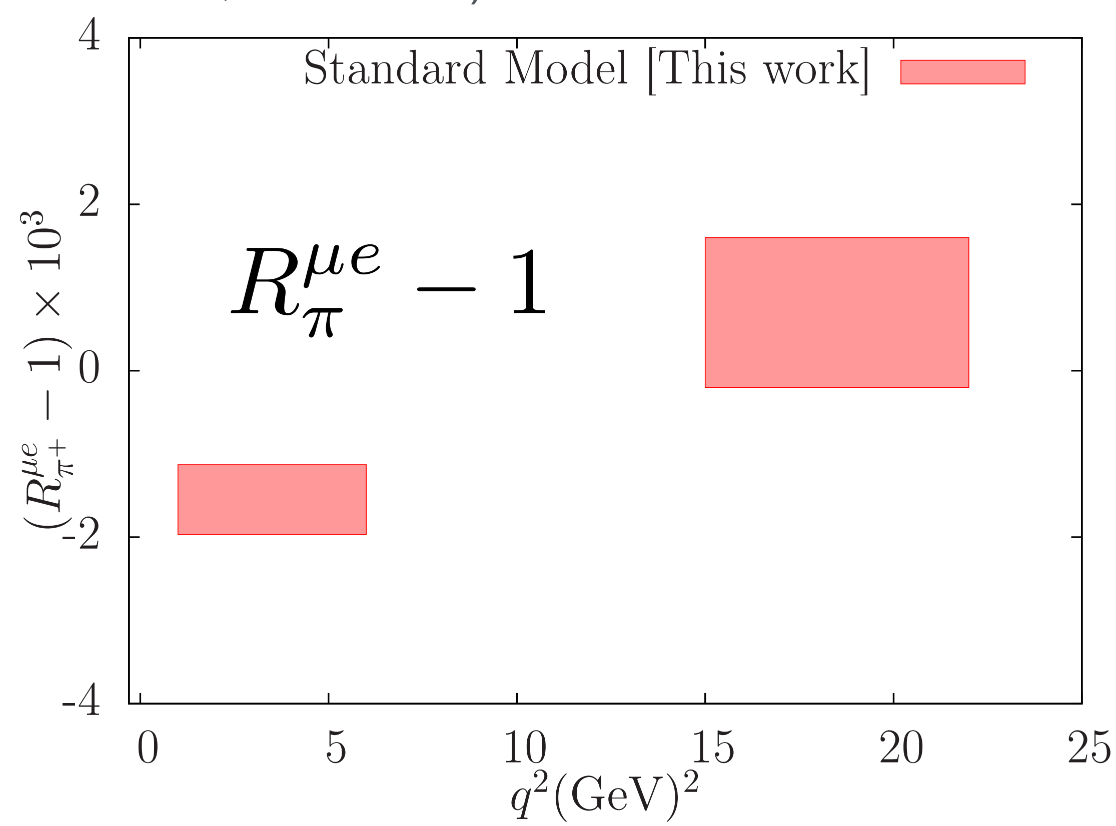
~2.6 σ tension between LHCb measurement and SM theory



BSM phenomenology: LFU μ/e

D. Du et al (arXiv:1510.02349, PRD 2016)





~2.6 σ tension between LHCb measurement and SM theory

In the SM these ratios are insensitive to the form factors
(see also C. Bouchard et al, arXiv:1303.0434, PRL 2013)

Summary

- ☆ Precise LQCD results for semileptonic form factors for $B \to \pi, K, D$ transitions (and $\Lambda_b \to \Lambda, \Lambda_c, p$ transitions)
- ☆ SM pre/postdictions with errors that are commensurate with experimental uncertainties
  - ➠ $\sim 2\sigma$ tensions between exp. and SM theory
- ☆ new LQCD results for neutral B meson mixing matrix elements with significantly smaller uncertainties
  - ➠ emerging $\sim 2\sigma$ tensions between loop processes and CKM unitarity
- ☆ observed LFU violating effects $\sim$2-4σ level:
  - ✦ still need LQCD form factors for $B \to D^*$ at nonzero recoil
  - ✦ also needed for exclusive $|V_{cb}|$ determination


Thank you!
Farah Willenbrock

Backup slides

Introduction to Lattice QCD

$$\langle \mathcal{O} \rangle \sim \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \, \mathcal{O}(\psi, \bar{\psi}, A) \, e^{-S} \qquad S = \int d^4x \left[\bar{\psi}(\not{D} + m)\psi + \frac{1}{4}(F^a_{\mu\nu})^2 \right]$$

use monte carlo methods (importance sampling) to evaluate the integral.

Note: Integrating over the fermion fields leaves $\det(\not{D} + m)$ in the integrand. The correlation functions, $\mathcal{O}$, are then written in terms of $(\not{D}+m)^{-1}$ and gluon fields.

steps of a lattice QCD calculation:

1. generate gluon field configurations according to $\det(\not{D}+m)\, e^{-S}$
2. calculate quark propagators, $(\not{D}+m_q)^{-1}$, for each valence quark flavor and source point
3. tie together quark propagators into hadronic correlation functions (usually 2 or 3-pt functions)
4. statistical analysis to extract hadron masses, energies, hadronic matrix elements, …. from correlation functions
5. systematic error analysis

systematic error analysis

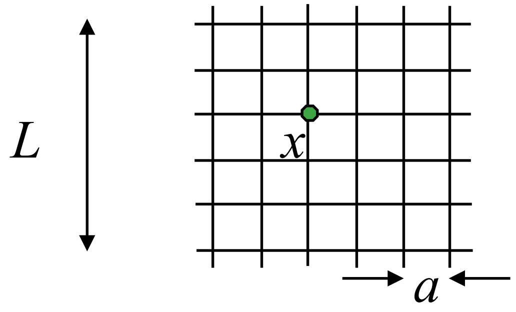
...of lattice spacing, chiral, and finite volume effects is based on EFT (Effective Field Theory) descriptions of QCD → ab initio

The EFT description:

- provides functional form for extrapolation (or interpolation)
- can be used to build improved lattice actions/methods
- can be used to anticipate the size of systematic effects

systematic error analysis

discretization effects

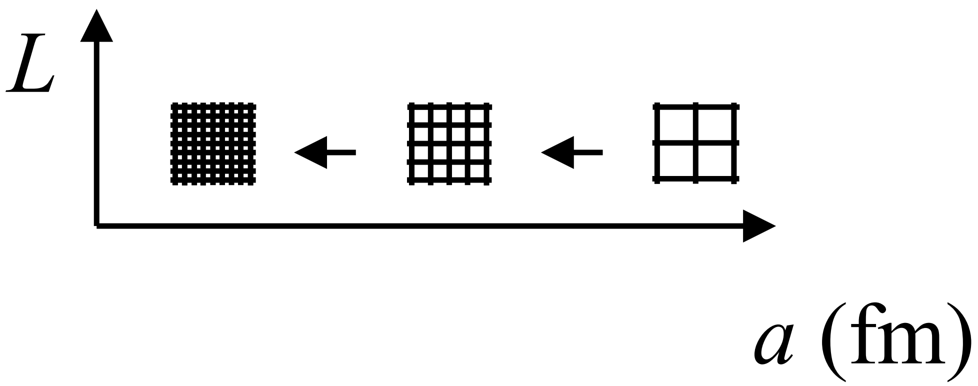


discrete space-time ➙ discrete QCD action

Symanzik EFT: $\langle \mathcal{O} \rangle^{\mathrm{lat}} = \langle \mathcal{O} \rangle^{\mathrm{cont}} + O(ap)^n$

p is the typical momentum scale associated with $\langle \mathcal{O} \rangle$

for light quark systems, $p \sim \Lambda_{\mathrm{QCD}}$

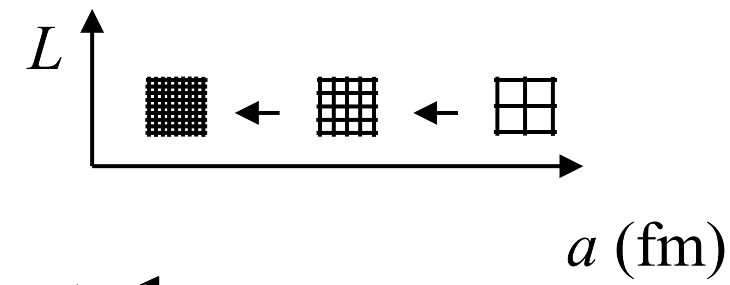


The form of $O(ap)^n$ depends on the details of the lattice action.

All modern light-quark actions start at $n = 2$
(improved Wilson, twisted-mass Wilson, asqtad, HISQ, Domain Wall, Overlap, ...).

systematic error analysis

discretization effects for b quarks



- If we use light quark actions for heavy quarks,
  discretization errors ~ $O(am_h)^2$,
  with currently available lattice spacings
  for charm $\boldsymbol{am_c \sim 0.15\text{-}0.6}$ and for b: $\boldsymbol{am_b > 1}$

➡ need effective field theory methods for b quarks
for charm lattice spacings are sufficiently small so that we can use improved light quark methods

- avoid errors of $(am_b)^2$ by using EFT in the formulation/matching of lattice action/currents:
  - ✦ relativistic HQ actions (Fermilab, Columbia, Tsukuba)
  - ✦ HQET
  - ✦ NRQCD

or

- use the same improved light quark action as for charm (HISQ, twisted mass Wilson, NP imp. Wilson, Overlap, ...)
  - ✦ keep $am_h < 1$
  - ✦ use HQET and/or static limit to extrapolate to the physical b quark mass

systematic error analysis

light quark mass effects

Simulations with $m_{\text{light}} = \ 1/2\ (m_u + m_d)$ **at the physical $\mathbf{u/d}$ quark masses are now available**, but many results still have

$$m_{\text{light}}\ >\ 1/2\ (m_u + m_d)_{\text{phys}}$$

χPT can be used to extrapolate/interpolate to the physical point.

- Can include discretization effects (for example, staggered χPT)
- It is now common practice to perform a combined continuum-chiral extrapolation/interpolation

systematic error analysis

finite volume effects

One stable hadron (meson) in initial/final state:

If L is large enough, FV error $\sim e^{-m_\pi L}$

- keep $m_\pi L \gtrsim 4$

To quantify residual error:

- include FV effects in CPT
- compare results at several Ls (with other parameters fixed)

The story changes completely with two or more hadrons in initial/final state!
(or if there are two or more intermediate state hadrons)

systematic error analysis

other effects

✓ statistical errors: from monte carlo integration
consider/include systematic errors from correlator fit procedure

✓ n_f dependence: realistic sea quark effects: use $n_f = 2+1$ or $n_f = 2+1+1$
Note: $n_f = 2$ (effects due to quenching the strange quark appear to be small)

❖ renormalization (and matching):
⇒ with lattice perturbation theory: need to include PT errors
⇒ nonperturbative methods
⇒ use absolutely normalized currents where possible

systematic error analysis

...of lattice spacing, chiral, and finite volume effects is based on EFT (Effective Field Theory) descriptions of QCD → ab initio

The EFT description:

- provides functional form for extrapolation (or interpolation)
- can be used to build improved lattice actions/methods
- can be used to anticipate the size of systematic effects

To control and reliably estimate the systematic errors

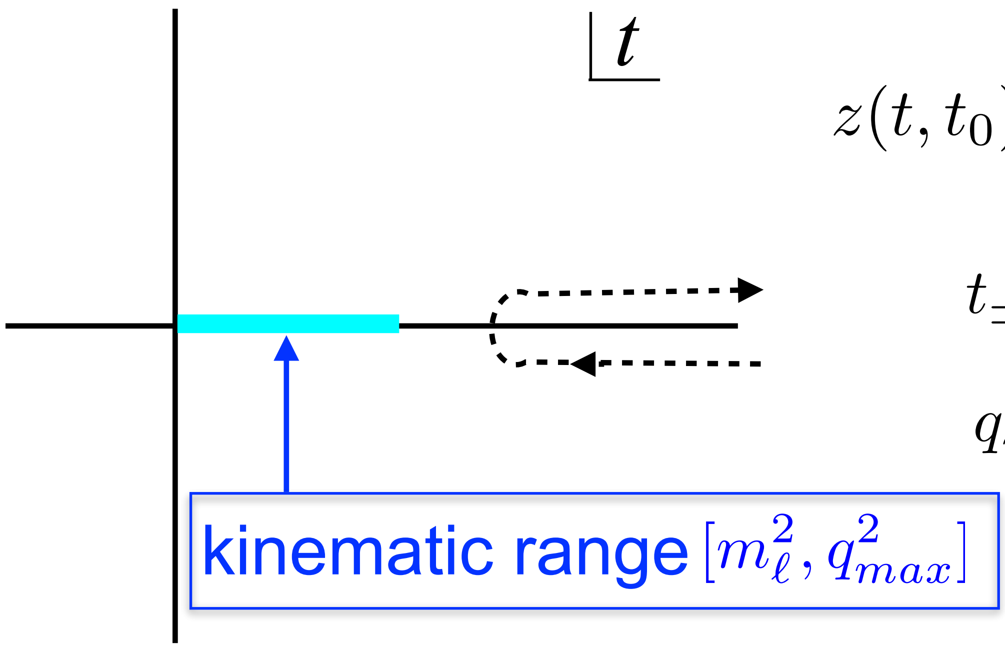
- repeat the calculation on several lattice spacings, light quark masses, spatial volumes, ...

CKM determinations

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \to \mu\nu & \begin{array}{c} K \to \pi \ell\nu \\ K \to \mu\nu \end{array} & \begin{array}{c} B \to \pi \ell\nu,\ B_s \to K \ell\nu \\ \Lambda_b \to p\, \ell\nu \end{array} \\ V_{cd} & V_{cs} & V_{cb} \\ \begin{array}{c} D \to \pi \ell\nu \\ D \to \ell\nu \end{array} & \begin{array}{c} D \to K \ell\nu \\ D_s \to \ell\nu \end{array} & B_{(s)} \to D_{(s)}, D^*_{(s)}\ \ell\nu \\ V_{td} & V_{ts} & V_{tb} \\ \begin{array}{c} B^0 - \overline{B^0} \\ B \to \pi \ell\ell \end{array} & \begin{array}{c} B^0_s - \overline{B^0_s} \\ B \to K \ell\ell \end{array} & \end{pmatrix}$$

(ρ, η) $K^0 - \overline{K^0}$

The z-expansion

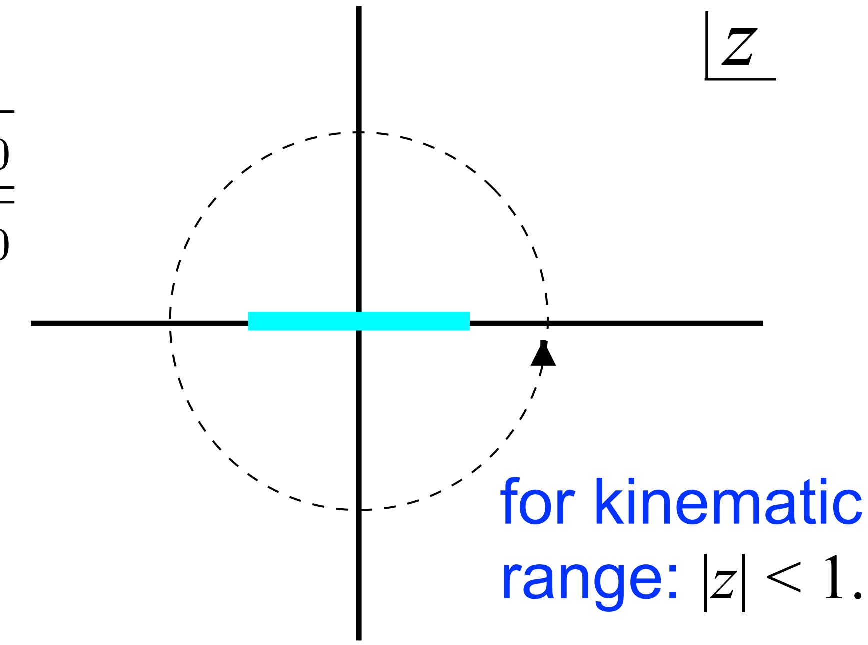


$$t = q^2$$

$$z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

$$t_\pm = (m_B \pm m_\pi)^2$$

$$q^2_{max} = t_-$$



Bourrely at al (Nucl.Phys. B189 (1981) 157)
Boyd et al (hep-ph/9412324,PRL 95)
Lellouch (arXiv:hep- ph/9509358, NPB 96)
Boyd & Savage (hep-ph/9702300, PRD 97)
Bourrely at al (arXiv:0807.2722, PRD 09)

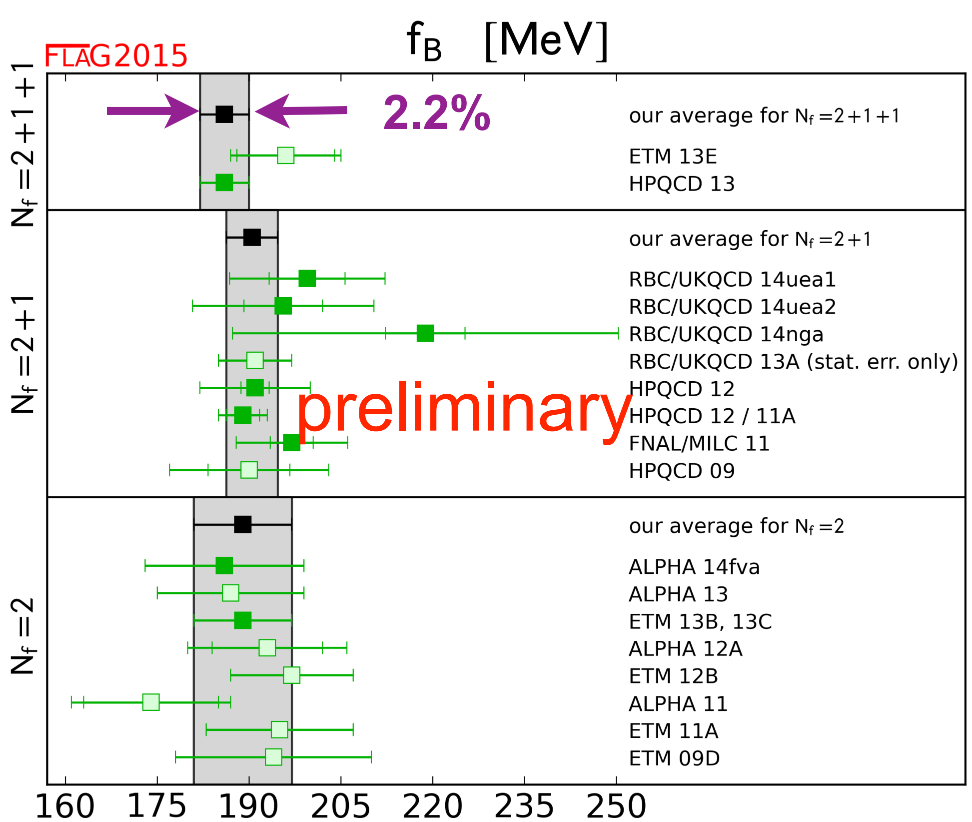
The form factor can be expanded as:

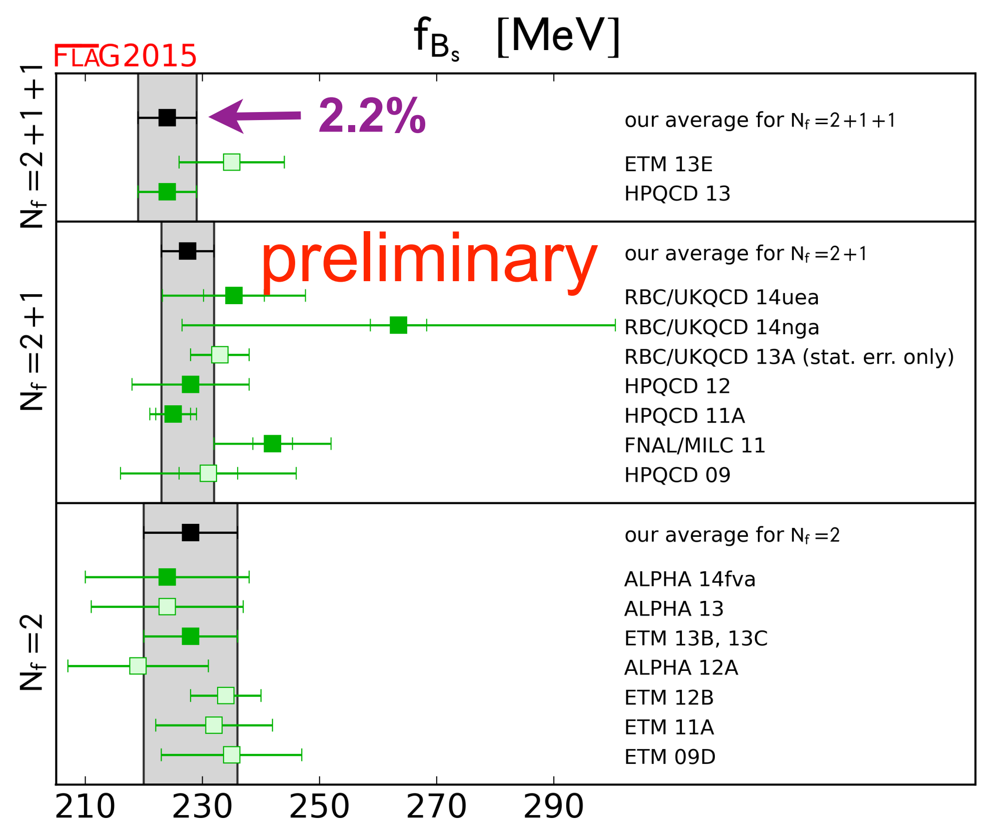
$$f(t) = \frac{1}{P(t)\phi(t, t_0)} \sum_{k=0} a_k(t_0) z(t, t_0)^k$$

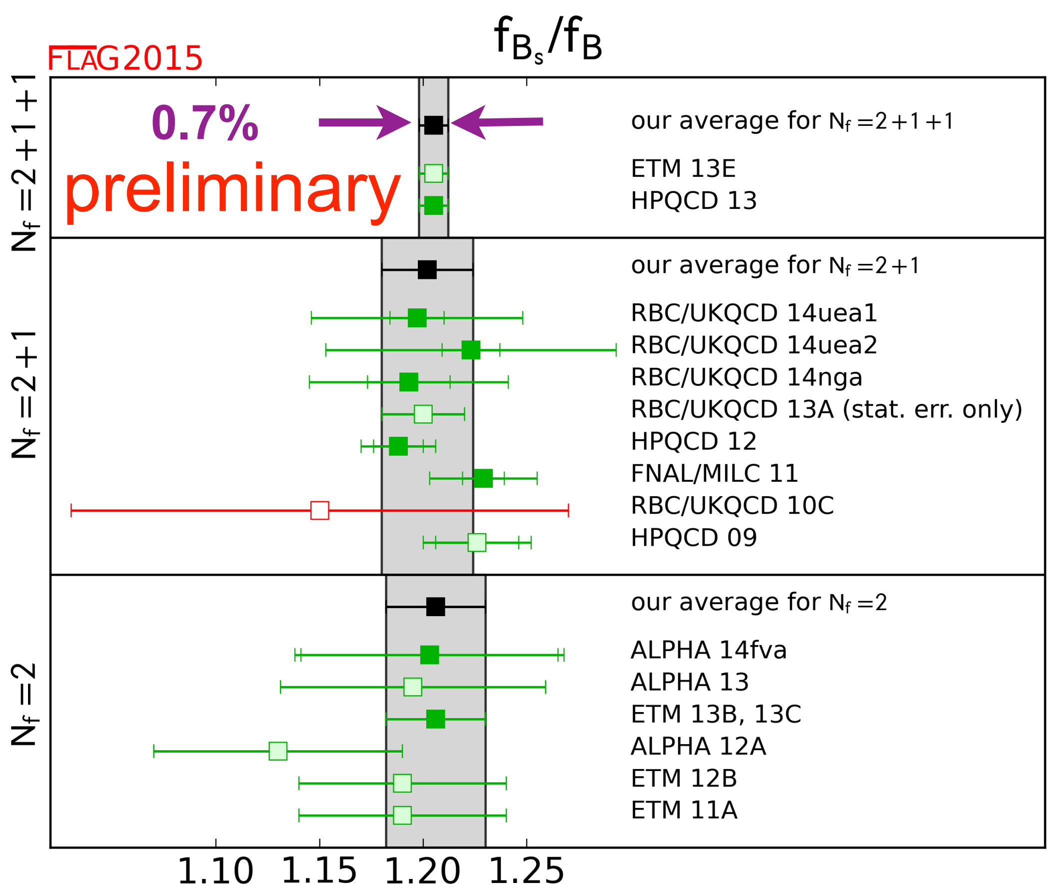
- $P(t)$ removes poles in $[t_-, t_+]$
- The choice of outer function ϕ affects the unitarity bound on the a_k.
- In practice, only first few terms in expansion are needed.

B meson decay constant summary

S. Aoki et al (FLAG-2 review, arXiv:1310.8555, FLAG-3 update)





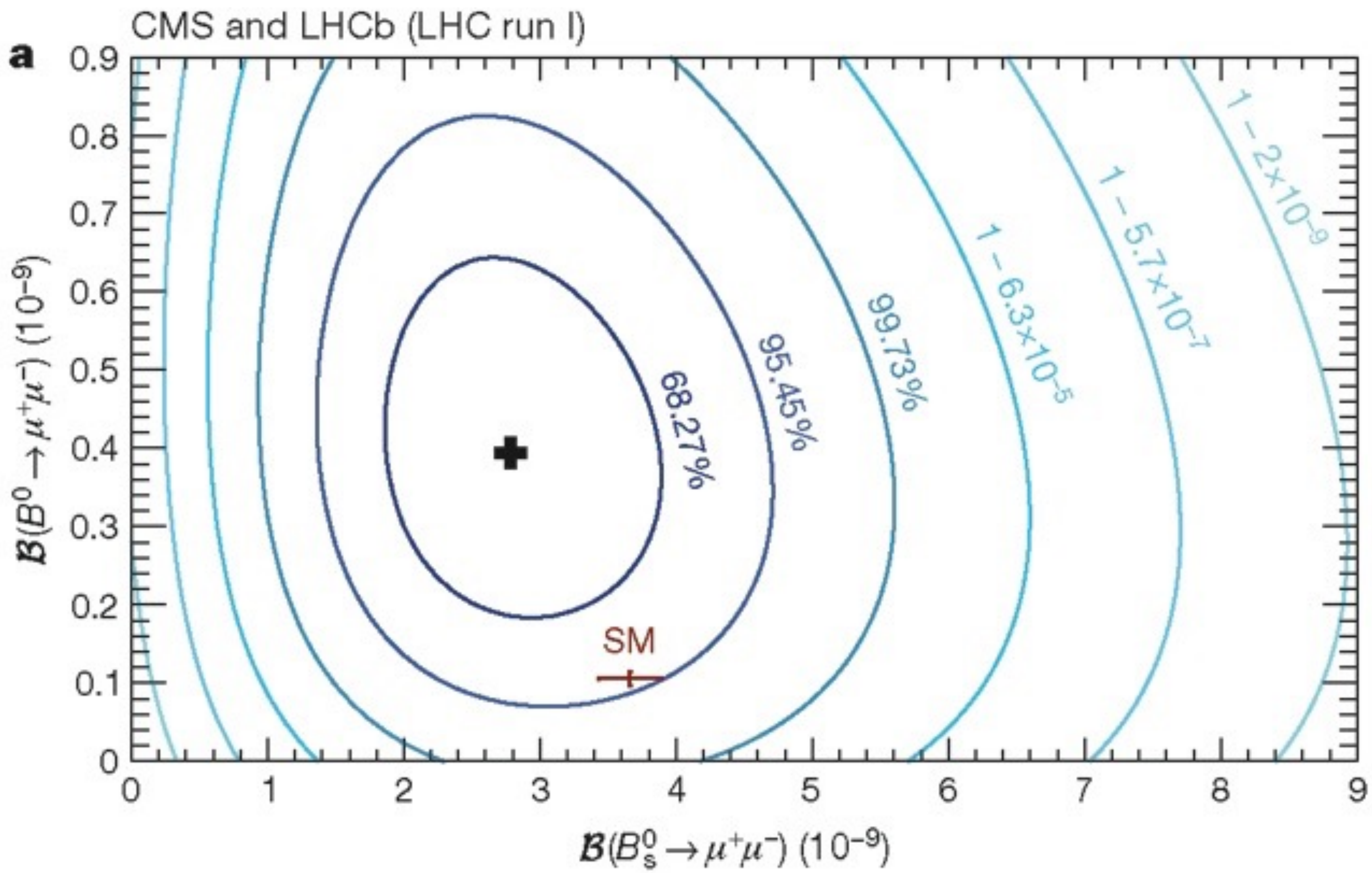


courtesy of
M. Della Morte
(HQ working group)

preliminary
status
mid 2015

BSM phenomenology $B_{s(d)} \to \mu^+\mu^-$

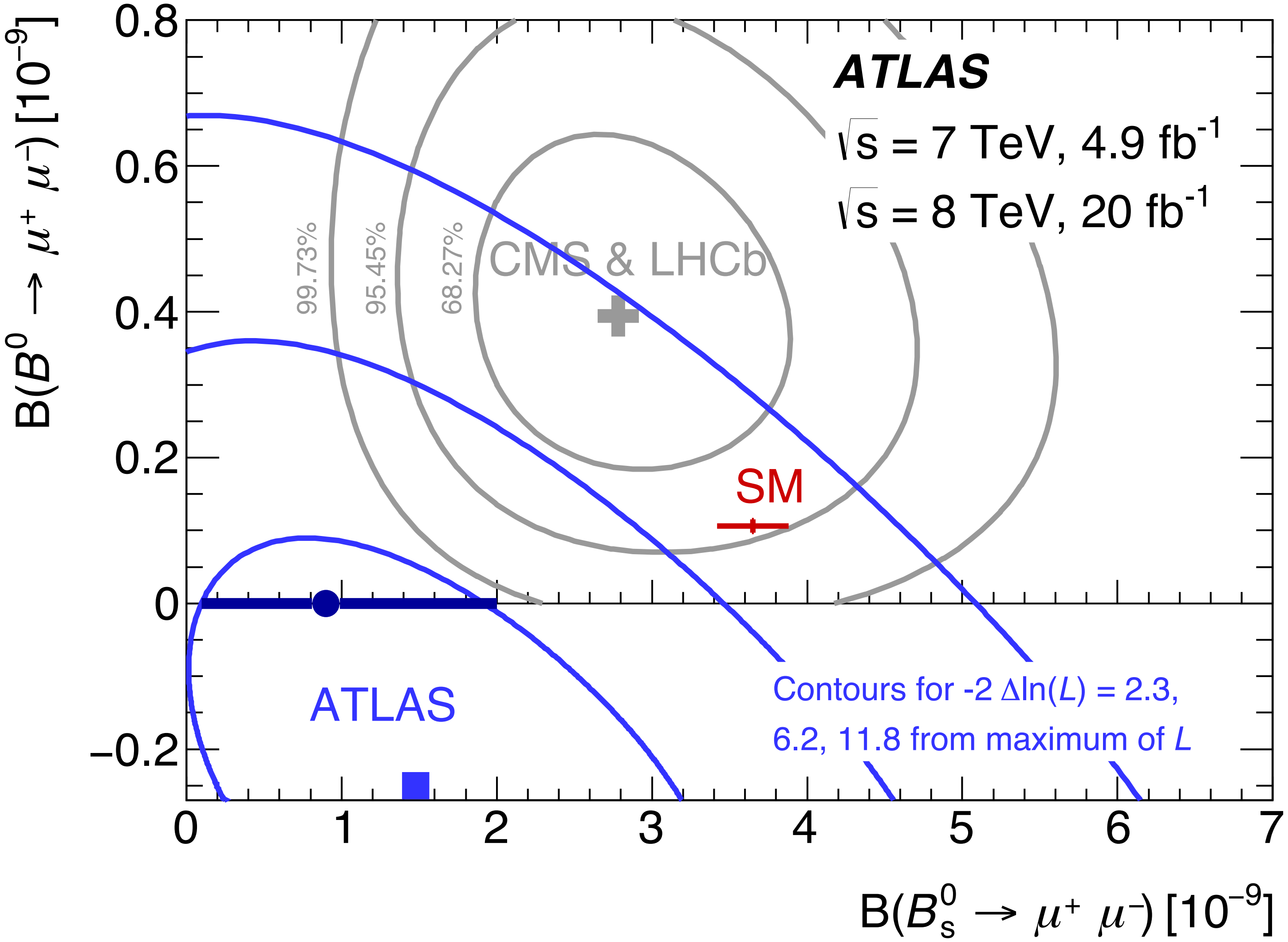
CMS+LHCb combined (arXiv:1411.4413, Nature 2015)



SM predictions depend on $f_{B(s)}$ or $\hat{B}_{B_s}$

BSM phenomenology $B_{s(d)} \to \mu^+\mu^-$

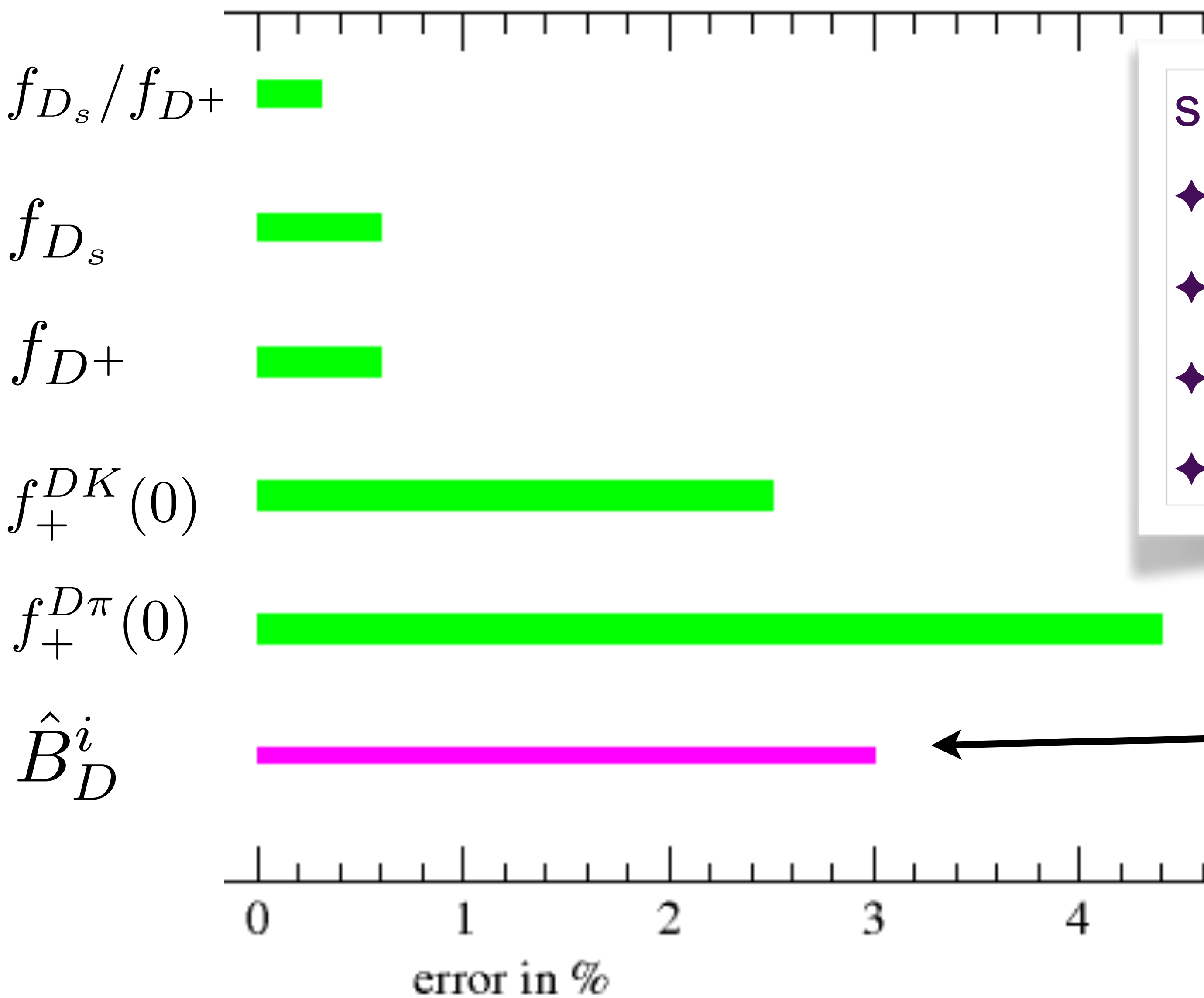
CMS+LHCb combined (arXiv:1411.4413, Nature 2015) and ATLAS (arXiv:1604.04263)



SM predictions depend on $f_{B(s)}$ or $\hat{B}_{B_s}$

D meson summary

errors (in %) **(preliminary) FLAG-3 averages** + **new results**



small errors due to

- ✦ **physical light quark masses** ($f_{D(s)}$)
- ✦ improved charm-quark action
- ✦ ensembles with small lattice spacings
- ✦ PCAC or NPR

- **First results for D mixing bag parameters** (all five) of local operators by ETM (2013, 2014) $n_f = 2,\ 2{+}1{+}1$

- work in progress: FNAL/MILC (J. Chang thesis), see backup slides