MATRIX PRODUCT STATES FOR LATTICE GAUGE THEORIES

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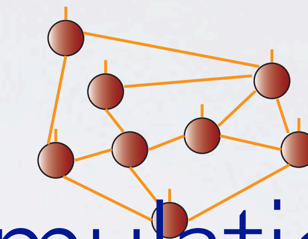
Max-Planck-Institut
für Quantenoptik
(Garching b. München)

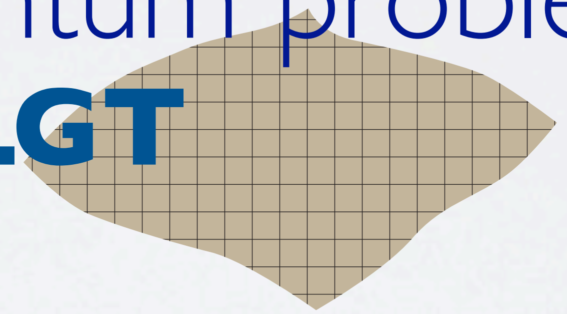
IAS (TUM) 19.5.2016



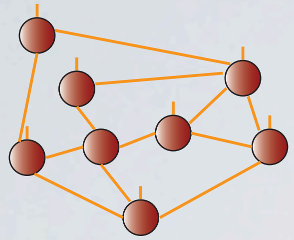
TNS

about classical simulations of a quantum problem

LGT



WHY?



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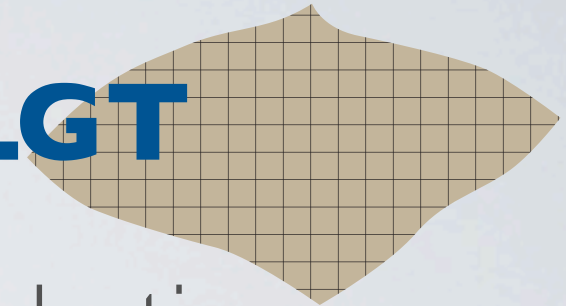


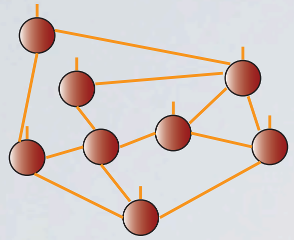
Non-perturbative way of solving QFT (QCD)

Mostly path-integral formalism & MC

4D lattice

spectrum
finite T
32^3x64
chemical potential
time evolution

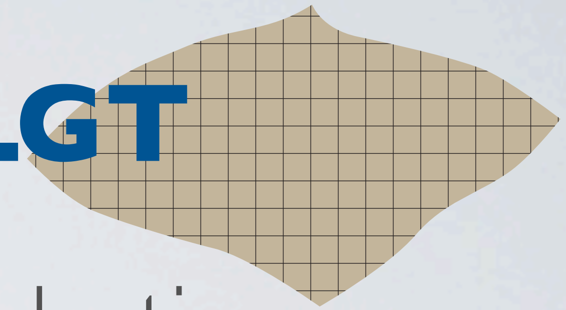
WHY?



TNS

Non-perturbative for Hamiltonian systems

LGT



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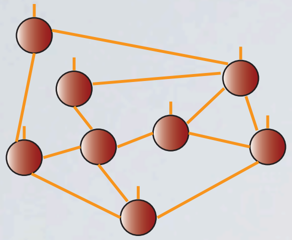
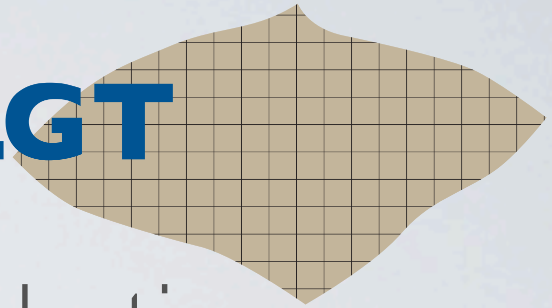
 **TNS**

- Non-perturbative for Hamiltonian systems
- Extremely successful for 1D systems (MPS)

LGT 

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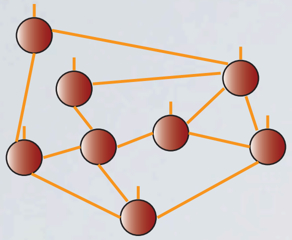
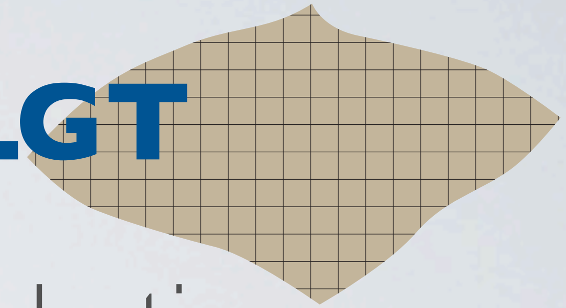


TNS

Non-perturbative for Hamiltonian systems

Extremely successful for 1D systems (MPS)

Promising improvements for higher dimensions



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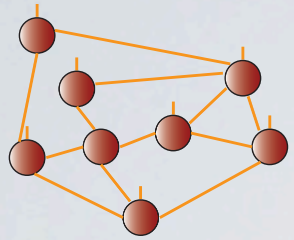


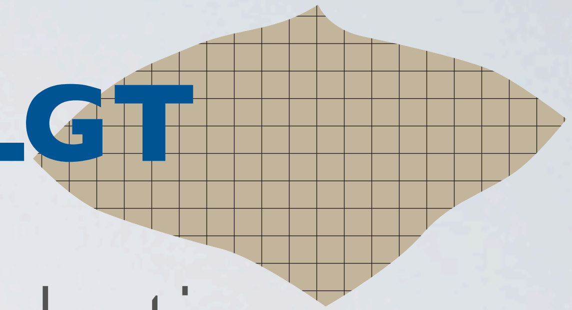
TNS

Non-perturbative for Hamiltonian systems

Extremely successful for 1D systems (MPS)

Promising improvements for higher dimensions

ground states
low-lying excitations
thermal states
time evolution



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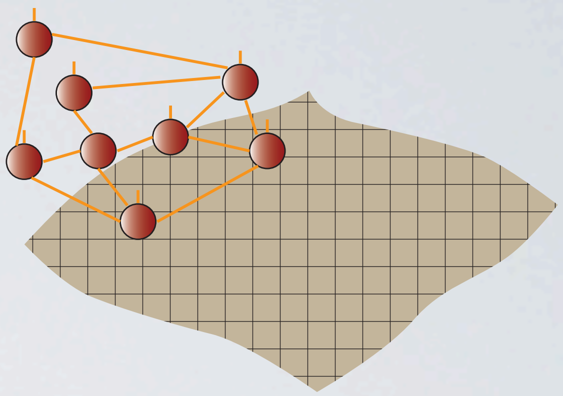
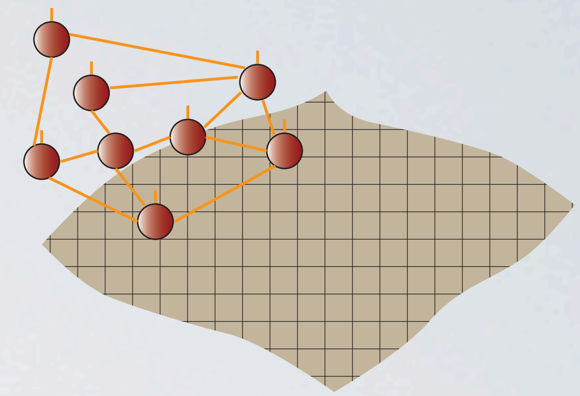
LGT WITH TNS



Different perspectives

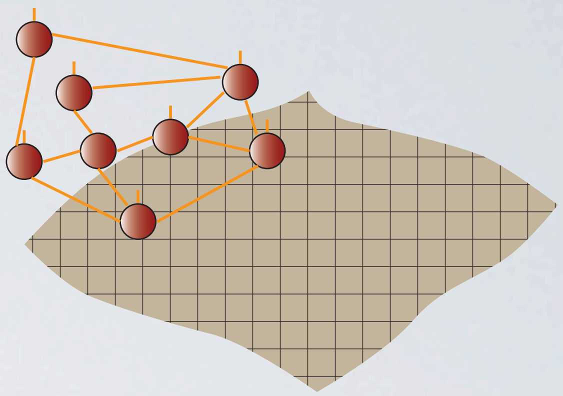
LGT WITH TNS



Different perspectives

TNS as alternative algorithms for LGT

LGT WITH TNS

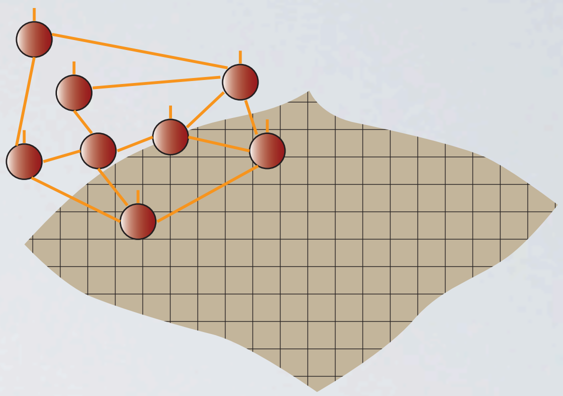


Different perspectives

TNS as alternative algorithms for LGT

ultimate goal: quantum simulation

LGT WITH TNS



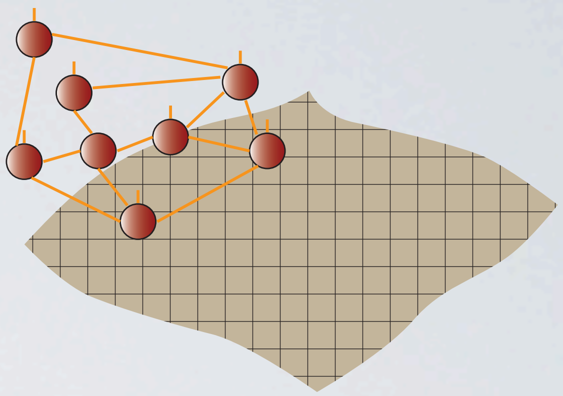
Different perspectives

TNS as alternative algorithms for LGT

ultimate goal: quantum simulation

TNS to explore and
validate schemes

LGT WITH TNS



Different perspectives

➡ TNS as alternative algorithms for LGT

ultimate goal: quantum simulation

TNS to explore and validate schemes

WHAT ARE TNS?

- TNS = Tensor Network States

Context: quantum many body systems

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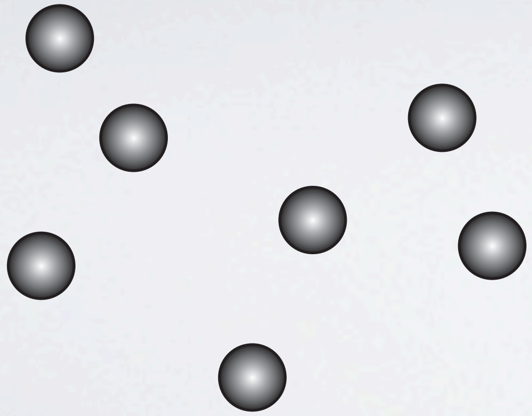
$\{|i\rangle\}_{i=0}^{d-1}$

N



WHAT ARE TNS?

- TNS = Tensor Network States

Context: quantum many body systems

interacting with each other

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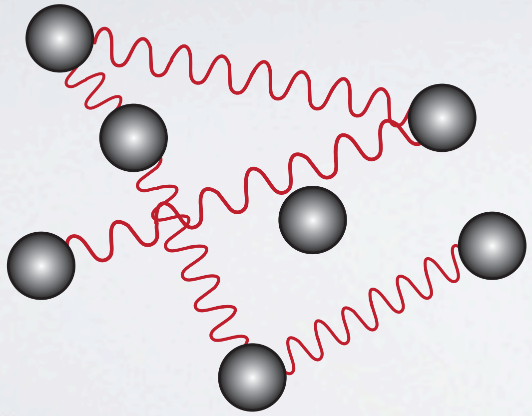

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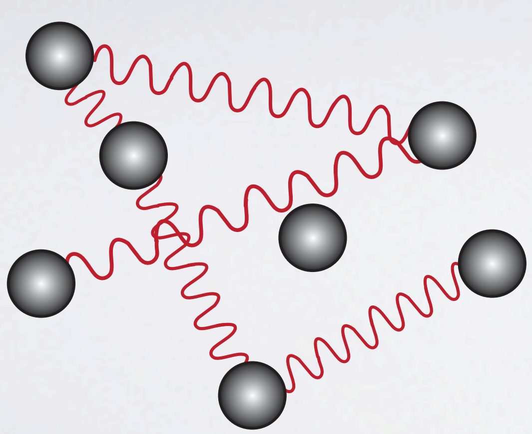

Goal: describe equilibrium states

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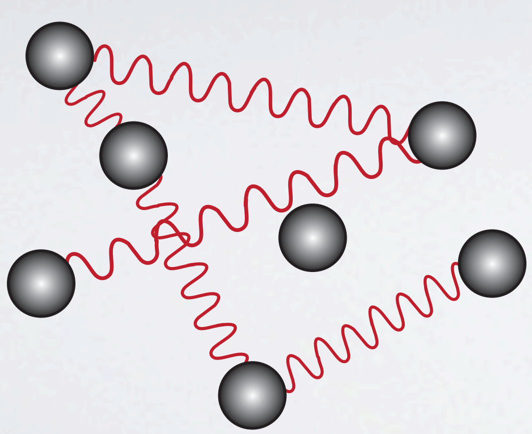


Goal: describe equilibrium states

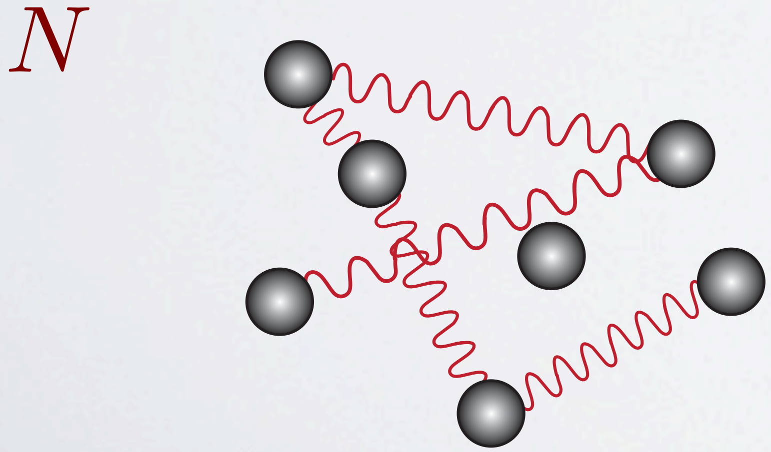
ground, thermal states

WHAT ARE TNS?

- TNS = Tensor Network States

A general state of the N-body Hilbert space has exponentially many coefficients

$$|\Psi\rangle = \sum_{i_j} c_{i_1 \dots i_N} |i_1 \dots i_N\rangle$$



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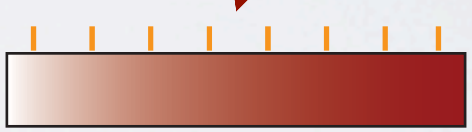
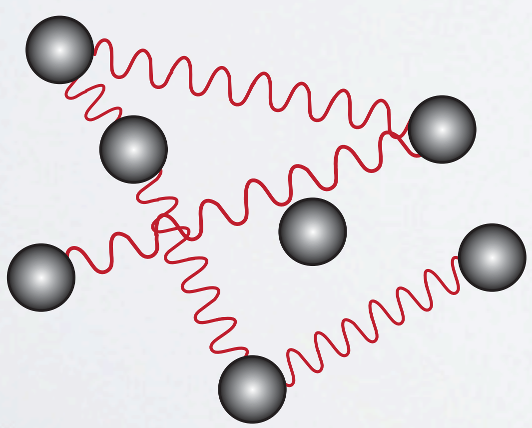


N-legged tensor

d^N

N



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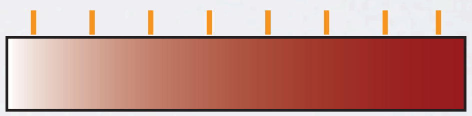

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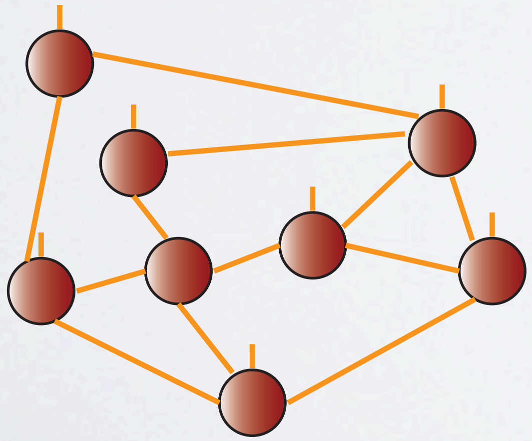
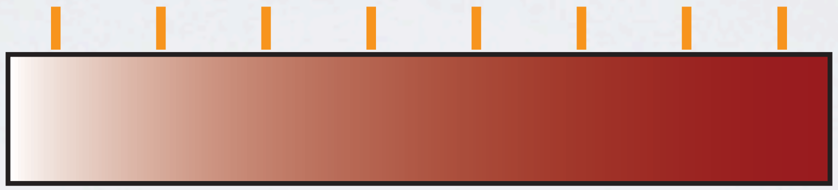


$\mathrm{poly}(N)$

A TNS has only a polynomial number of parameters

MPS

- MPS = Matrix Product States



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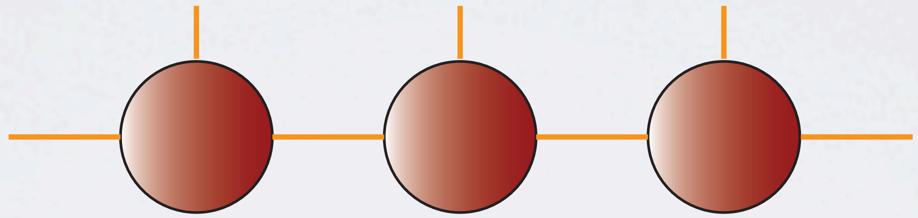

$$|\Psi\rangle = \sum_{i_1 \ldots i_N} \mathrm{tr}(A_1^{i_1} A_2^{i_2} \ldots A_N^{i_N}) |i_1 \ldots i_N\rangle$$

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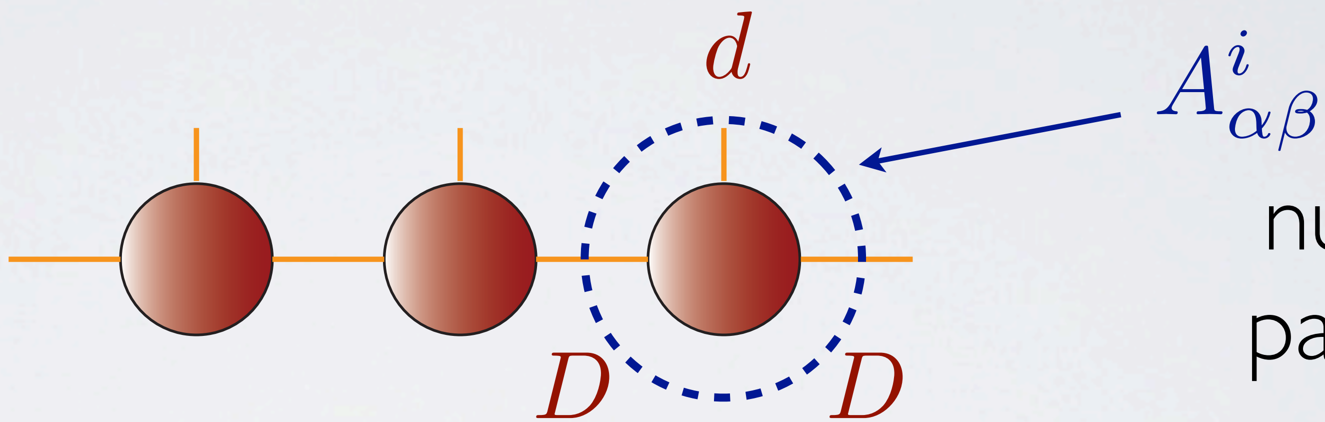

number of parameters

NdD^2

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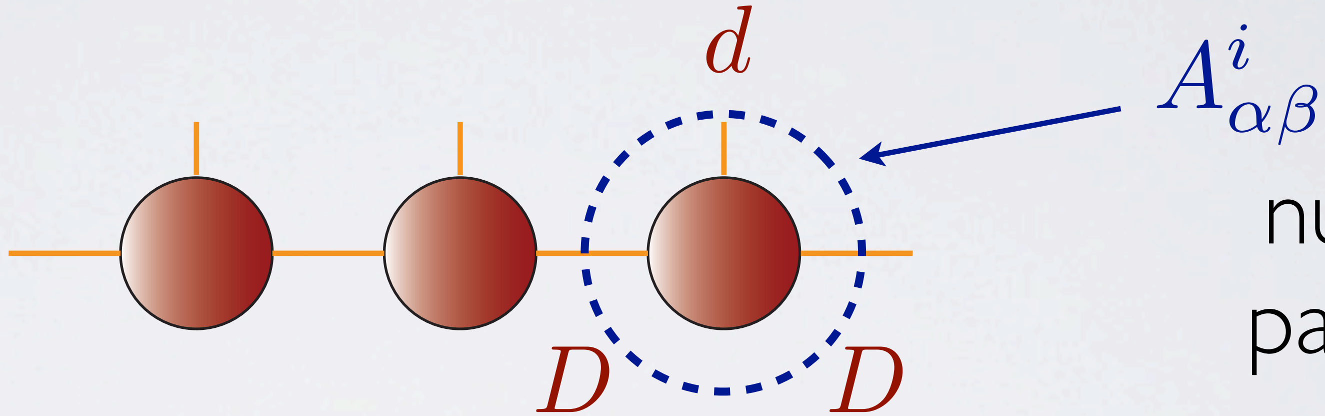

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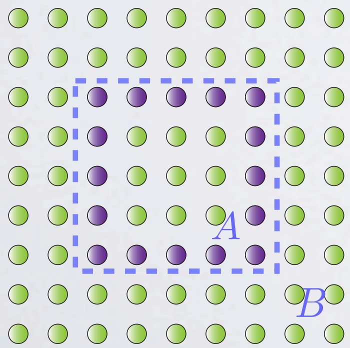

Area law by construction

MPS

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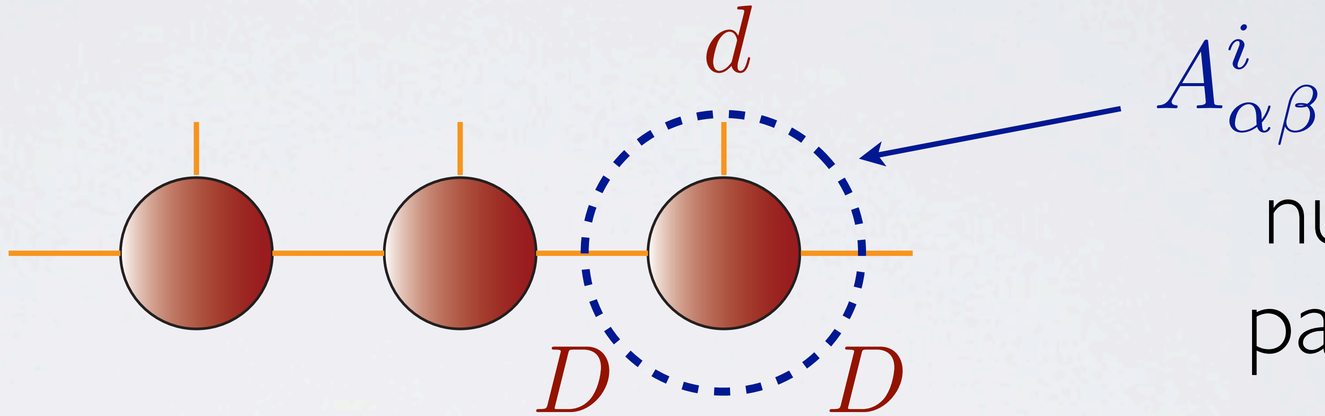

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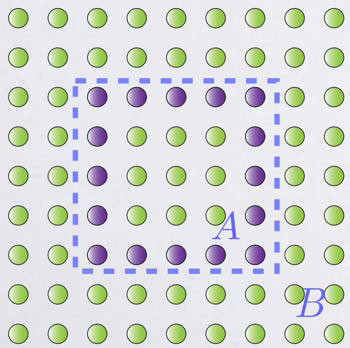

Area law by construction

Bounded entanglement $S(L/2) \leq \log D$

BASIC PROBLEMS

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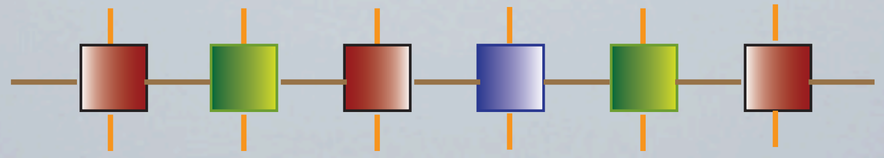
HAMILTONIAN



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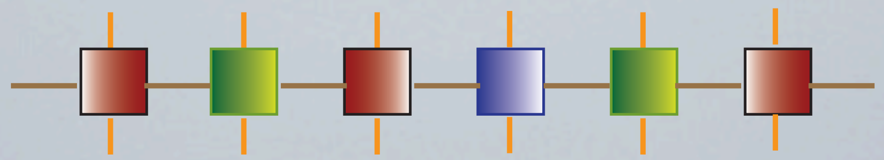

find ground states

BASIC PROBLEMS

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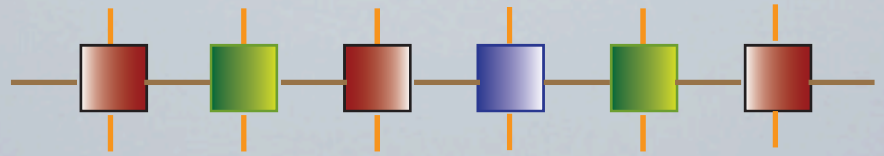

find ground states

→ variational search

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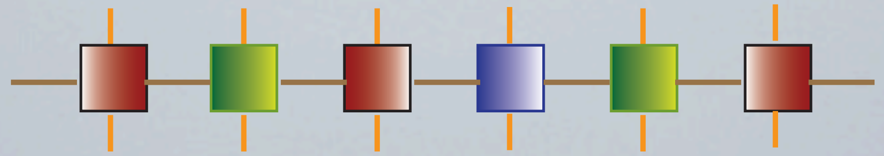

find ground states

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→ imaginary time evolution

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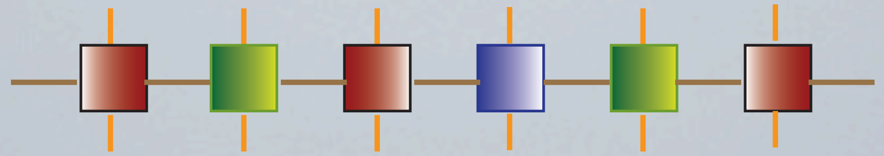

find ground states

→ variational search

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time-dependence → real time evolution

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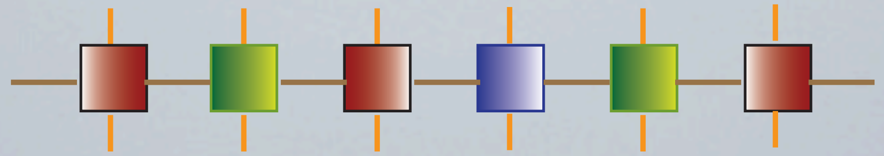

find ground states

→ variational search — typically faster and more precise

→ imaginary time evolution — also for thermal states

time-dependence → real time evolution — works for short times or close to equilibrium

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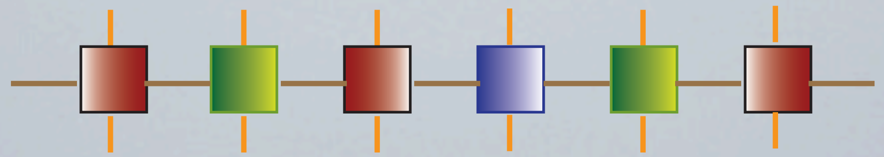

find ground states

produce an ansatz for the state

→ variational search — typically faster and more precise

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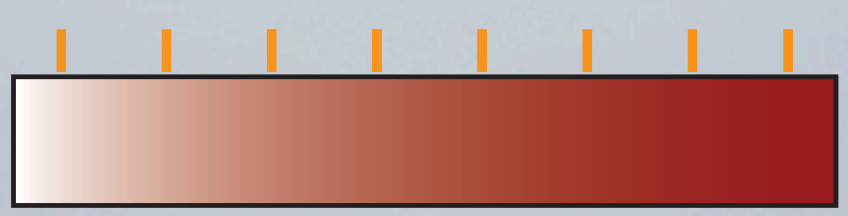
time-dependence → real time evolution — works for short times or close to equilibrium

MIXED STATES

- MPO = Matrix Product Operator



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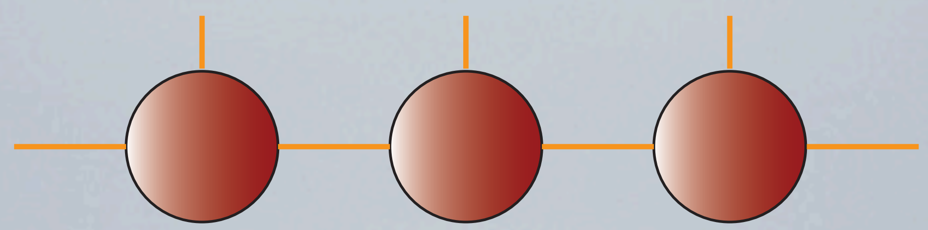

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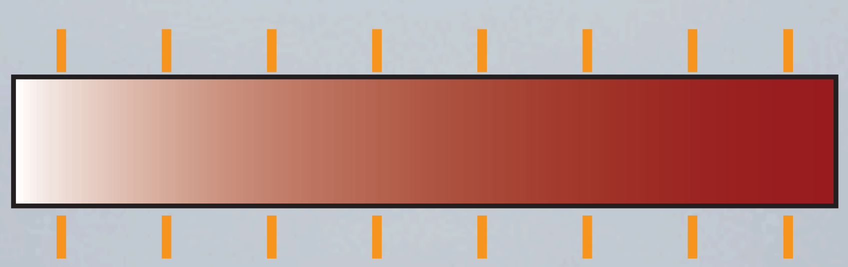
Same kind of ansatz for operators



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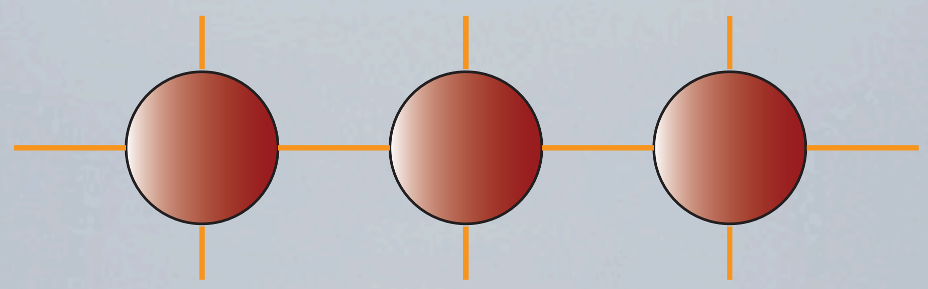

$$\hat{M} = \sum_{i_1, j_1 \ldots i_N, j_N} \mathrm{tr}(M_1^{i_1 j_1} M_2^{i_2 j_2} \ldots M_N^{i_N j_N}) |i_1 \ldots i_N\rangle\langle j_1 \ldots j_N|$$

Verstraete et al., PRL 2004
Pirvu et al., NJP 2010

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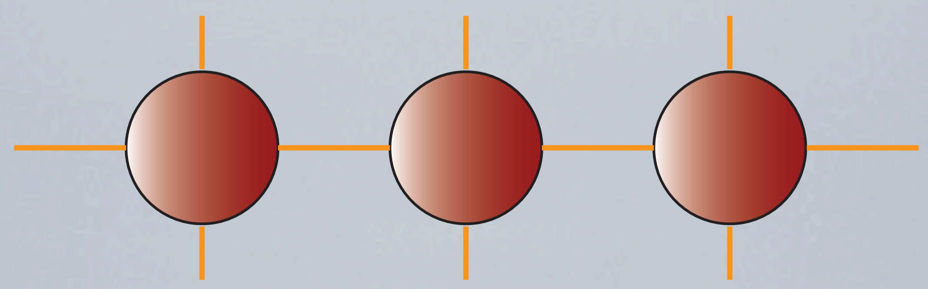

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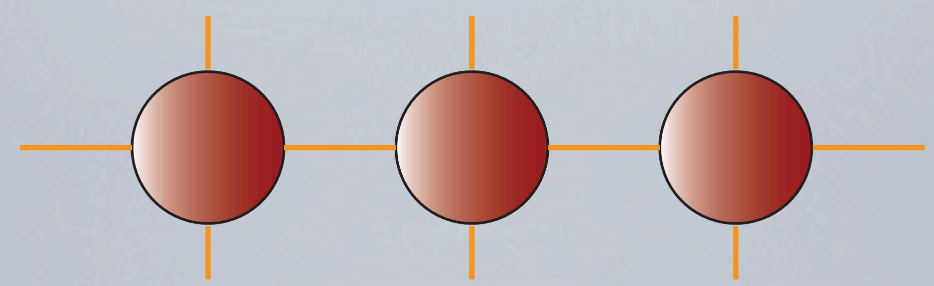

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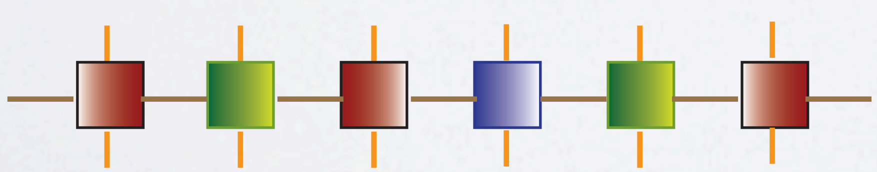

Also used for H and $U(t)$

Verstraete et al., PRL 2004

Pirvu et al., NJP 2010

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equilibrium → thermal states

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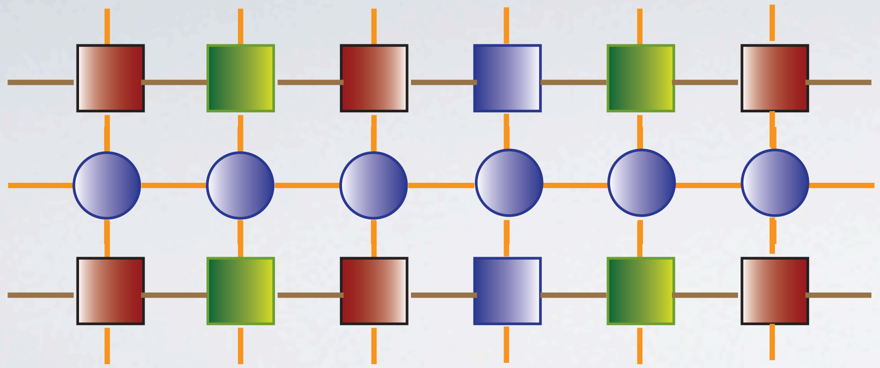

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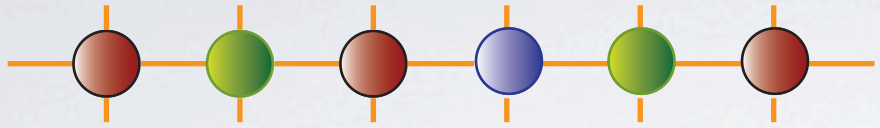

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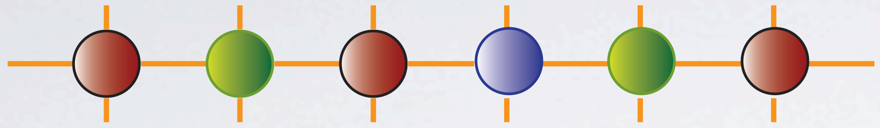
unitary $\rho(t) = U(t)\rho(0)U(t)^\dagger$



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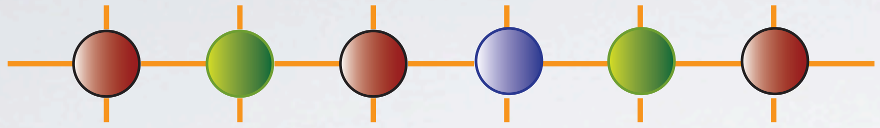

unitary $\rho(t) = U(t)\rho(0)U(t)^\dagger$

non-unitary $\frac{d\rho(t)}{dt} = \mathcal{L}(\rho)$

Verstraete et al., PRL 2004
Prosen, Znidaric PRL 2008
Cai, Barthel, PRL 2013,...

USING TNS FOR LGT: SCHWINGER MODEL AS LABORATORY

SCHWINGER MODEL

Schwinger '62

Simplest gauge theory with matter

QED in 1+1 dimensions

electrons & photons

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Shows some of the features of *full* QCD

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confinement → bound states (massive bosons)

fermion condensate

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Simplest gauge theory with matter

QED in 1+1 dimensions
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Shows some of the features of *full* QCD

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A testbench for lattice techniques

Precedents / Related work

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DMRG on Schwinger model

Byrnes et al. PRD 2002

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TN→extensions

time evolution, finite T

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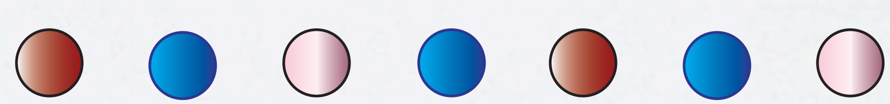
TNS for classical gauge models

Meurice et al. 2013

SCHWINGER MODEL

discrete Hamiltonian (staggered) formulation

Kogut, Susskind '75

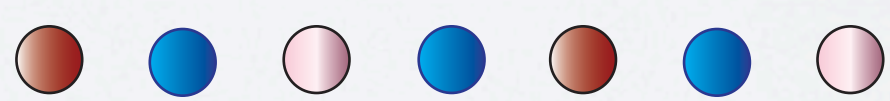


SCHWINGER MODEL

relativistic in the continuum limit

discrete Hamiltonian (staggered) formulation

Kogut, Susskind '75



SCHWINGER MODEL

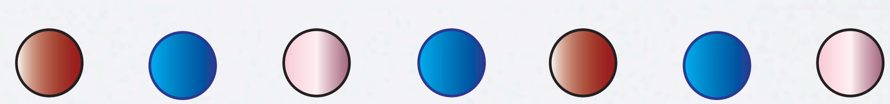
discrete Hamiltonian (staggered) formulation

Kogut, Susskind '75

Jordan-Wigner → spin model

rescaled: adimensional

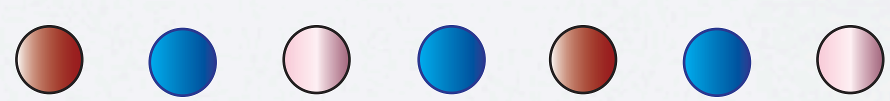
$$H = \frac{1}{g^2 a^2} \sum_n \left(\sigma_n^+ e^{i\theta_n} \sigma_{n-1}^- + \sigma_{n+1}^+ e^{-i\theta_n} \sigma_n^- \right) + \frac{m}{a g^2} \sum_n \left(1 + (-1)^n \sigma_n^3\right) + \sum_n L_n^2$$



SCHWINGER MODEL

$$|\ldots s_e\,\ell\,s_o\,\ell\,s_e\,\ell\,s_o\ldots\rangle$$

$$H = \frac{1}{g^2a^2}\sum_n \left(\sigma_n^+ e^{i\theta_n}\sigma_{n-1}^- + \sigma_{n+1}^+ e^{-i\theta_n}\sigma_n^-\right) + \frac{m}{ag^2}\sum_n \left(1 + (-1)^n\sigma_n^3\right) + \sum_n L_n^2$$



SCHWINGER MODEL

$$|\ldots s_e \, \ell \; s_o \, \ell \; s_e \, \ell \; s_o \ldots\rangle$$

$$H = \frac{1}{g^2 a^2} \sum_n \left(\sigma_n^+ e^{i\theta_n} \sigma_{n-1}^- + \sigma_{n+1}^+ e^{-i\theta_n} \sigma_n^- \right)$$

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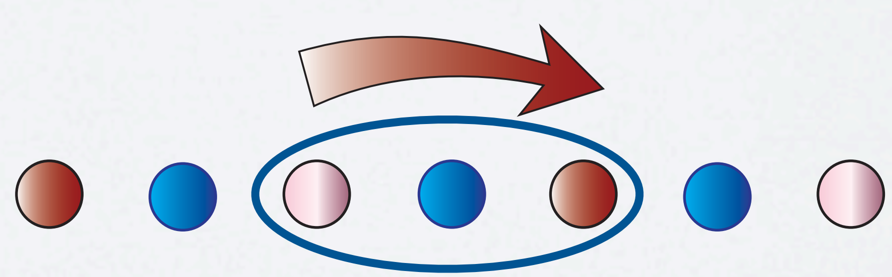

hopping

$\ell + 1$

SCHWINGER MODEL

$$|\ldots s_e\,\ell\;s_o\,\ell\;s_e\,\ell\;s_o\ldots\rangle$$

$$H = \frac{1}{g^2a^2}\sum_n\left(\sigma_n^+ e^{i\theta_n}\sigma_{n-1}^- + \sigma_{n+1}^+ e^{-i\theta_n}\sigma_n^-\right)$$
$$+\frac{m}{ag^2}\sum_n\left(1+(-1)^n\sigma_n^3\right)+\sum_n L_n^2$$

Gauss Law

$$L_n - L_{n-1} = \frac{1}{2}[\sigma_n^3 + (-1)^n]$$

SCHWINGER MODEL

MPS representation with OPEN BOUNDARIES

basis $\quad |\ldots s_e \, \ell \, s_o \, \ell \, s_e \, \ell \, s_o \ldots\rangle$

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Gauss' law fixes *photon* content

SCHWINGER MODEL

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SCHWINGER MODEL

MPS representation with OPEN BOUNDARIES

basis $|\dots s_e\,\ell\,s_o\,\ell\,s_e\,\ell\,s_o\dots\rangle$ all terms are local

Gauss' law fixes *photon* content

$$L_n = \ell_0 + \frac{1}{2}\sum_{k\leq n}\sigma_n^3 + \dots \longrightarrow \sum_n \sum_{k<n}(N-n)\sigma_k^3\sigma_n^3$$

$|\ell_0\dots s_e\,s_o\,s_e\,s_o\dots\rangle$ non-local terms

SCHWINGER MODEL AS TESTBENCH

Relevant states can be described as MPS

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TN allow reliable continuum limit

SCHWINGER MODEL AS TESTBENCH

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- Mass spectrum

  TN allow reliable continuum limit

  MCB, Cichy, Jansen, Cirac, JHEP11(2013)158

- Chiral condensate (order parameter of chiral symmetry breaking)

  PoS 2014 arXiv:1412.0596

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Thermal equilibrium states well approximated by MPO

Temperature dependence of chiral condensate

MCB, Cichy, Cirac, Jansen, Saito, PRD 92, 034519 (2015);
arXiv:1603.05002

COMPUTING THE SPECTRUM WITH MPS

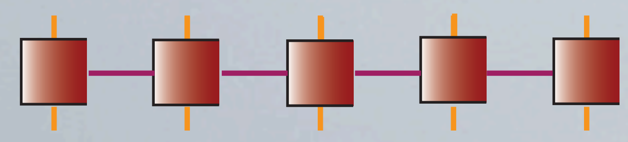
White, PRL 1992

Verstraete, Porras, Cirac, PRL 2004

Schollwöck, RMP 2005, Ann. Phys. 2011

COMPUTING THE SPECTRUM WITH MPS

Variational minimization of energy

$H =$ 

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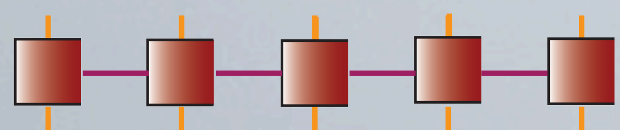
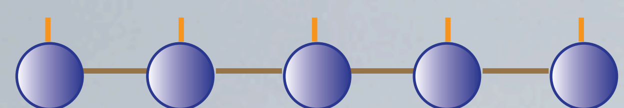
$H =$ 

$|E_0\rangle \simeq$ 

$$\min_{\{A\}} \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}$$

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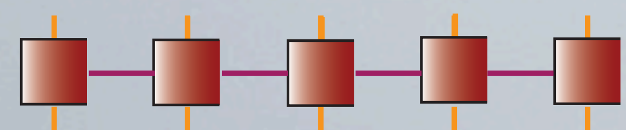
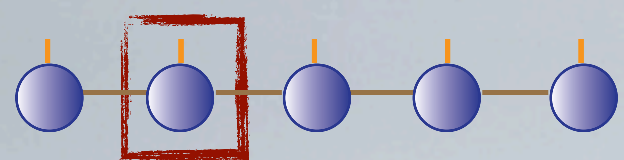
$H =$ 

$|E_0\rangle \simeq$ 

$$\min_{\{A\}} \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle} \longrightarrow \min_{A} \frac{\bar{A} H_{\text{eff}} A}{\bar{A} N_{\text{eff}} A}$$

White, PRL 1992
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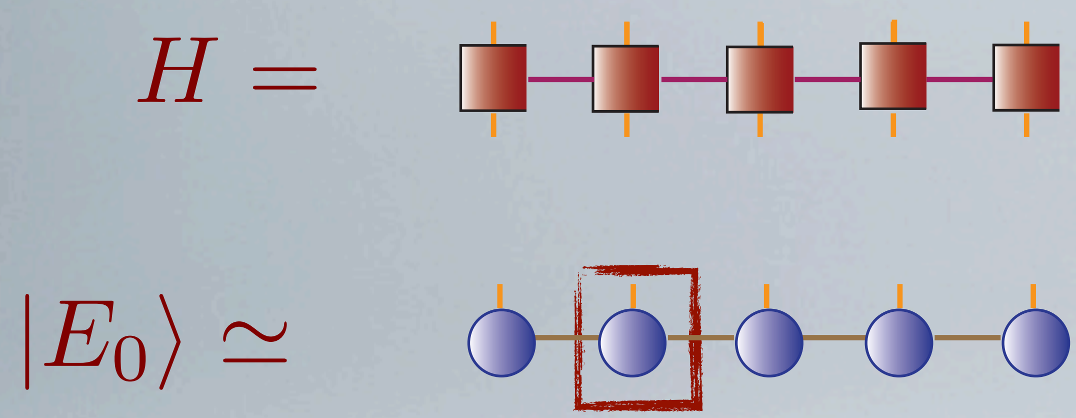

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sweep back and forth
over tensors

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Variational minimization of energy

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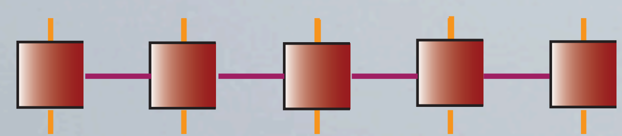

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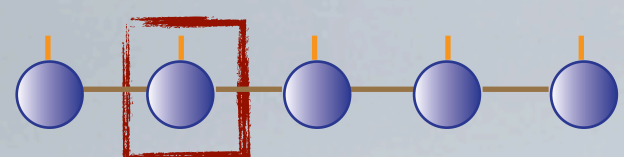
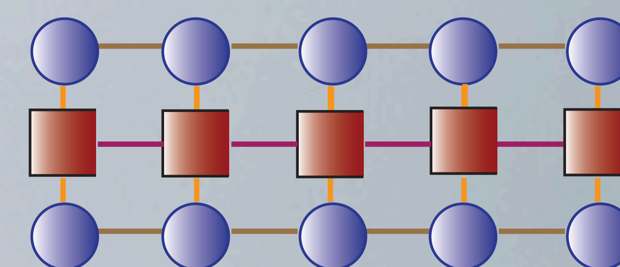


$\langle\Psi|H|\Psi\rangle$



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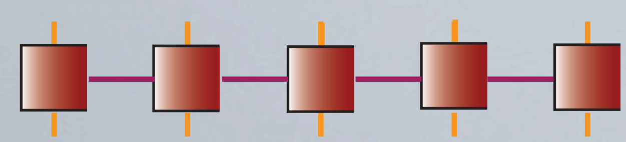
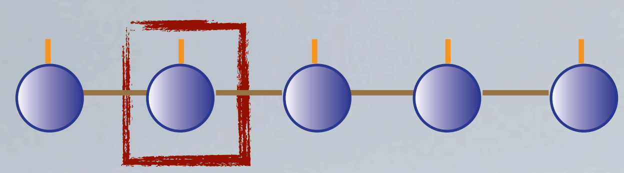
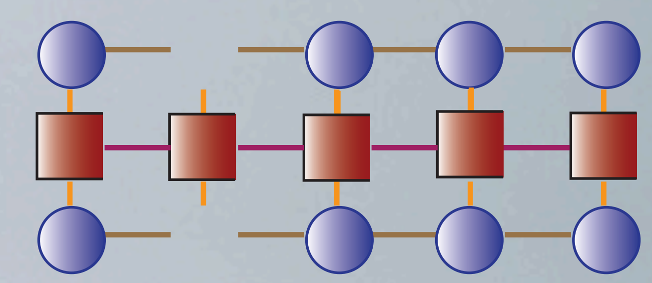
$H =$ 

$|E_0\rangle \simeq$ 



H_{eff}

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COMPUTING THE SPECTRUM WITH MPS

JHEP11(2013)158
arXiv:1310.4118

COMPUTING THE SPECTRUM WITH MPS

Scan parameters

JHEP11(2013)158
arXiv:1310.4118

COMPUTING THE SPECTRUM WITH MPS

Scan parameters

$$m/g$$

JHEP11(2013)158
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COMPUTING THE SPECTRUM WITH MPS

Scan parameters

m/g mass gaps and GS energy density
in the continuum $x \to \infty$

JHEP11(2013)158
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COMPUTING THE SPECTRUM WITH MPS

Scan parameters

m/g mass gaps and GS energy density in the continuum $x \to \infty$

x $x \in [5,\ 600]$

JHEP11(2013)158
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COMPUTING THE SPECTRUM WITH MPS

Scan parameters

m/g mass gaps and GS energy density in the continuum $x \rightarrow \infty$

x $x \in [5,\ 600]$

N $N \propto x$ (up to ~850)

JHEP11(2013)158
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COMPUTING THE SPECTRUM WITH MPS

Scan parameters

m/g	mass gaps and GS energy density in the continuum $x \to \infty$
x	$x \in [5,\ 600]$
N	$N \propto x$ (up to ~850)
D	$D \in [20,\ 120]$

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m/g mass gaps and GS energy density in the continuum $x \to \infty$

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convergence

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x $x \in [5,\ 600]$

finite-size

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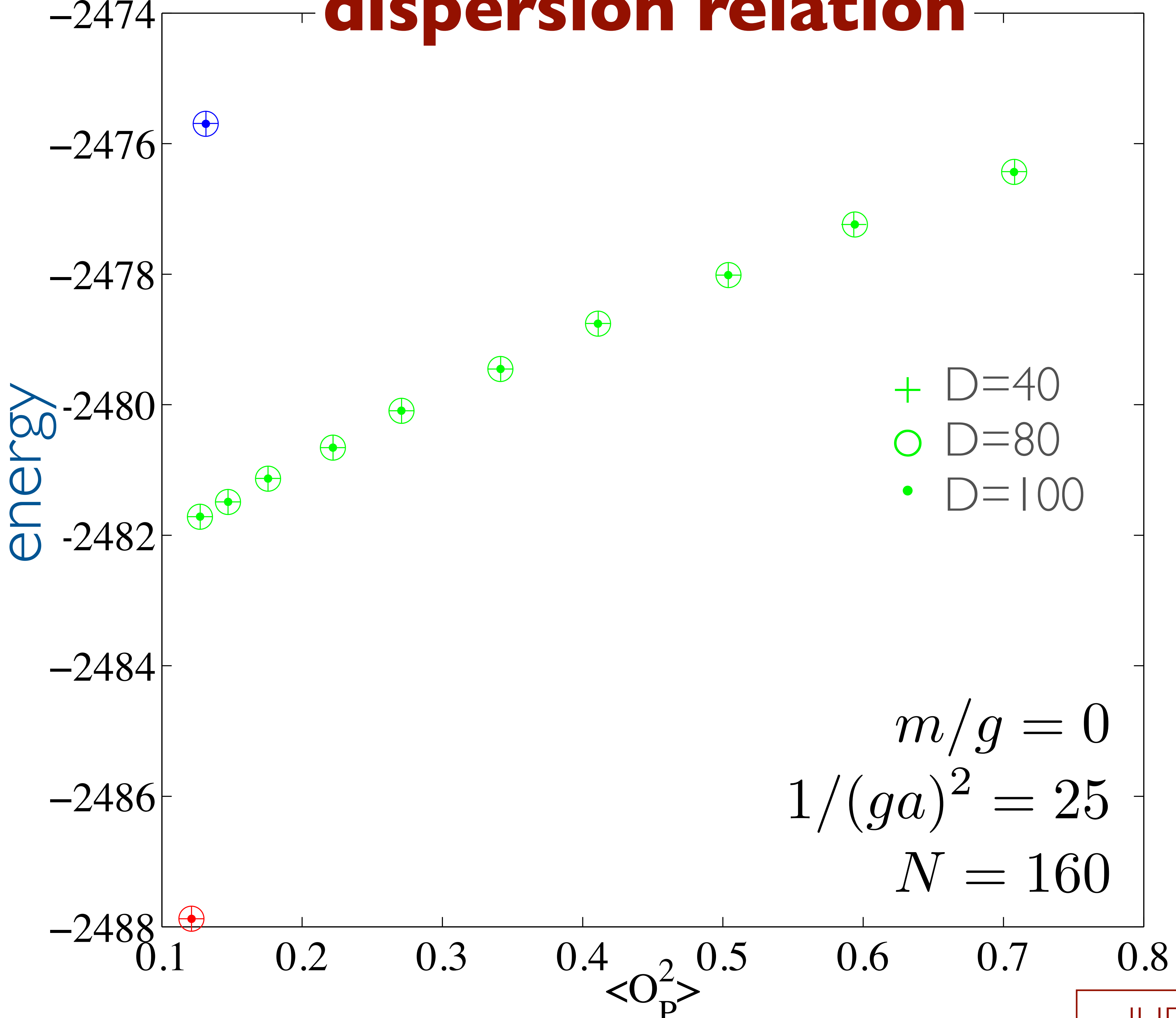
dispersion relation



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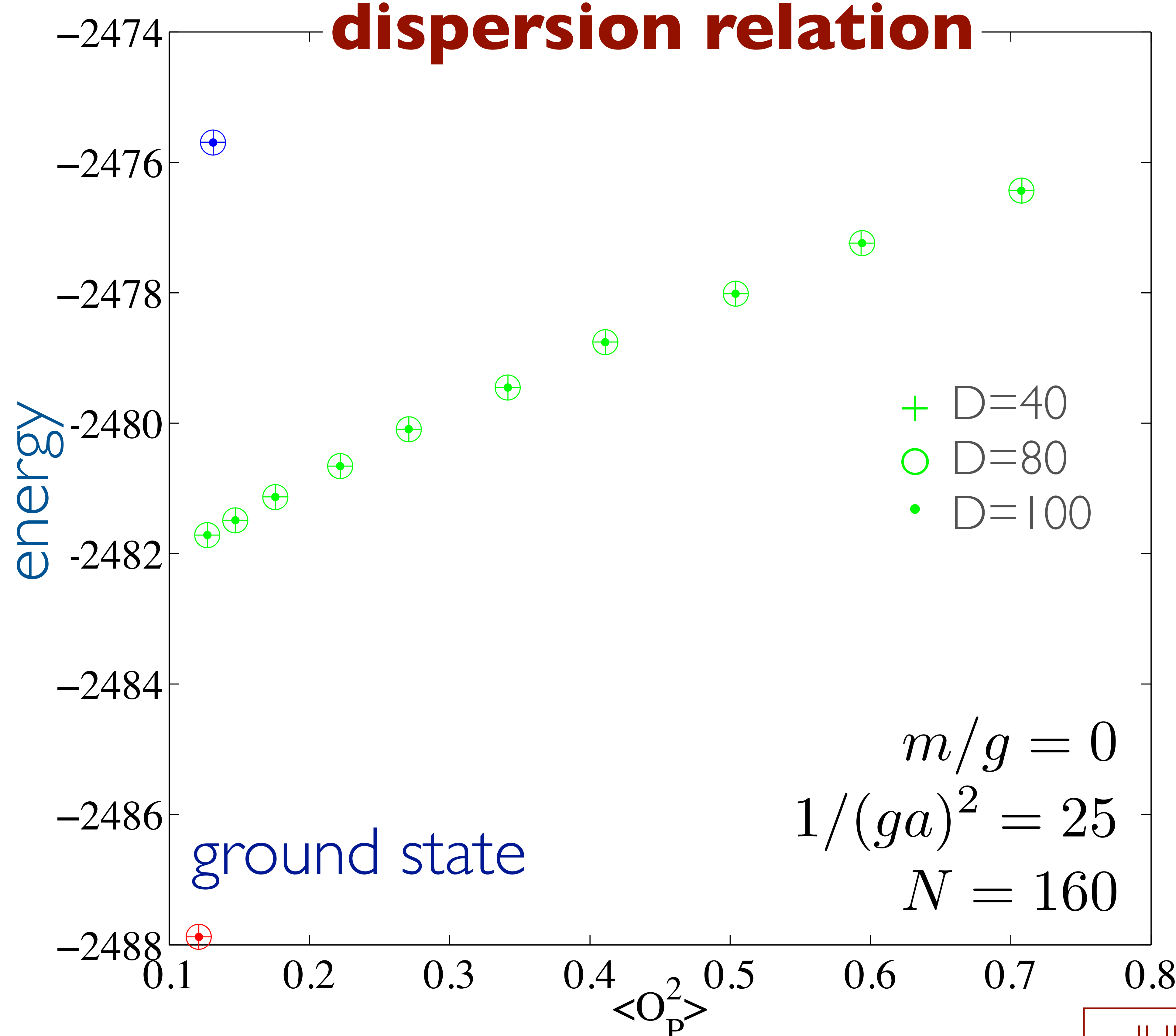

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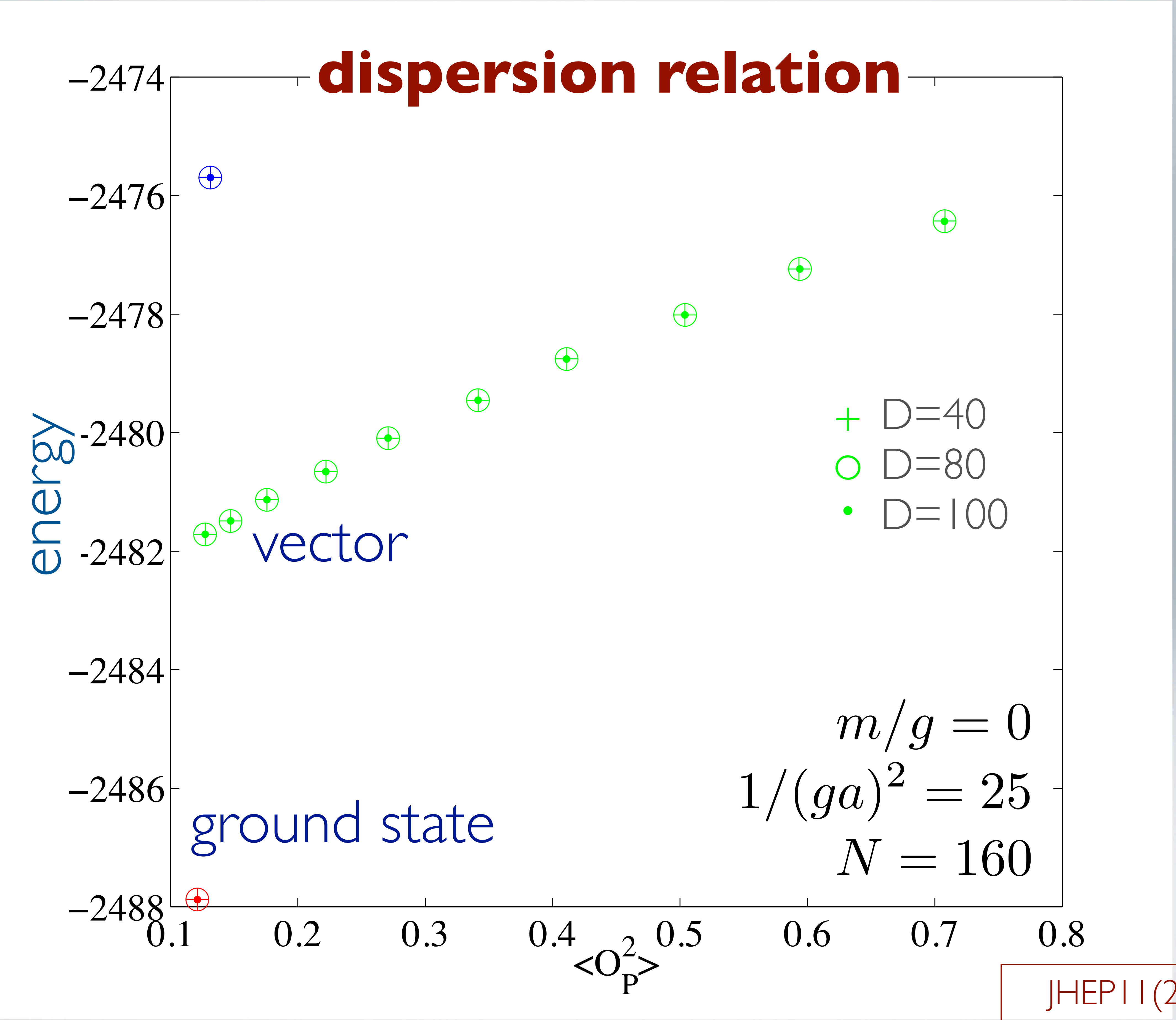

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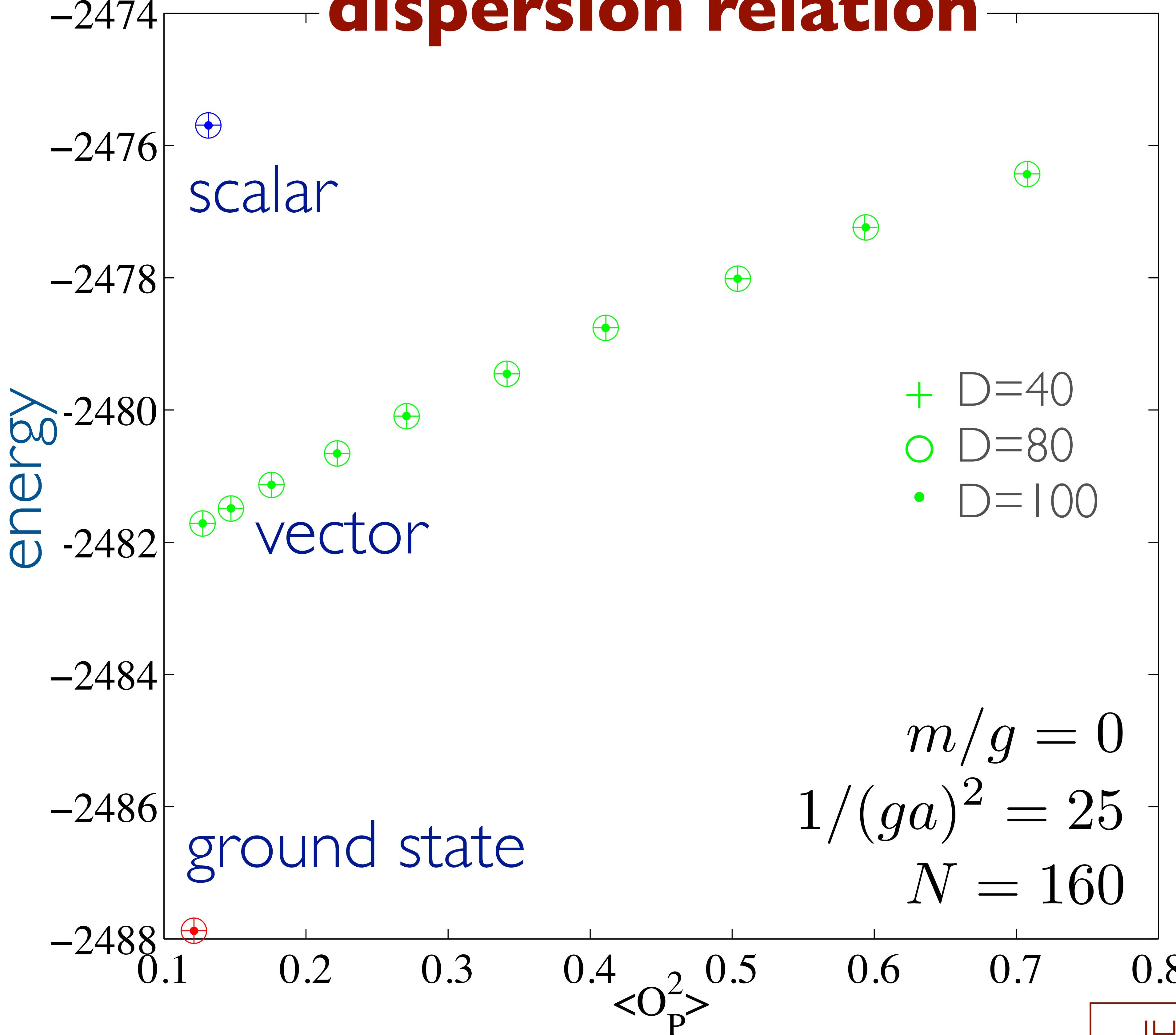

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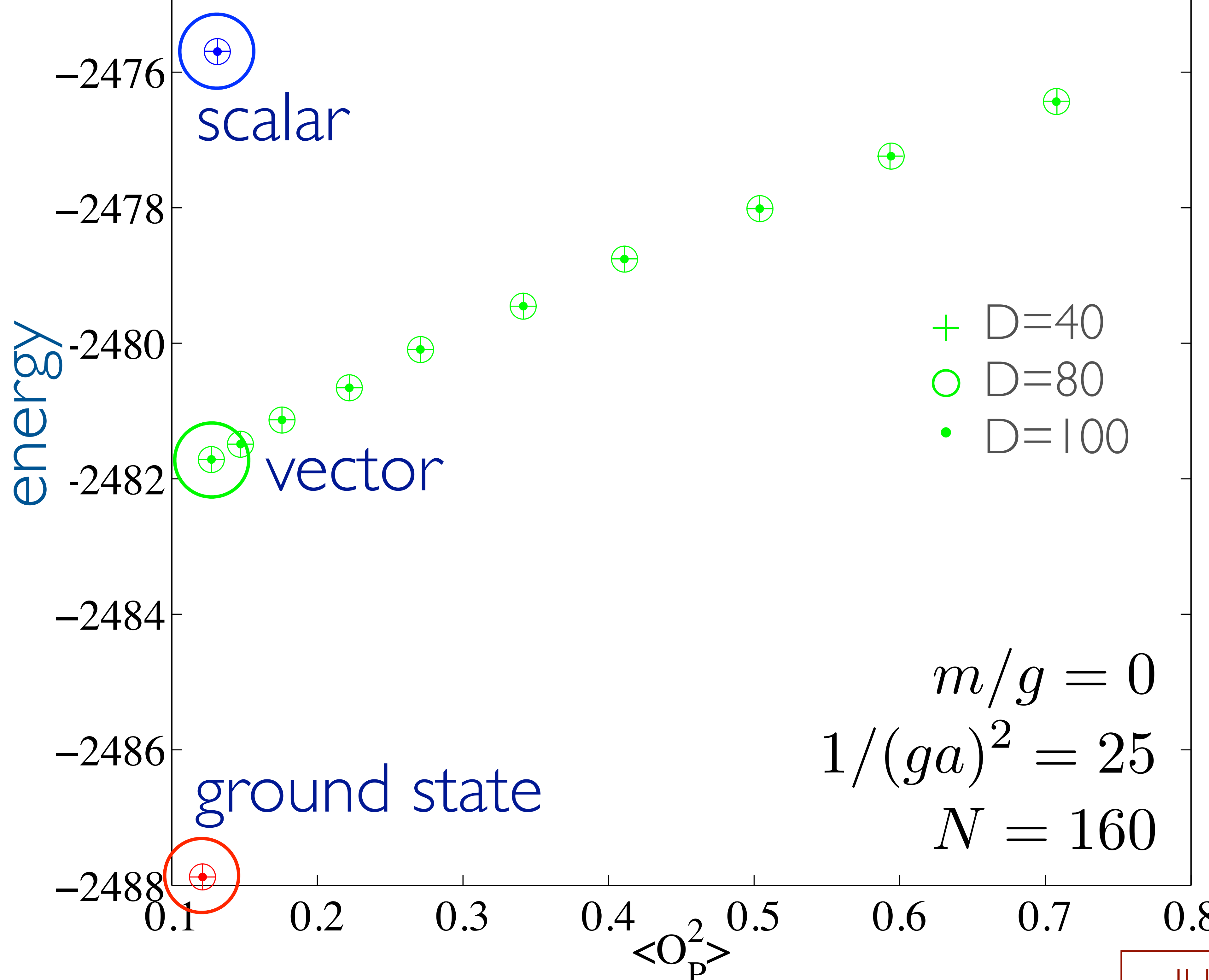

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some results

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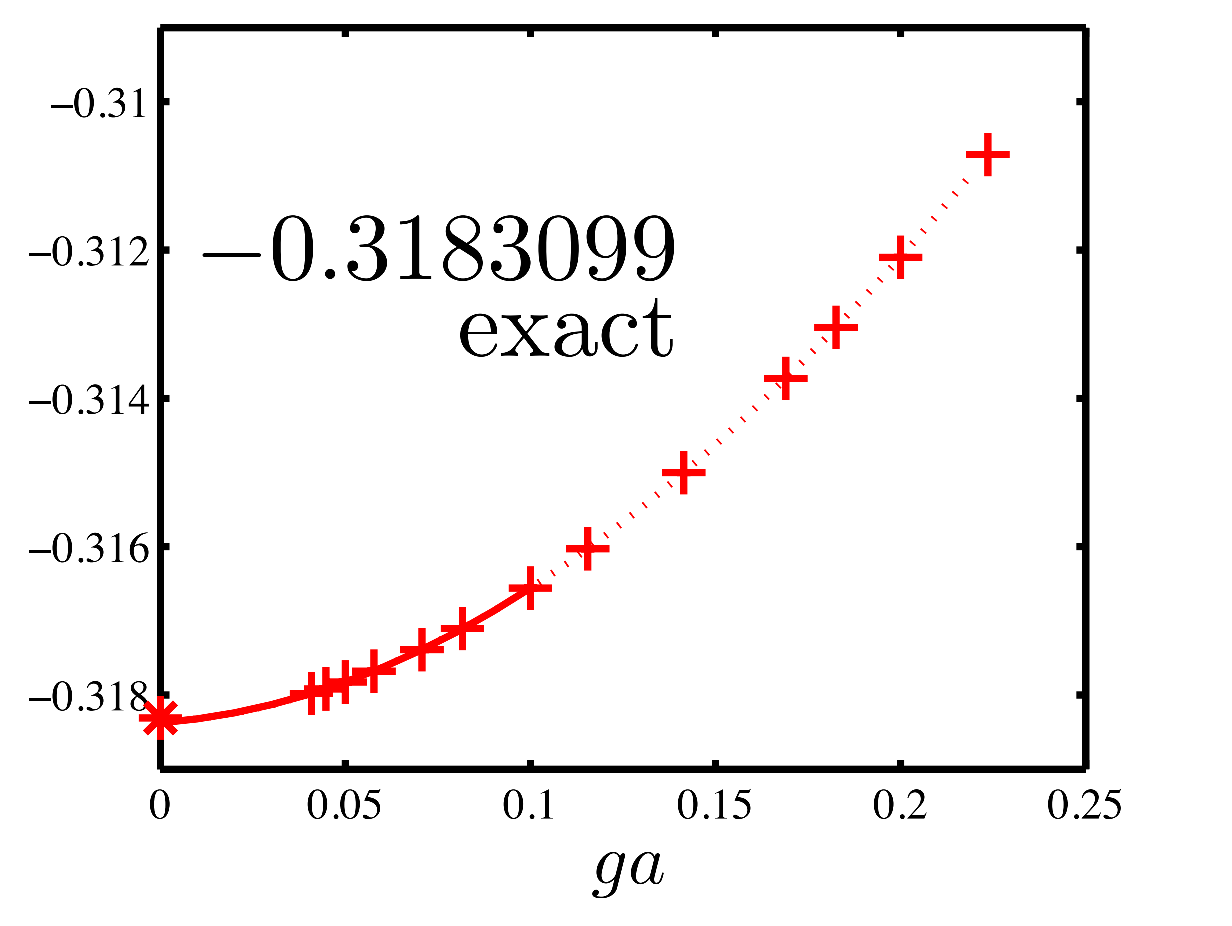

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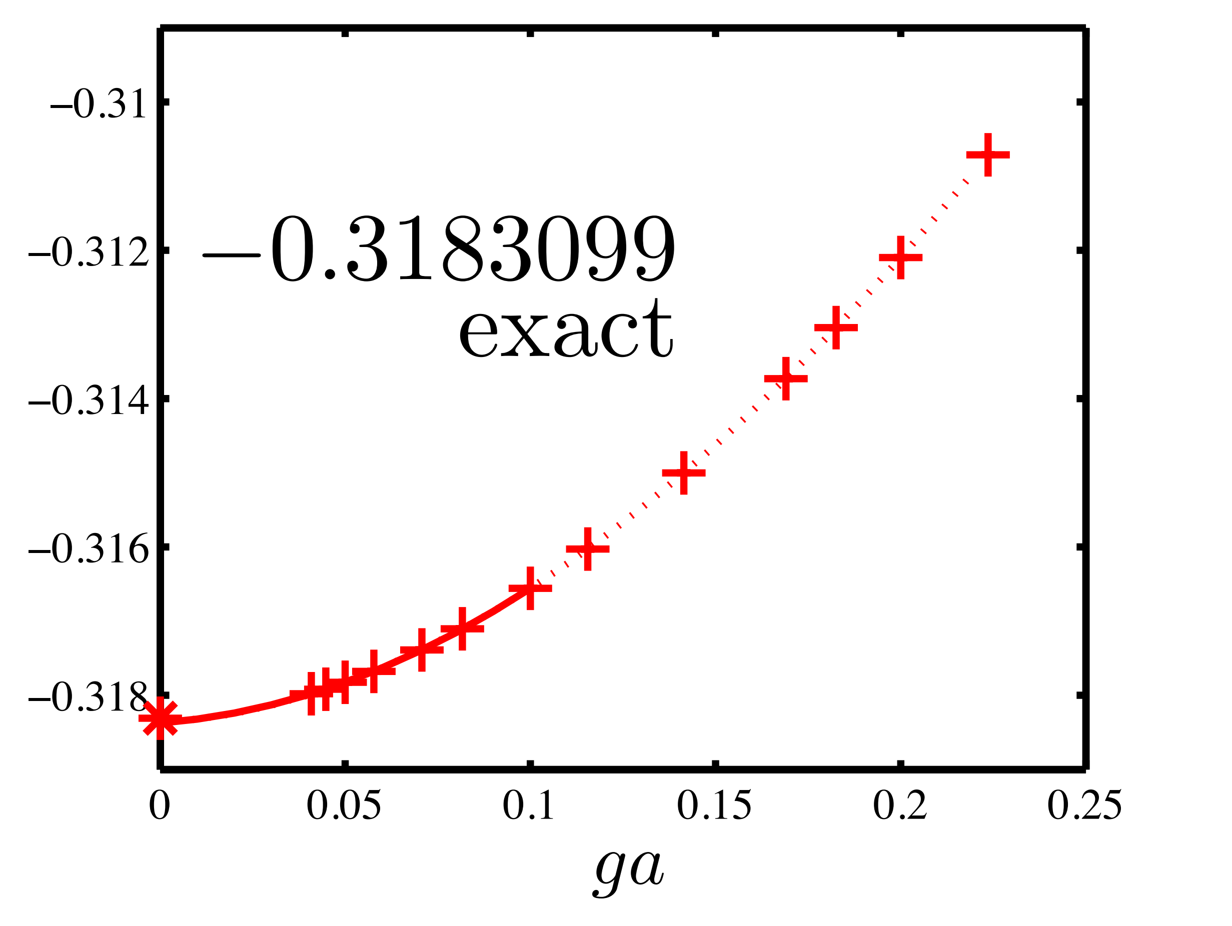

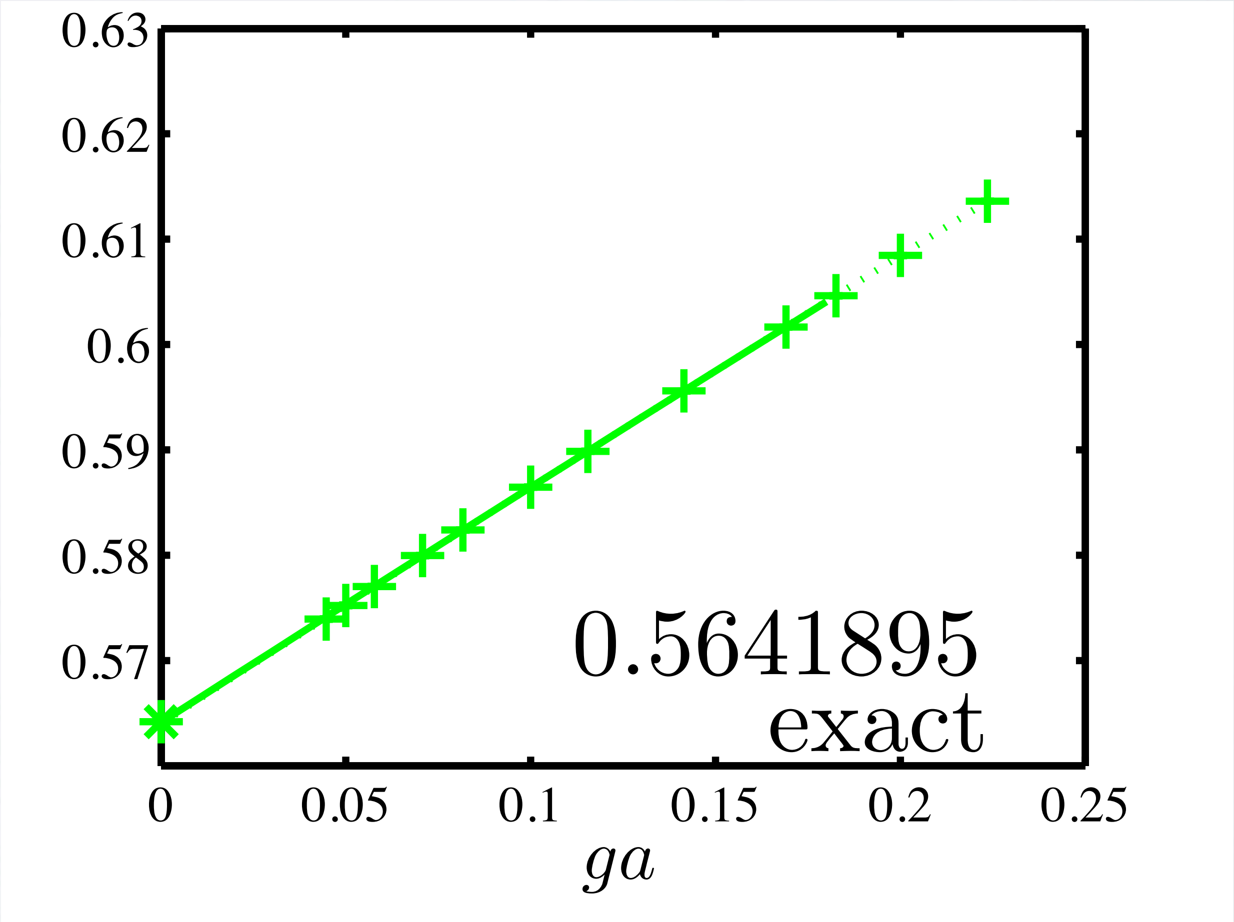

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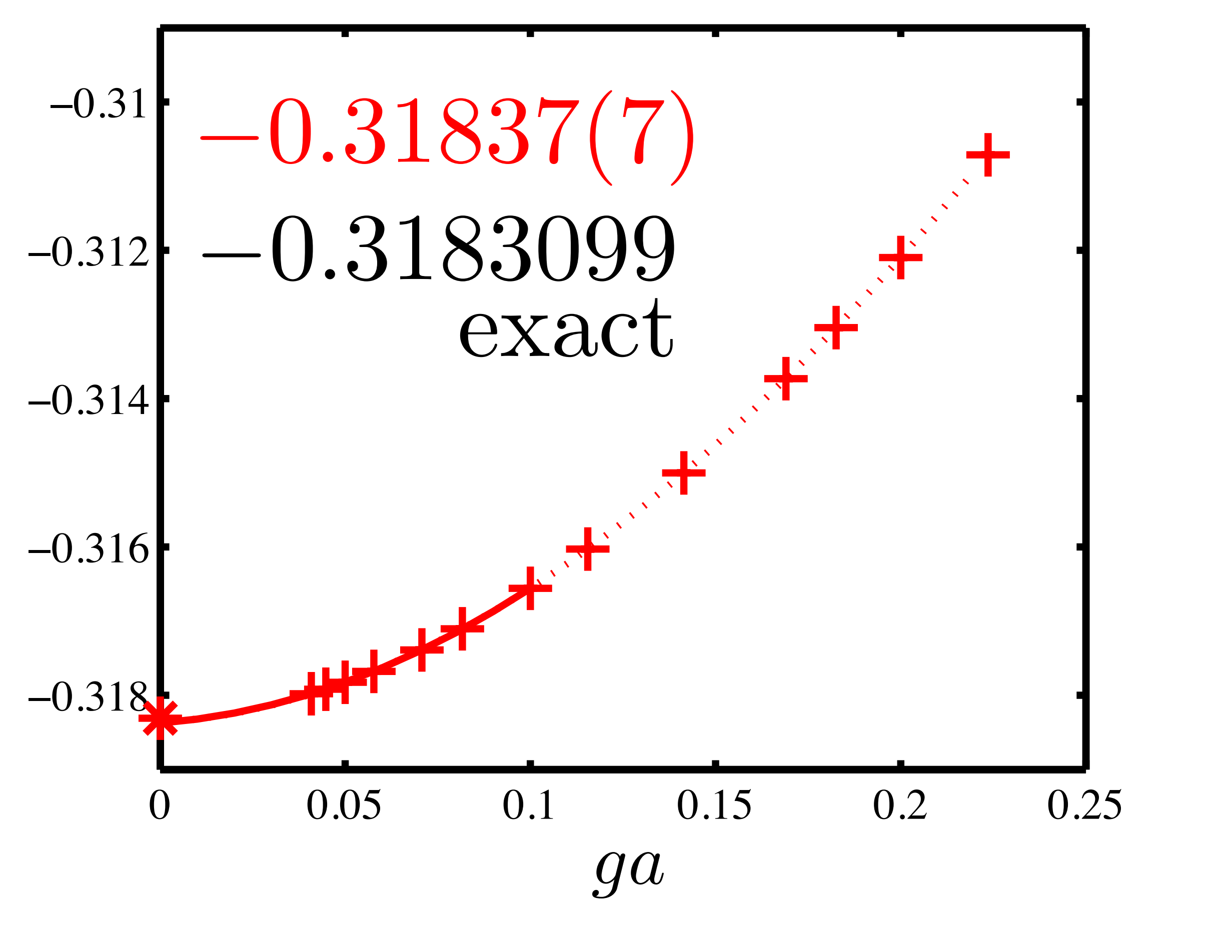

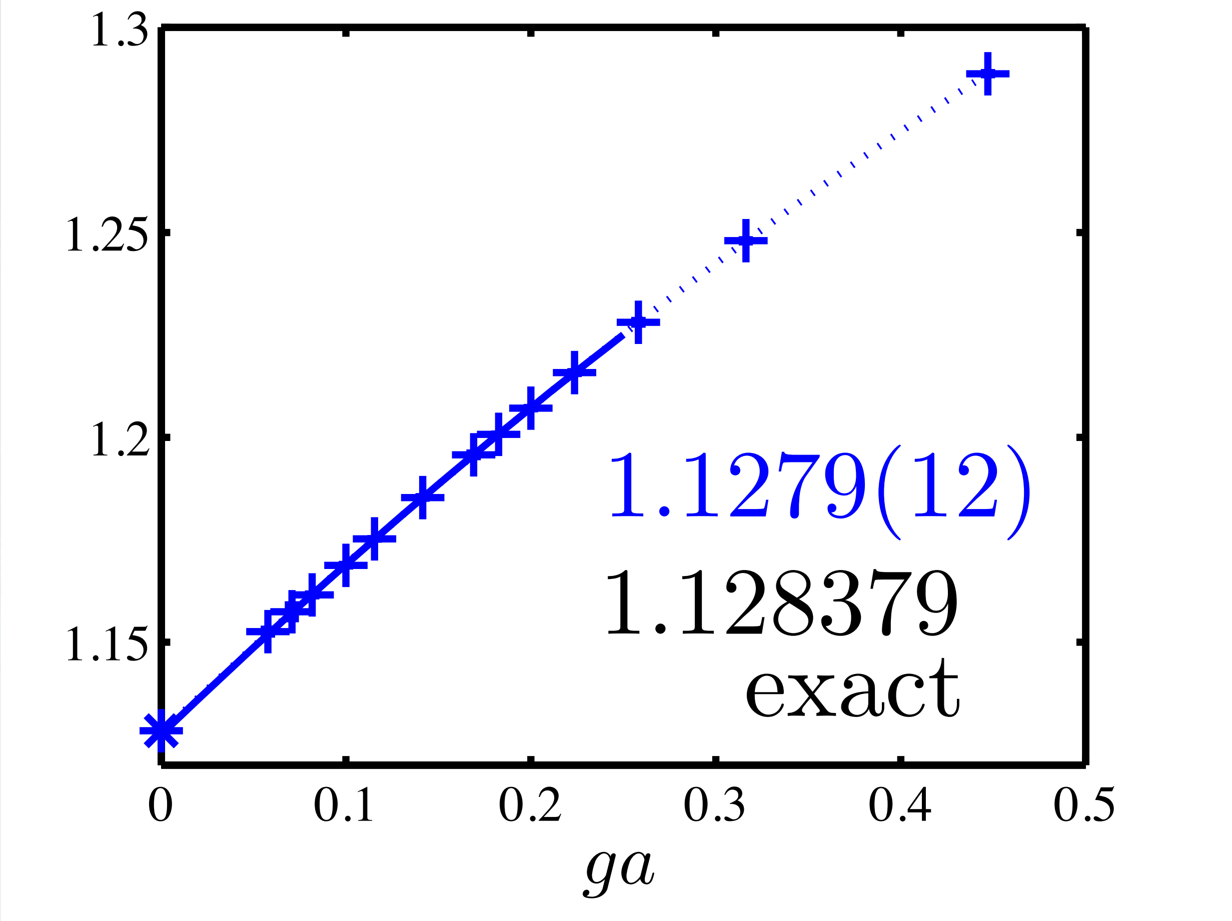

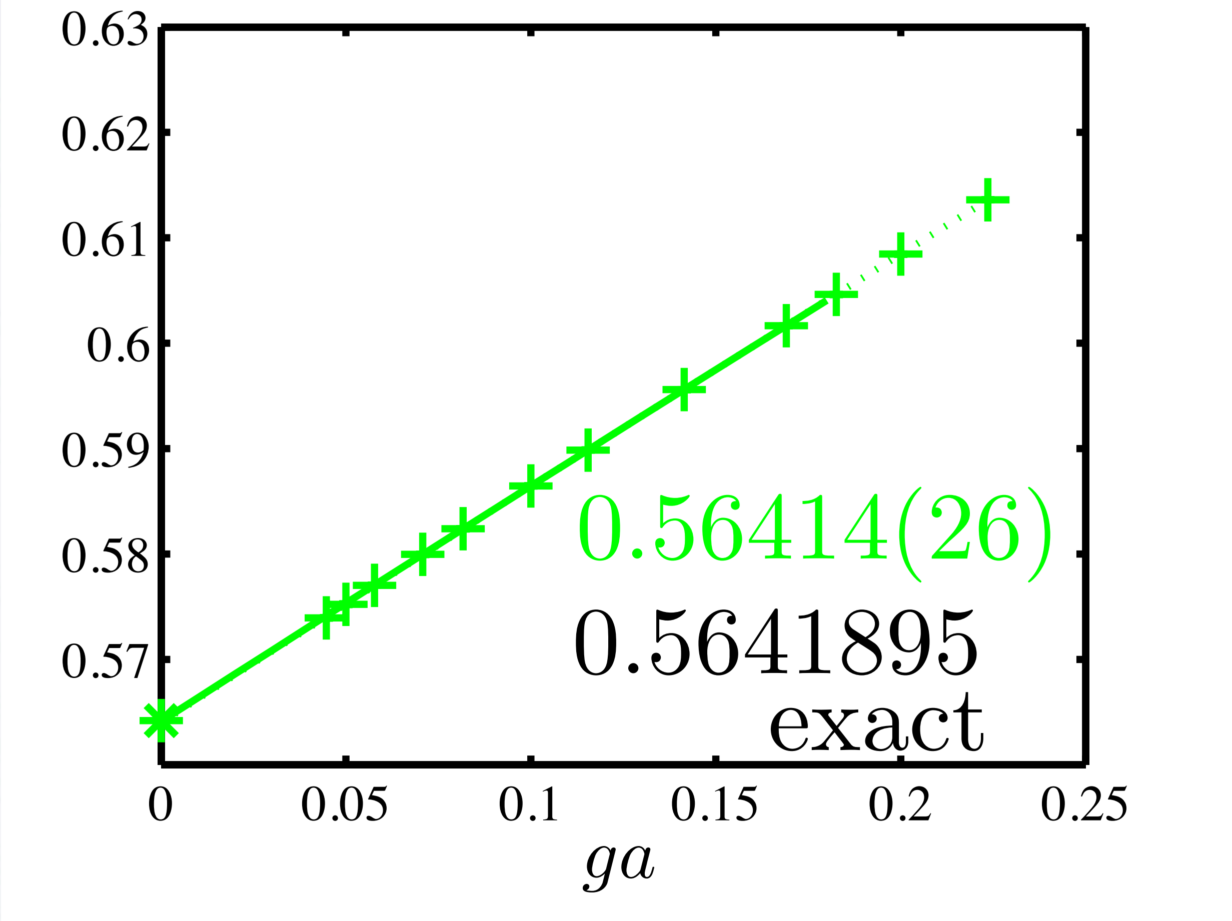

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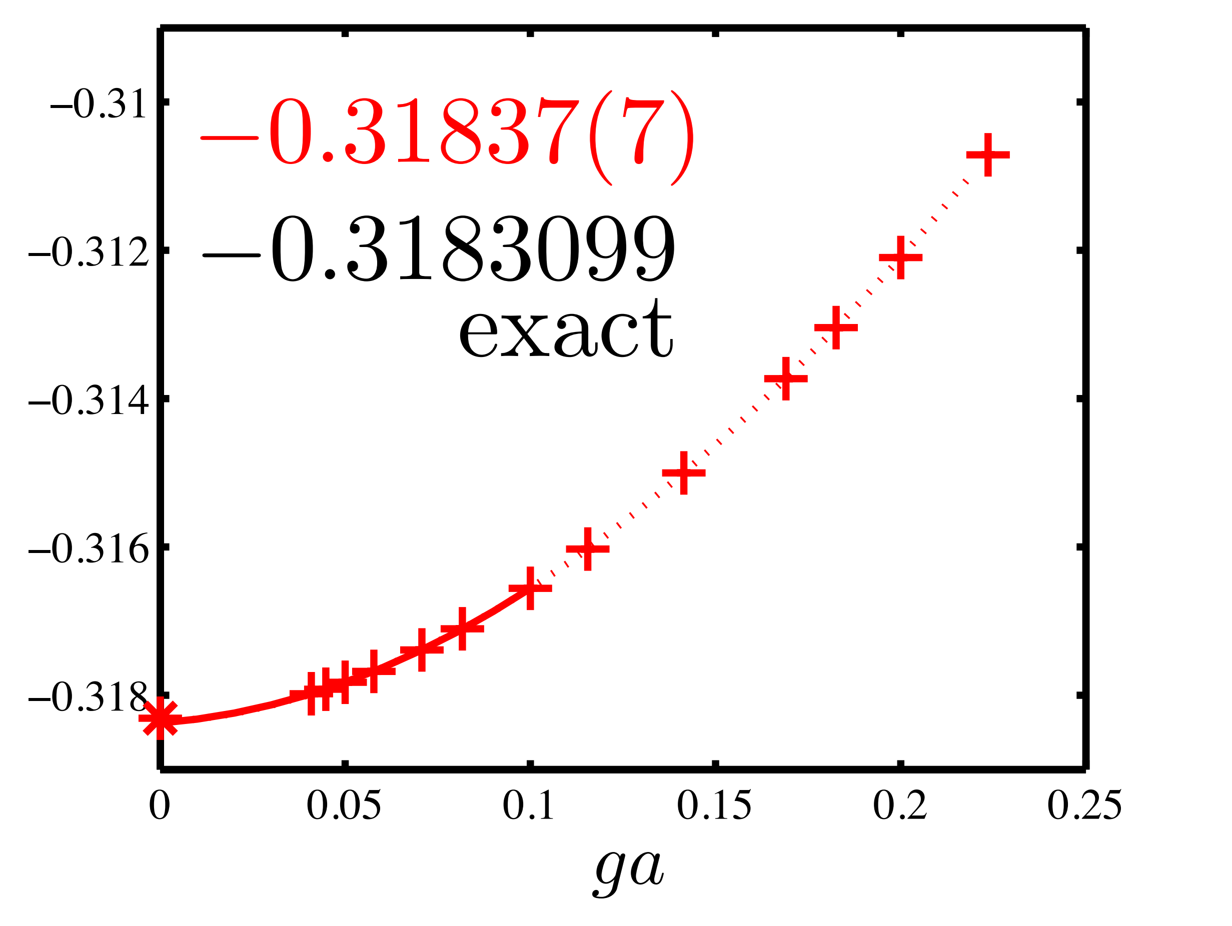
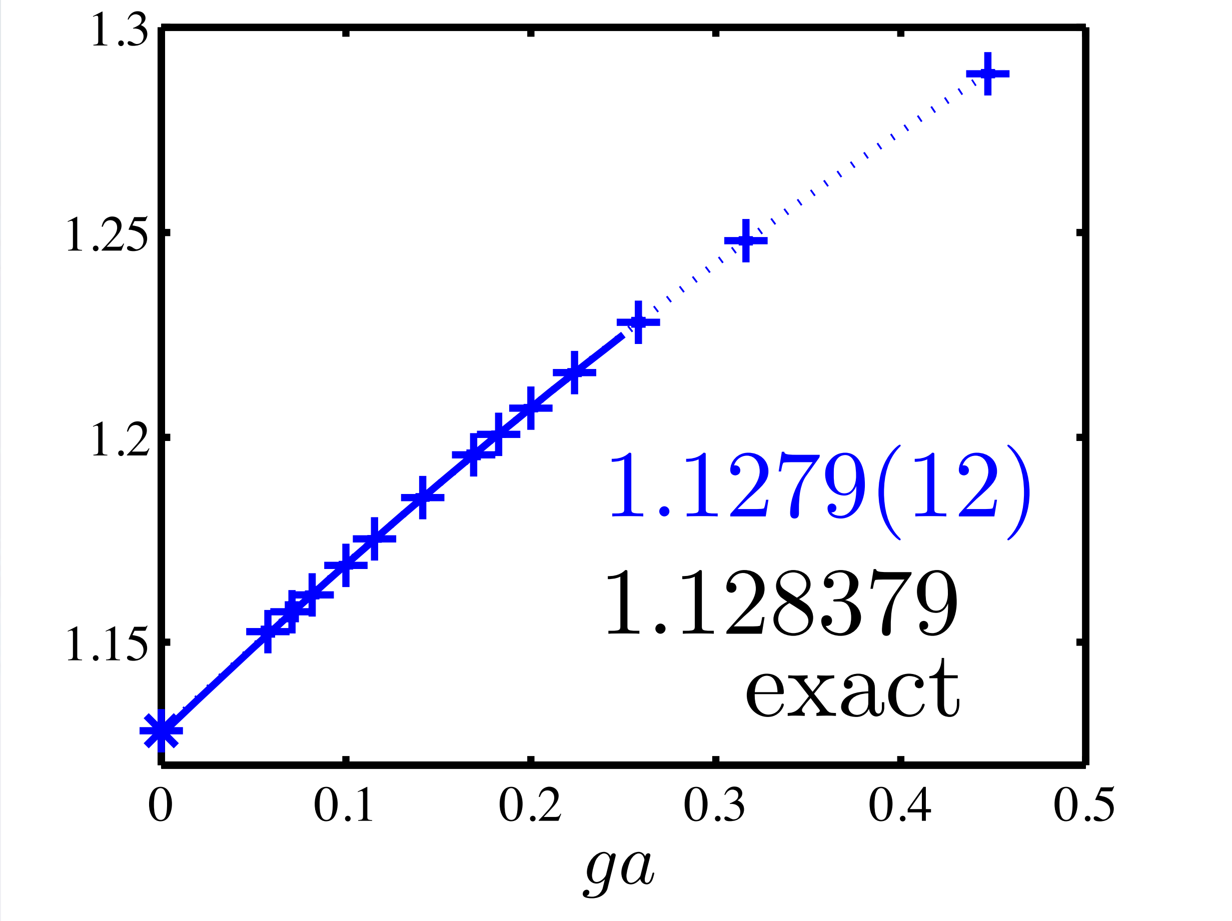
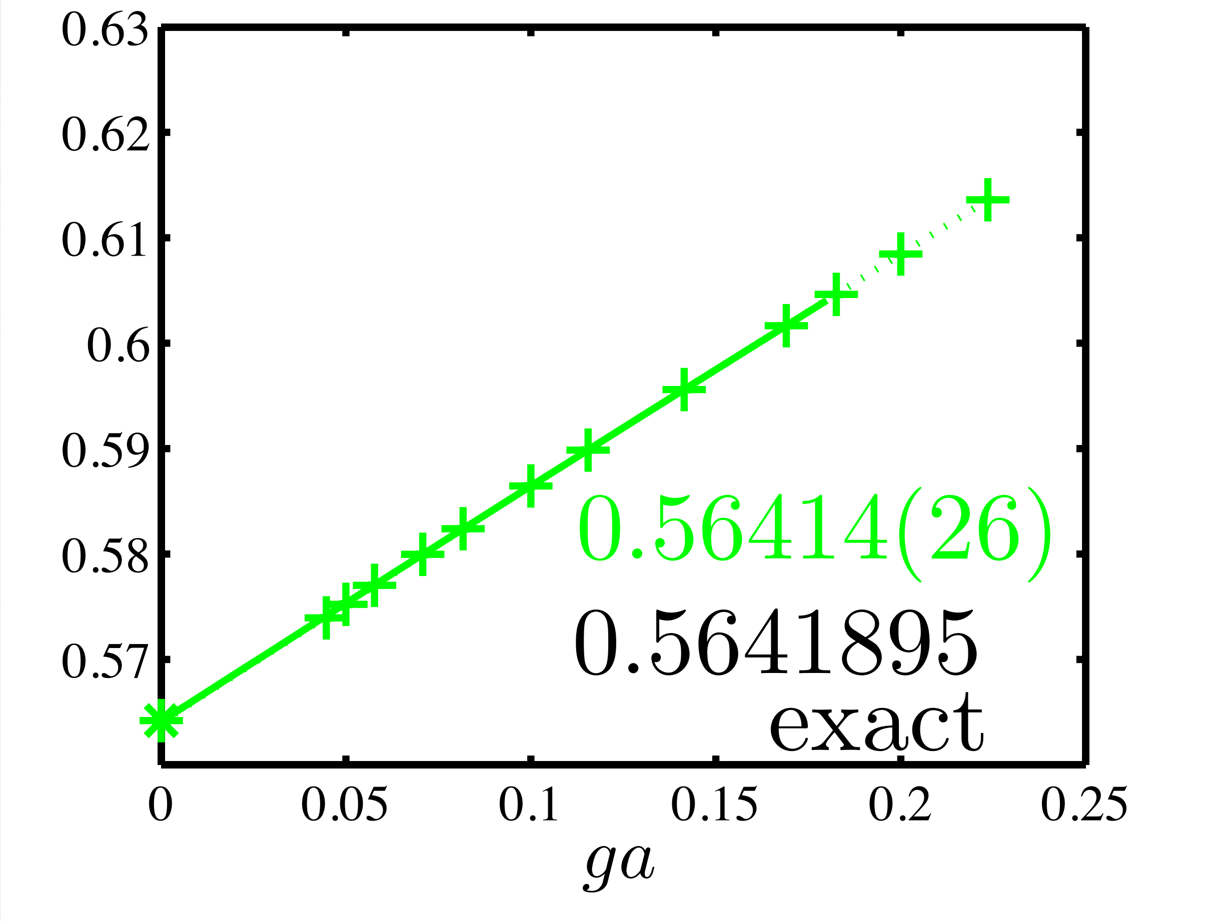




good agreement with exact values

precise extrapolations



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some results

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some results

m/g
0,125
0,25
0,5

JHEP11(2013)158
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some results

m/g	DMRG
0,125	0,53950(7)
0,25	0,51918(5)
0,5	0,48747(2)

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some results

m/g	DMRG	MPS with OBC
0,125	0,53950(7)	0,53946(20)
0,25	0,51918(5)	0,51915(14)
0,5	0,48747(2)	0,48748(6)

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some results

m/g	DMRG	MPS with OBC	SCE
0,125	0,53950(7)	0,53946(20)	1,22(2)
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SCE	MPS with OBC
1,22(2)	1,2155(28)
1,24(3)	1,2239(22)
1,20(3)	1,1998(17)

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comparable or better precision than available numerics

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SCE	MPS with OBC
1,22(2)	1,2155(28)
1,24(3)	1,2239(22)
1,20(3)	1,1998(17)

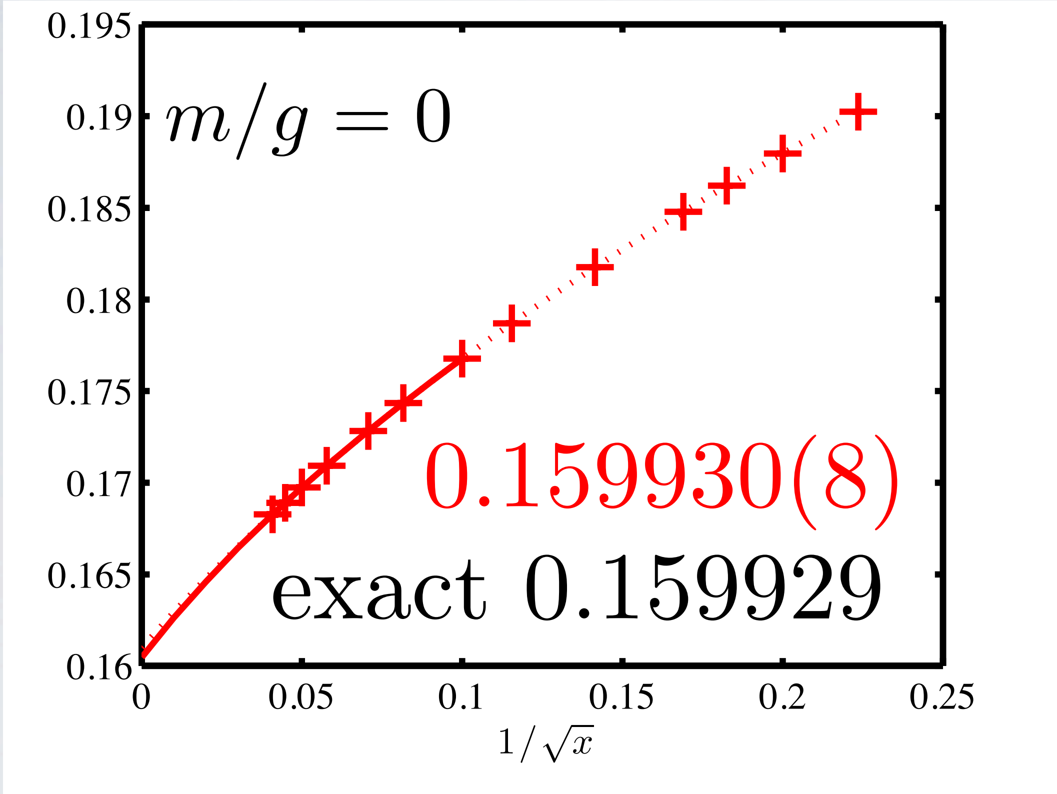
comparable or better precision than available numerics

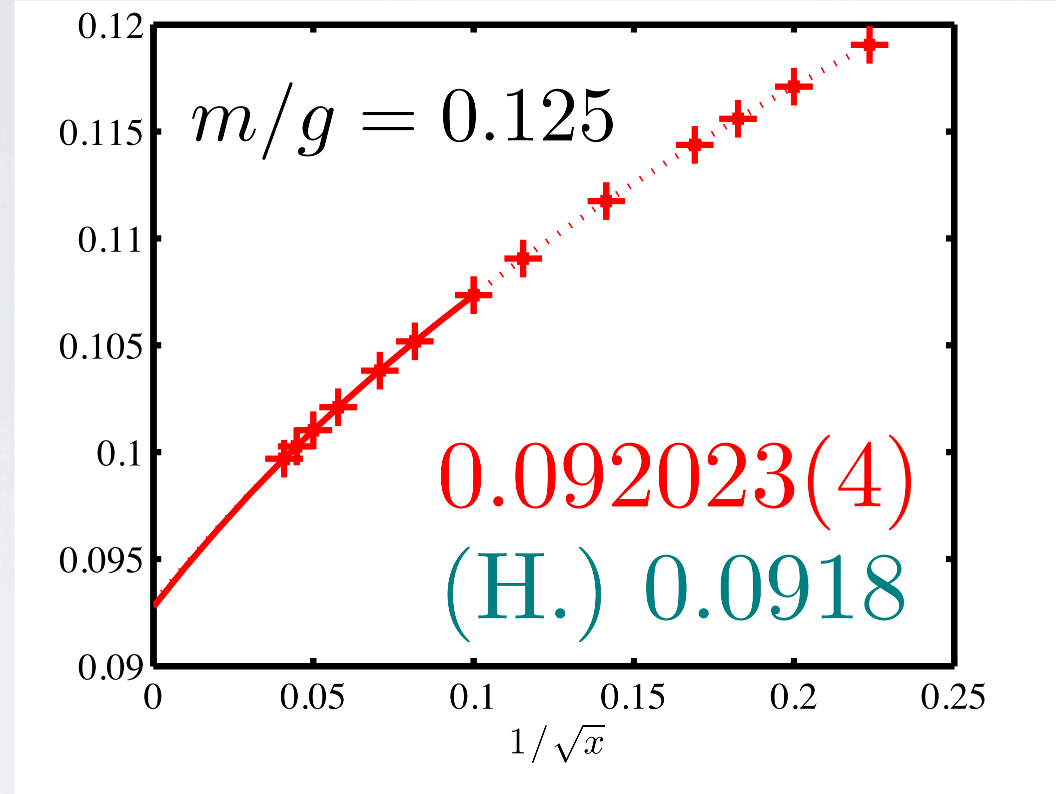
finite T, chemical potential also possible

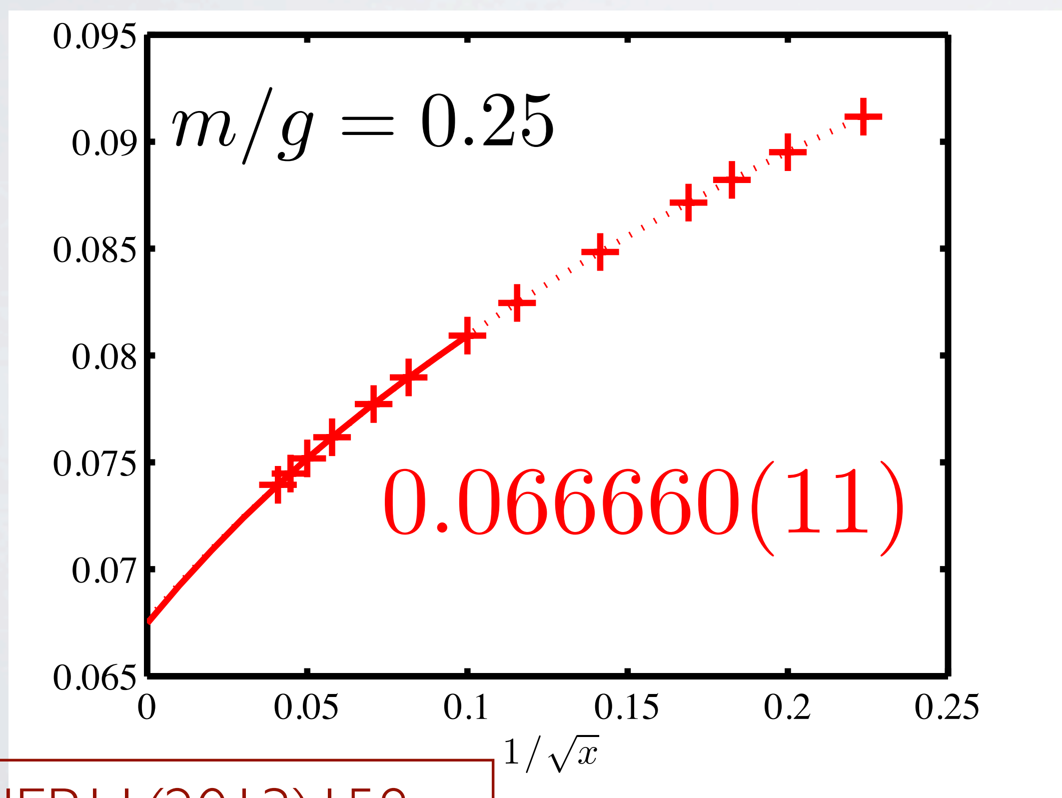
JHEP11(2013)158
arXiv:1310.4118

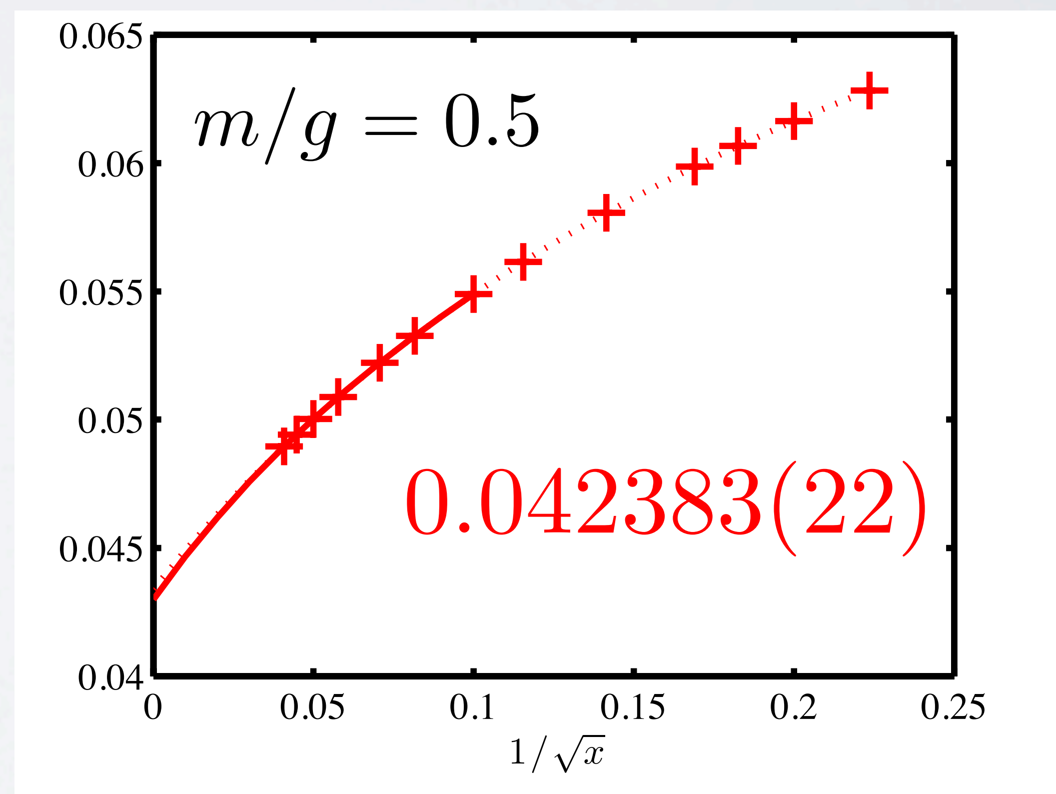
some results

CHIRAL CONDENSATE





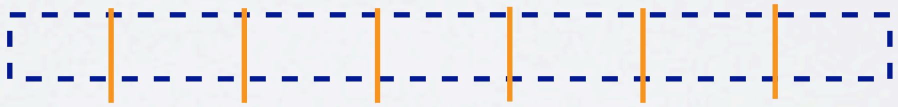




JHEP11(2013)158
arXiv:1310.4118

THERMAL PROPERTIES WITH MPO

$$\rho_{th}(\beta) = e^{-\frac{\beta}{2}H}\mathbf{1}e^{-\frac{\beta}{2}H}$$



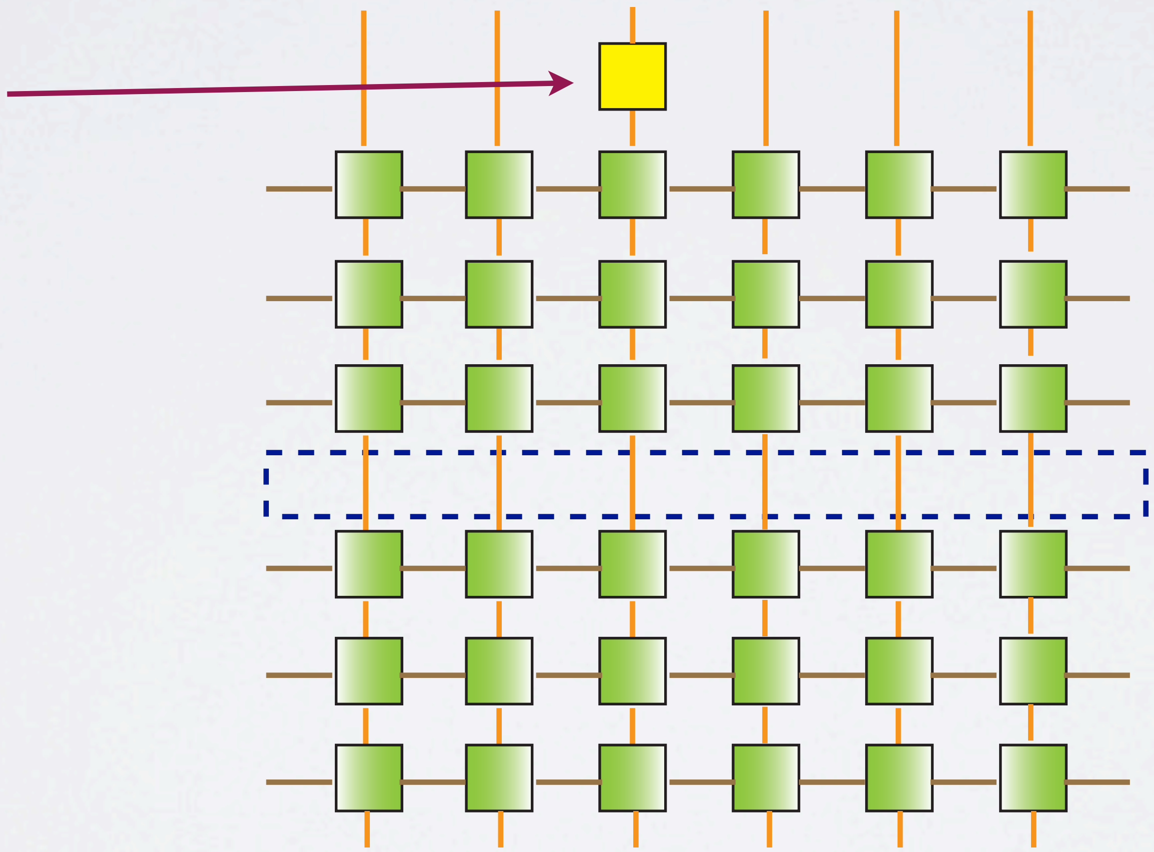
$$\mathbf{1} = \bigotimes \mathbf{1_i}$$

THERMAL PROPERTIES WITH MPO

$$\rho_{th}(\beta) = e^{-\frac{\beta}{2}H}\mathbf{1}e^{-\frac{\beta}{2}H}$$

$$\langle O \rangle_{th} = \mathrm{tr}(Oe^{-\frac{\beta}{2}H}\mathbf{1}e^{-\frac{\beta}{2}H})$$

local operator



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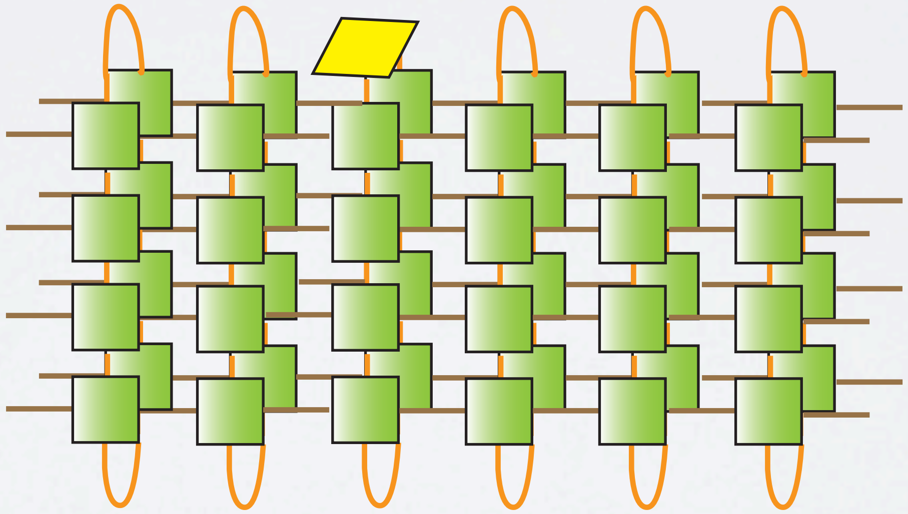

THERMAL PROPERTIES WITH MPO

Scan parameters; perform extrapolations for each β

arxiv:1603.05002

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m/g chiral condensate as a function of temperature, in the continuum $x \to \infty$

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x $x \in [9,\ 1024]$

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Scan parameters; perform extrapolations for each β

m/g chiral condensate as a function of temperature, in the continuum $x \to \infty$

$x \to \infty$

x $\quad x \in [9,\ 1024]$

finite-size

N $\quad N \propto \sqrt{x}$ (up to ~800)

extrapolation

δ $\quad$ sufficiently small for resolution

convergence

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arxiv:1603.05002

THERMAL PROPERTIES WITH MPO

Scan parameters; perform extrapolations for each β

m/g chiral condensate as a function of temperature, in the continuum $x \to \infty$

$x \to \infty$

x $x \in [9,\ 1024]$

plus L_{cut}

finite-size

N $N \propto \sqrt{x}$ (up to ~800)

extrapolation

δ sufficiently small for resolution

convergence

D $D \in [80,\ 160]$

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THERMAL PROPERTIES WITH MPO

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m/g — chiral condensate as a function of temperature, in the continuum $x \to \infty$

$x \to \infty$

x — $x \in [9,\ 1024]$

finite-size

N — $N \propto \sqrt{x}$ (up to ~800)

extrapolation

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convergence

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THERMAL PROPERTIES WITH MPO

Scan parameters; perform extrapolations for each β

chiral condensate as a function of temperature, in the continuum $x \to \infty$

m/g

$x \to \infty$

x

finite-size

N

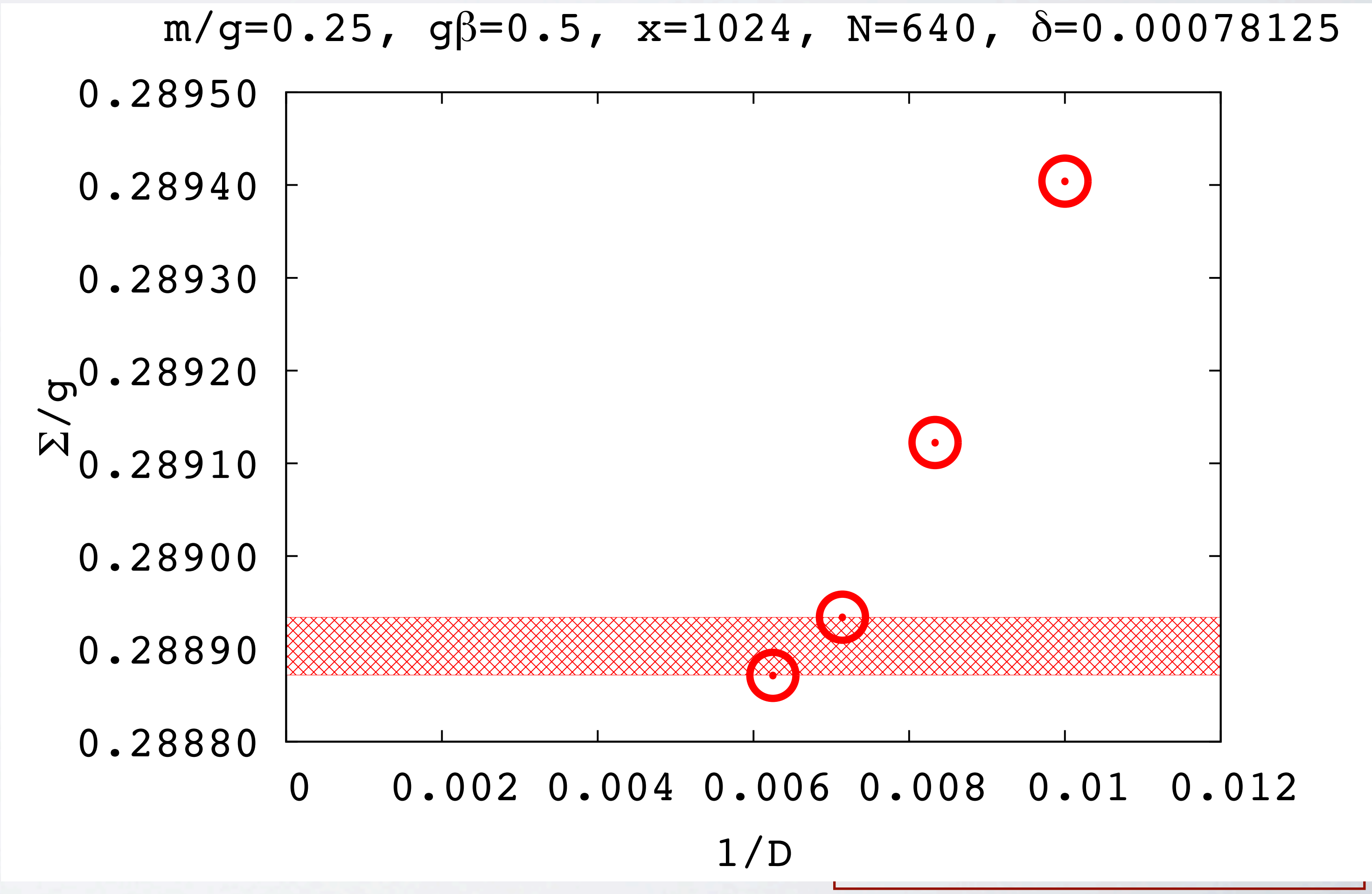
extrapolation

δ

convergence

D

x



THERMAL PROPERTIES WITH MPO

Scan parameters; perform extrapolations for each β

chiral condensate as a function of
temp ... $\rightarrow \infty$

m/g

$x \rightarrow \infty$

x

finite-size

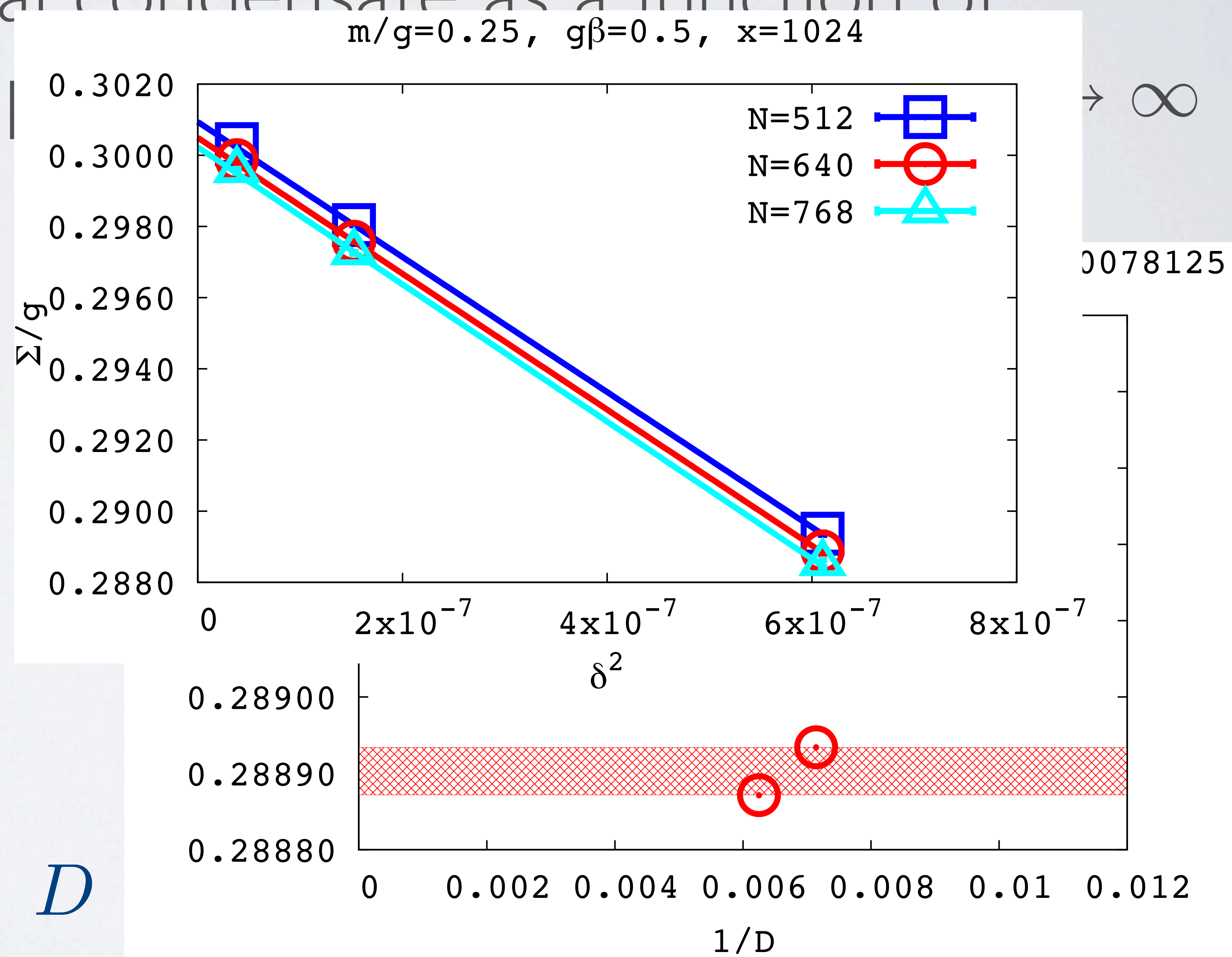
N

extrapolation

δ

convergence

D



THERMAL PROPERTIES WITH MPO

Scan parameters

m/g

$x \to \infty$

x

finite-size

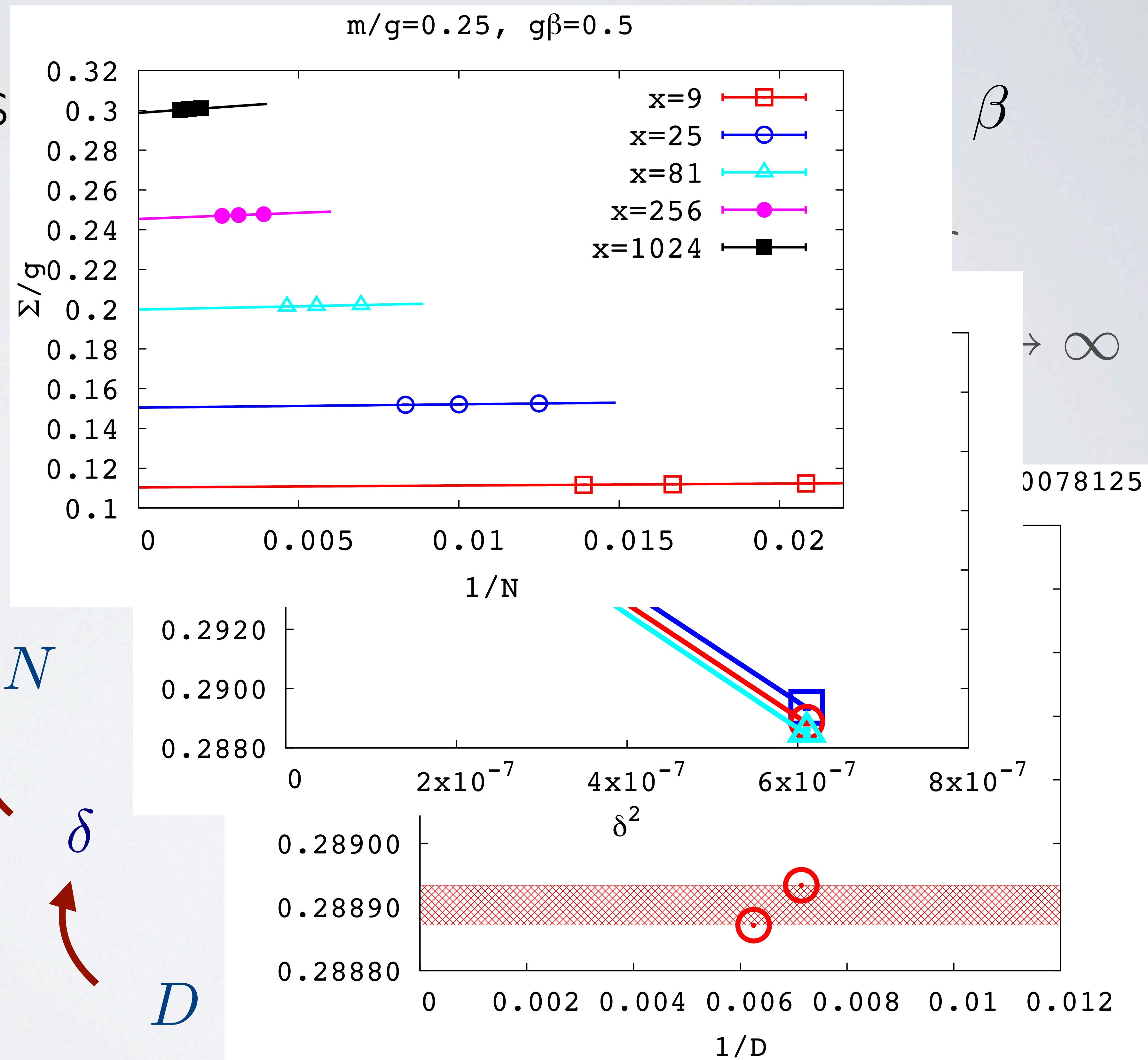
N

extrapolation

δ

convergence

D



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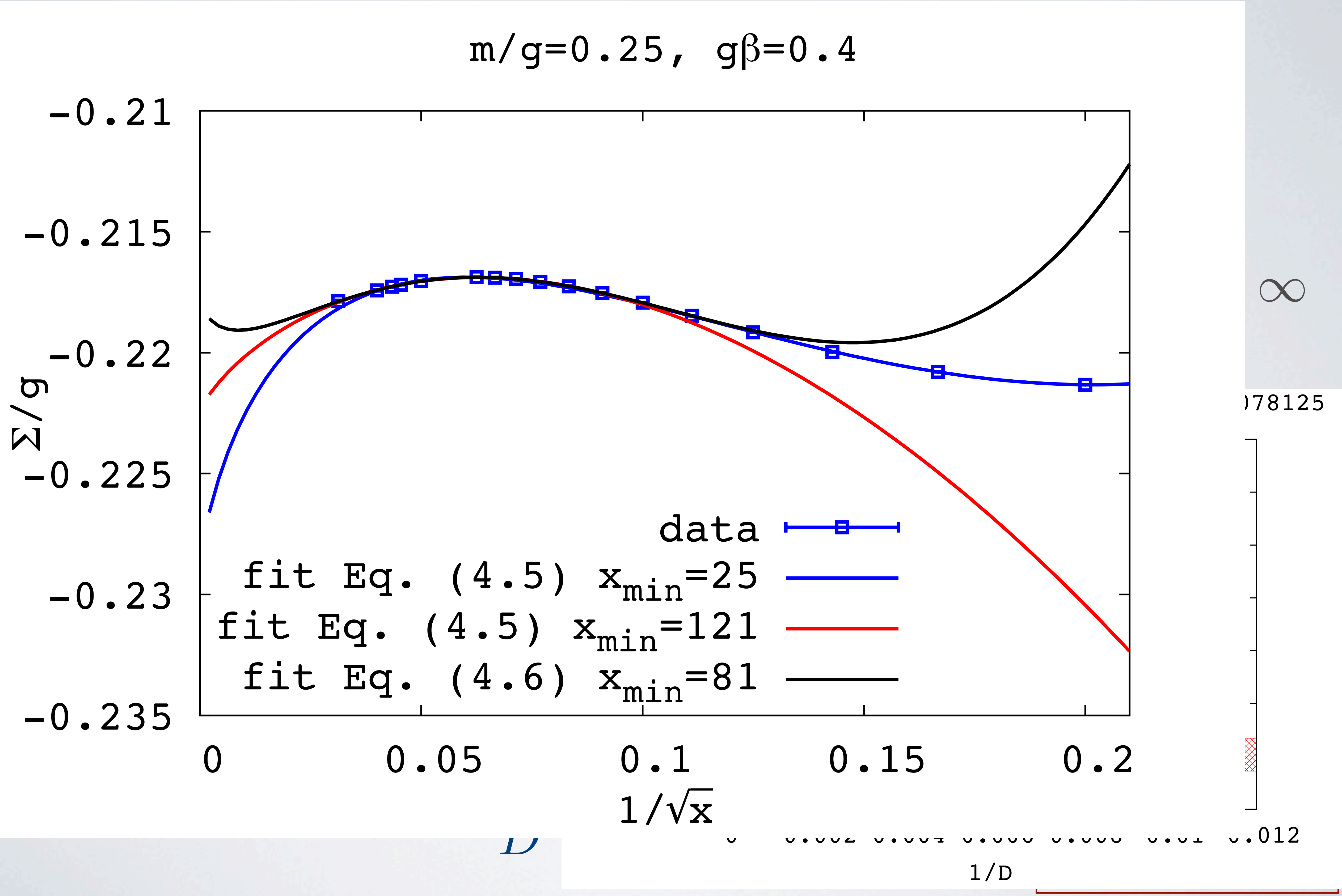

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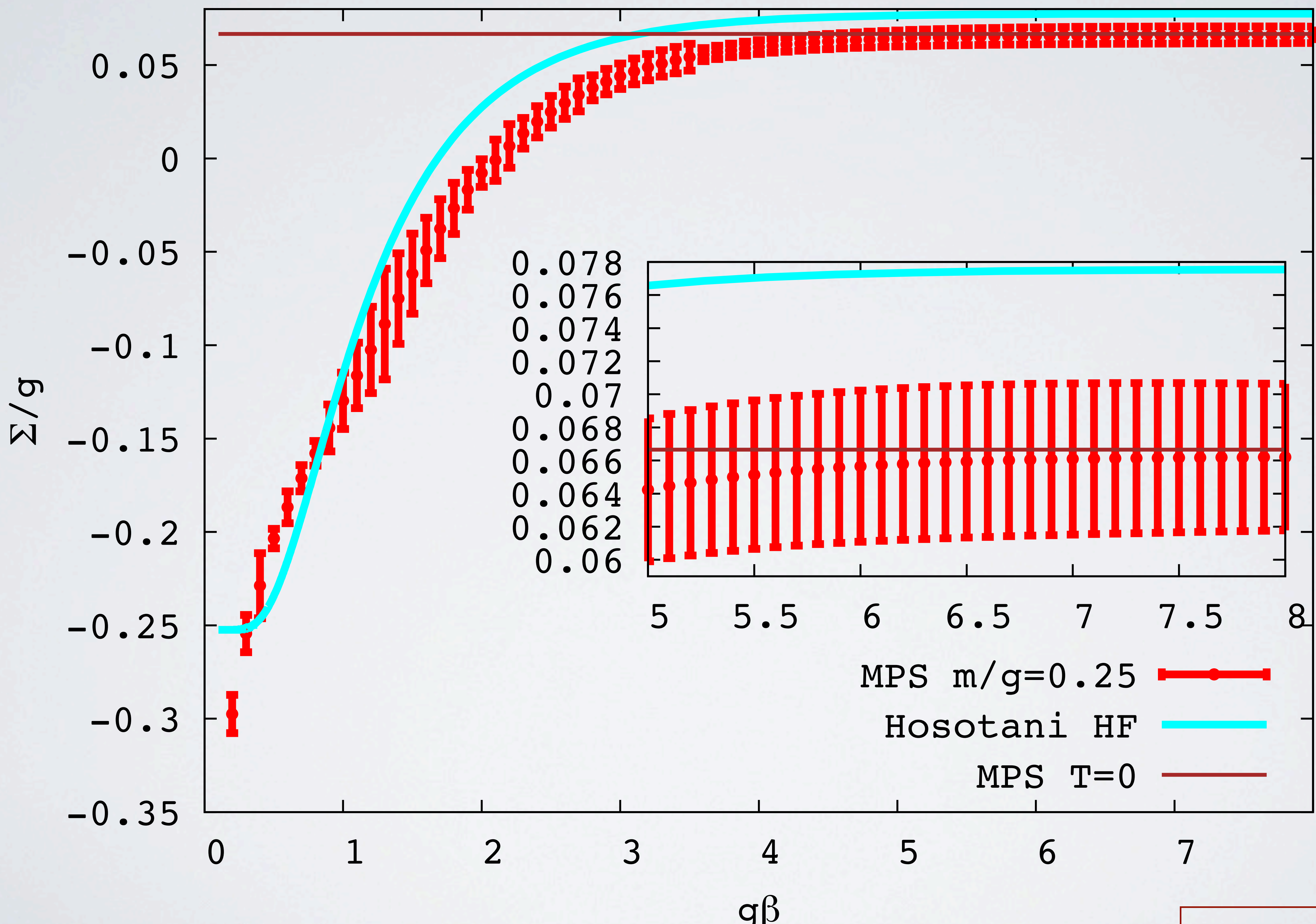
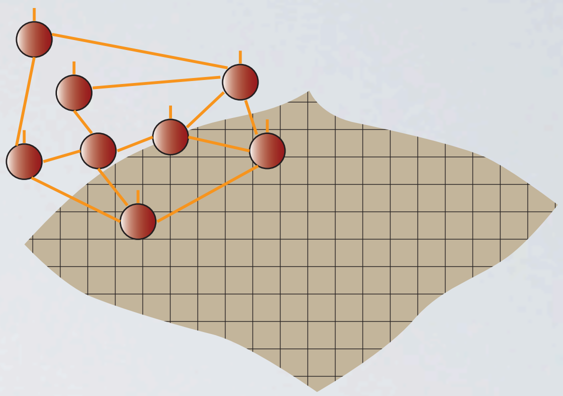


arxiv:1603.05002

PREPARING FOR QUANTUM SIMULATIONS OF LGT

LGT WITH TNS



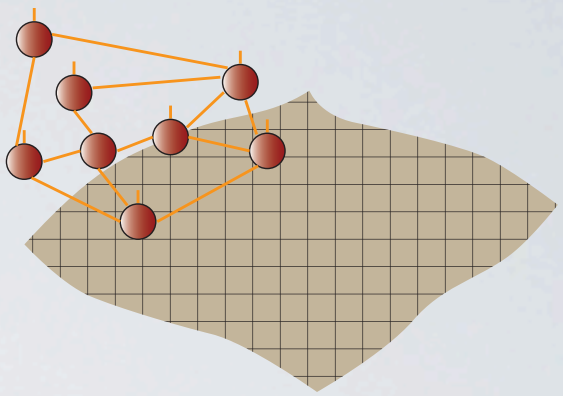
Different approaches

➡ TNS as alternative algorithms for LGT

ultimate goal: quantum simulation

TNS to explore and validate schemes

LGT WITH TNS



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QUANTUM SIMULATIONS

MPS (TNS) tool for classical simulation

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Ultimately wanted: quantum simulator of HEP models

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Several proposals exist using ultracold atoms

finite dimensional dof

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Banerjee et al., PRL 2012

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Zohar et al. PRL 2010, 2012

Tagliacozzo et al. Nat. Comm. 2013

Banerjee et al., PRL 2012

MPS can be very good to validate such schemes

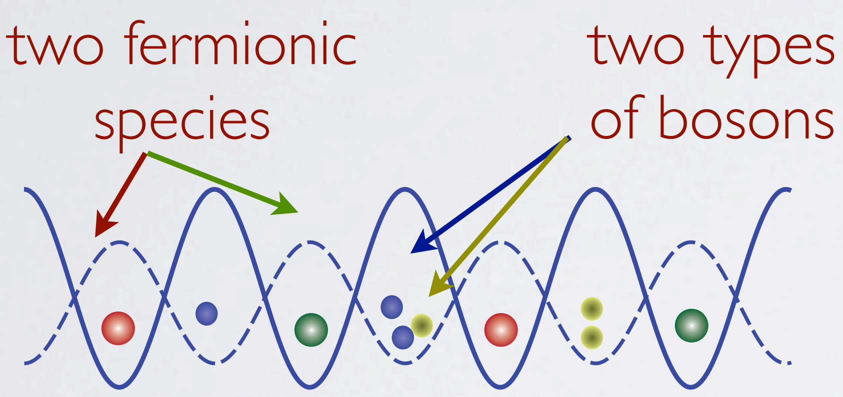
Rico et al. PRL 2014

Pichler et al, PRX 2016

PREPARING FOR QUANTUM SIMULATIONS

PREPARING FOR QUANTUM SIMULATIONS

A feasibility study for Schwinger model



Zohar et al. PRA 2013

S. Kühn et al., Phys. Rev. A 90, 042305 (2014)

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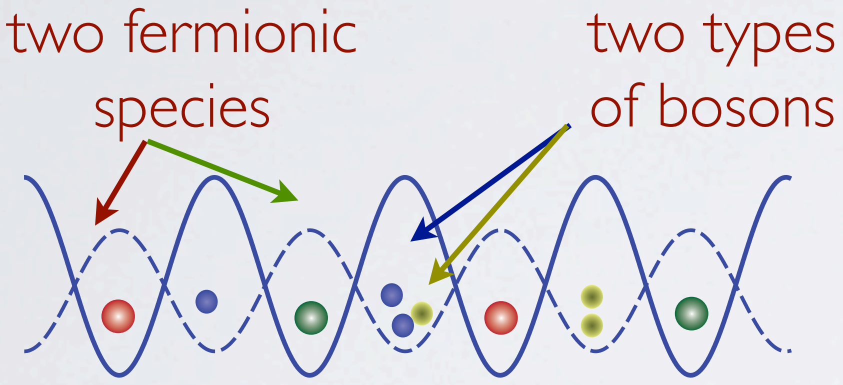

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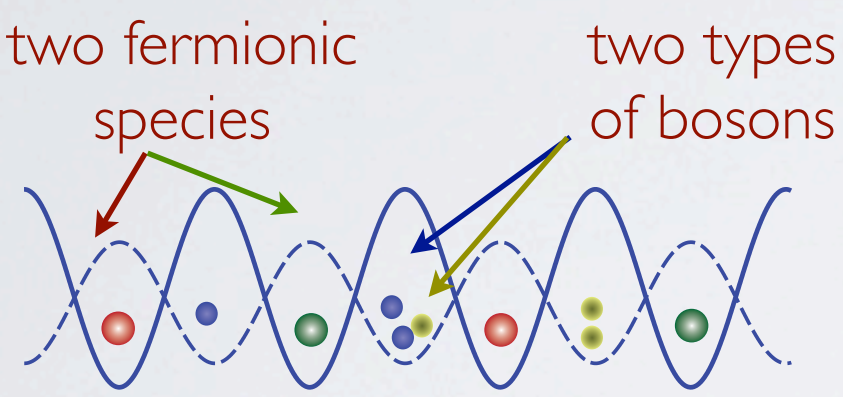

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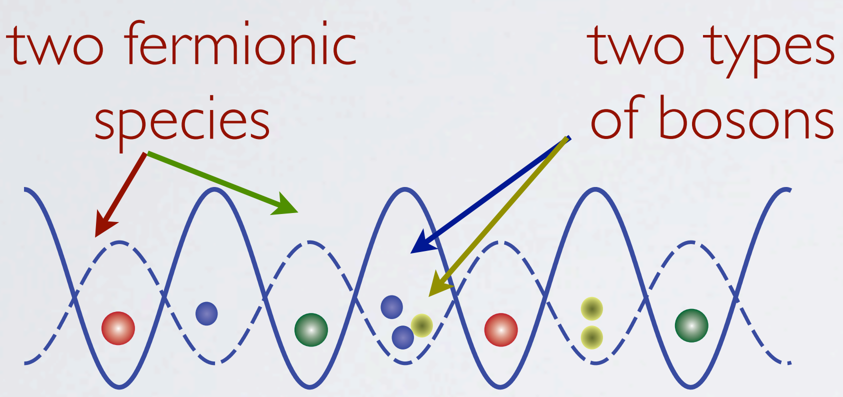

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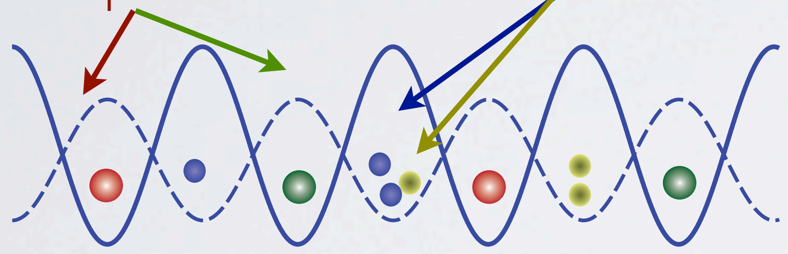

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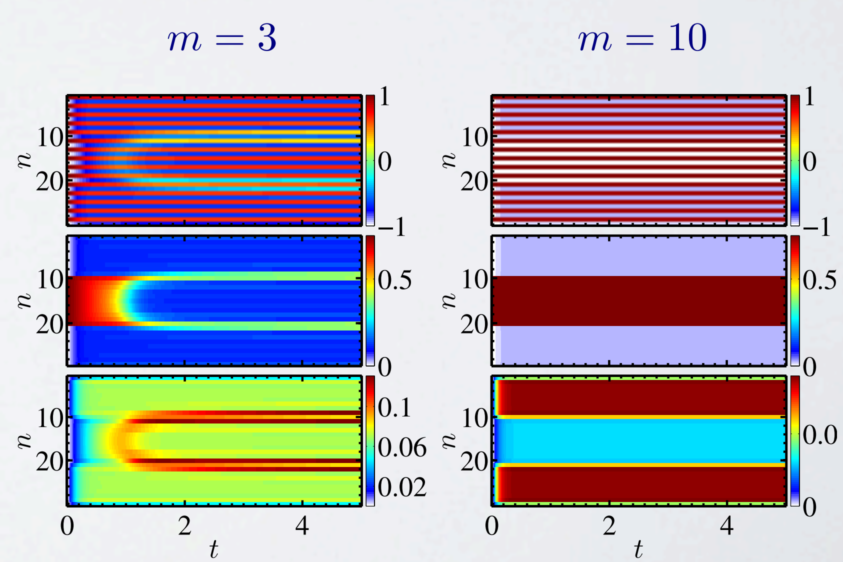
SU(2) in 1+1D



S. Kühn et al., JHEP 07 (2015) 130

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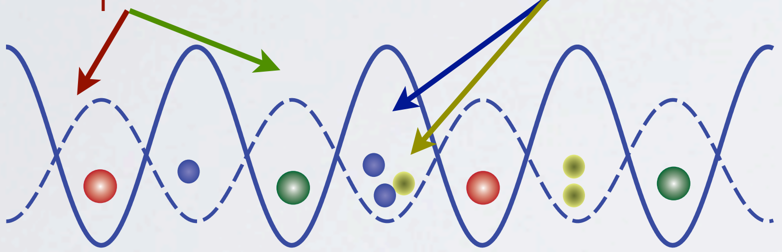

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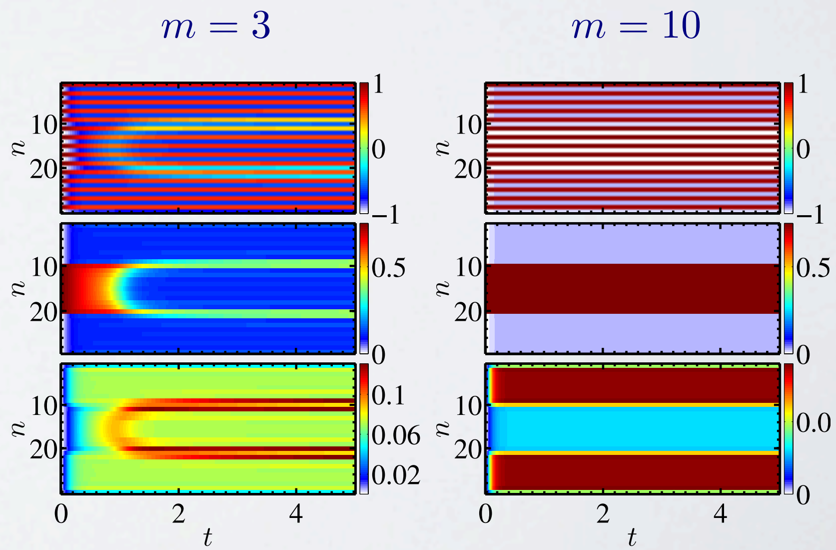

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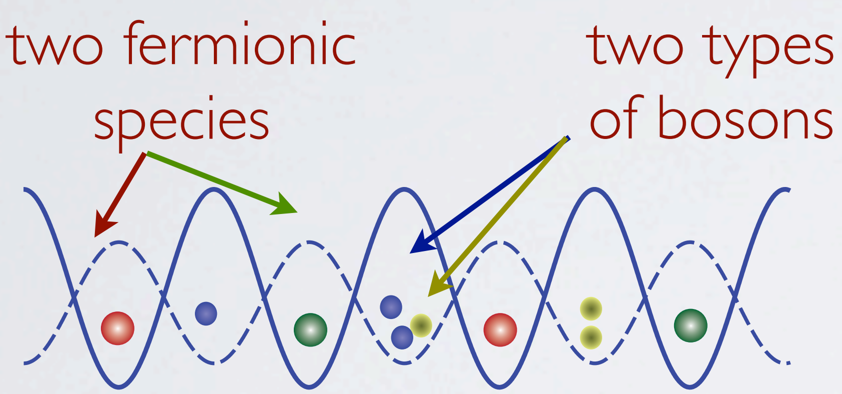

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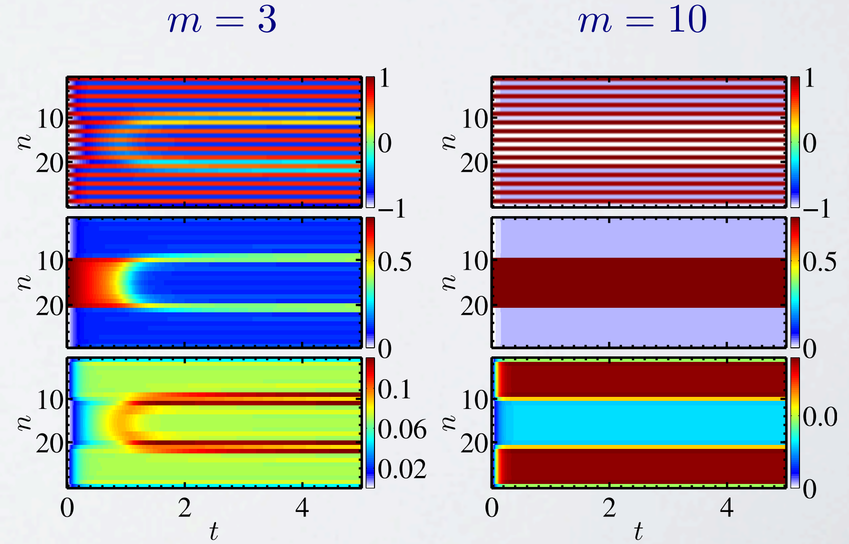
String breaking: statical and dynamical study



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CONCLUSION

Versatile TNS tools can be used for different problems

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Proof of feasibility of TNS for LQFT

JHEP11(2013)158
PRD 92, 034519 (2015)
arXiv:1603.05002

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Very useful for Q Simulators

PRA90, 042305 (2014)
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Very useful for Q Simulators

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THANKS

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PRA90, 042305 (2014)
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THANKS!

MPQ Max Planck Institut
of Quantum Optics
(Garching)