Axion cosmology (how lattice contributes)

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Outline

Strong CP Problem

Full QCD can include an effective CP breaking θ term:

$$\mathcal{L}_{QCD} = \sum_f \bar{\psi}_f (D_\mu \gamma^\mu + m_f)\psi_f + \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} - i\theta \frac{g^2}{32\pi^2} \tilde{F}^a_{\mu\nu} F^a_{\mu\nu}$$

with $-\pi < \theta \leq \pi$, so naturally $\theta \sim \mathcal{O}(1)$

From experiments: $|\theta| < 10^{-10}$, unnatural $\rightarrow$ fine-tuning?

Antrophic principle does not help: $|\theta| < 10^{-2}$ would be still fine

Peccei-Quinn solution

interpret/introduce θ as a dynamical field with minimum at 0

- as phase of a global $U(1)$ symmetric scalar field ϕ
- with spontaneous symmetry breaking potential

redefinition of the angular mode as $\arg(\phi) := \theta_{eff}$

$$\mathcal{Z} = \int \mathcal{D}A_\mu exp(-S_{QCD} - i\theta_{eff} \cdot g^2/32\pi^2 \cdot \tilde{F}^a_{\mu\nu}F^a_{\mu\nu})$$

Z reduced, F raised by phase cancellation unless θ_{eff}=0

one can get the mass of the axion: $m_A^2 \propto \langle Q^2 \rangle \propto \chi_t$

effective potential for ϕ has a tilt & a minimum for $0 = \theta_{eff} = \arg(\phi) = 0$

$$\mathcal{L}_a = \partial_\mu\phi^*\partial^\mu\phi - \frac{\lambda}{8}\left(\phi^*\phi - f_a^2\right)^2 + \chi_t\frac{|\phi|}{f_a}\cos(\theta_{eff})$$

Massive Modes

Two massive oscillations of ϕ

- heavy "string" mode in magnitude; with mass $m_s \approx \sqrt{\lambda} f_a$
- light "axion" mode in phase; with mass $m_a \approx \sqrt{\chi_t}/f_a$

Given χ_t, cosmology gives an abundance of axions

Axions can provide substantial/total amount of dark matter

Two axion production mechanisms:

- dynamics and decay of string/wall networks
- misalignment (sole ingredient in the pre-inflation case)

Topological Structures

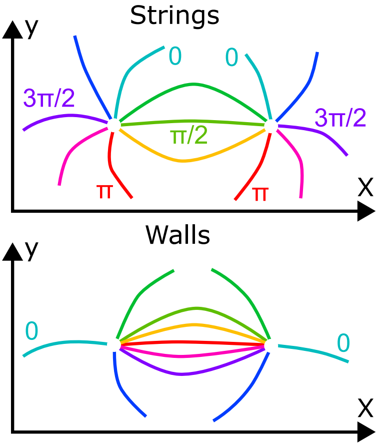
Spontaneous symmetry breaking + causality:

different θ_{eff} in causally disconnected patches

$\Rightarrow$ Strings

with QCD potential $\theta_{eff} \rightarrow 0$ everywhere

$\Rightarrow$ Walls between Strings



String/Wall Networks

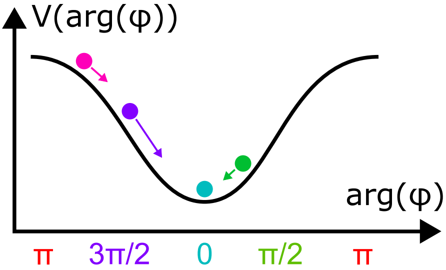
- string-like defects arise and form networks
  $\rightarrow$ axion radiation
- when χ_t becomes relevant, formation of walls between strings
  $\rightarrow$ axion radiation
- walls accelerate annihilation of topological defects
  $\rightarrow$ axion radiation

χ_t influences string dynamics, needed as input for total axion production
only in case of a post-inflationary Peccei-Quinn symmetry breaking

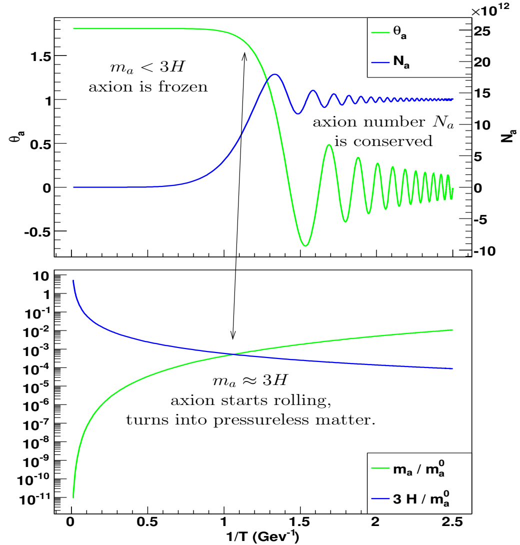
Misalignment

- alignment of misaligned neighbouring patches
  $\rightarrow$ axion radiation
- when χ_t becomes relevant, θ_{eff} "rolls" down to $\theta = 0$
  $\rightarrow$ axion radiation

χ_t influences field dynamics, needed as input for total axion production



Evolution in the expanding universe



Cosmological Models

Both production mechanisms

- depend on χ_t
- depend on the dynamics over cosmological time scales

$\Rightarrow$ need $\chi_t(t)$ over cosmological time scales

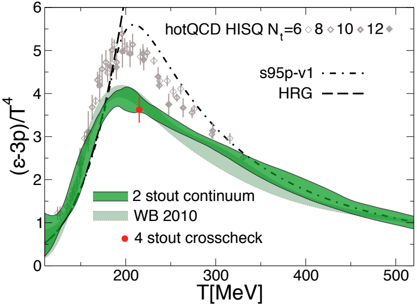
- χ_t is temperature dependent (not explicitly on time)
- the equation of state of QCD gives $T(t)$ for cosmology

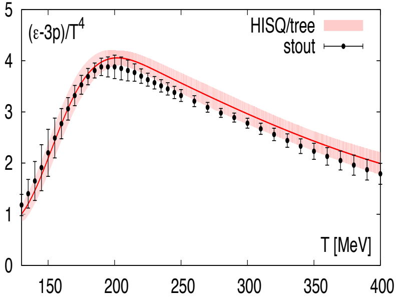
$\Rightarrow$ need $\chi_t(T)$, $p(T)$ for cosmologically relevant temperatures
as we will see T up to few GeV is required: lattice QCD

Trace anomaly continuum result

all of our point with various lattice spacings
comparison with hotQCD (which is the basis of s95p-v1)

result until 2014 ⇓ | result after 2014 ⇓





long standing discrepancy (since 2005) finally disappeared

What We Know About $\chi_t(T)$

Low $T \ll T_c$: χPT

- $\chi_t(T) \approx \chi_0$
- $\chi_t \propto m_f$

  $\rightarrow$ very small χ_t

High $T \gg T_c$: dilute instanton gas approximation (DIGA)

- $\chi_t(T) \sim (T/T_c)^{-b}$, $b \sim 7-8$

  $\rightarrow$ even smaller χ_t

DIGA is a factor of 10 off for the cosmologically relevant region (we observe it aposteriori) $\Rightarrow$ lattice is needed

Quenched Study

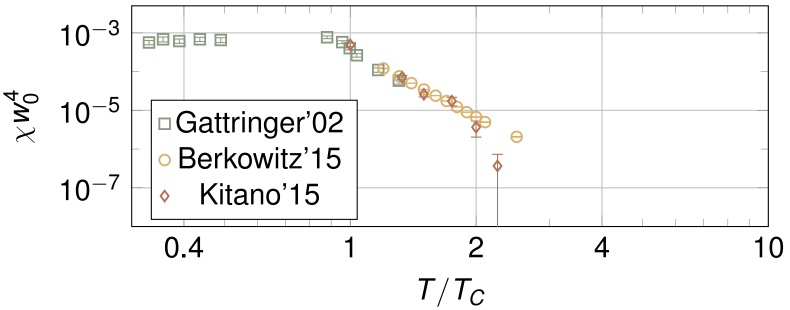
How far can we go with conventional brute force?

$\rightarrow$ test it in the "cheap" quenched case

- learn how to control all errors and apply it for full QCD
- test bed to improve on the brute force strategy
- roughly the same temperature scaling as for full QCD
- estimate the costs for the full result

Previous lattice studies

- [Alles:1996nm,Gattringer:2002mr] etc. 1st gen results
- [Berkowitz:2015aua] large volume/statistics up to $2.5T_c$
- [Kitano:2015fla] HMC up to $2T_c$



Lattice Setup

Pure SU(3)

- Symanzik improved gauge action
- gluonic $q(x)$ from clover field strength tensor $F_{\mu\nu}$
- update sweep: 1 heatbath + 4 overrelaxation

Parameters

- $0.1\ T_c \leq T \leq 4.0\ T_c$
- $n_t = 5, 6, 8$
- spatial volume fixed in physical units $L_{x,y} = 2/T_c$
- $L_z = 2L_{x,y}$ to enable subvolume analysis

Simulations on the Wuppertal-QPACE machine

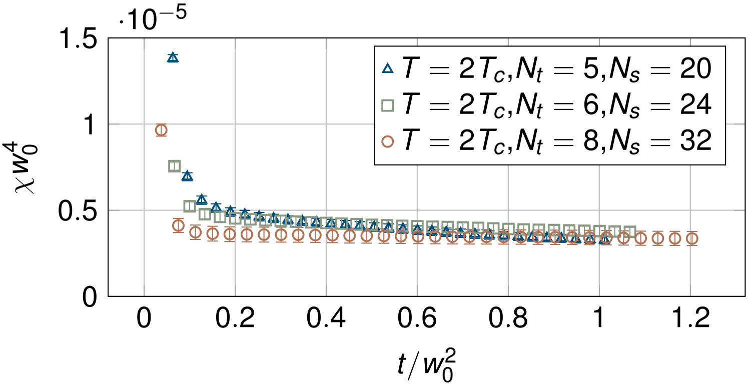
Renormalization of χ

$\chi(t)$ at finite Wilson-flow time is already renormalized [Luscher:2010iy]

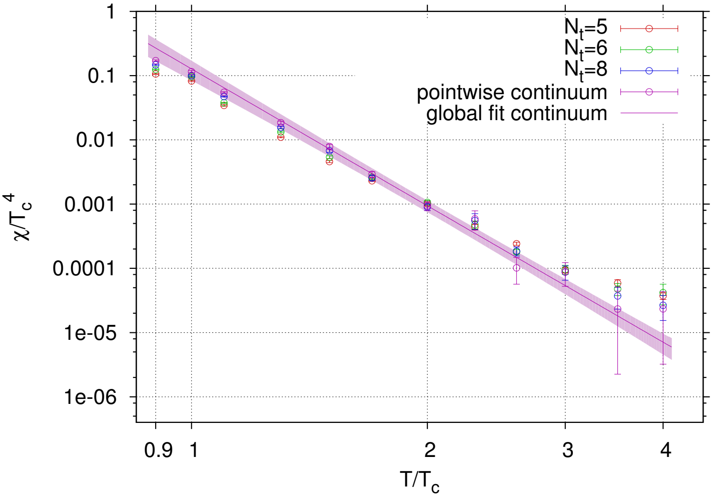
- sufficient to perform a continuum limit at flow time fixed in physical units, e.g. $t = w_0^2$ (w_0^2: flow time at which $td/dt \cdot [t^2 E(t)] = 0.3$ [Borsanyi:2012zs])
- the choice of t influences the size of the lattice artefacts

Flow dependence of $\chi(t)$

- $\chi(t)$ has weak dependence on the choice of t
- we choose $t = w_0^2 \approx (0.176\ \text{fm})^2$
- the finer the lattice the weaker the t-dependence



Continuum result: b=7.1(4)(2) & $\chi(4T_c)^{1/4}$=17 MeV



Quenched Lattice $\leftrightarrow$ DIGA

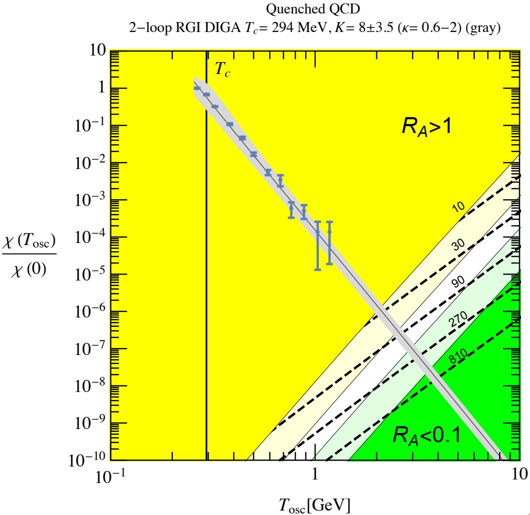
correct T dependence
normalization off by $\mathcal{O}(10)$
fixed by comparison to lattice

how $\chi_t(T)$ determines m_A?
start with an m_A e.g. 30μeV
m_A(T=0) gives the value of f_A

known: Hubble constant H(T)
fix T_{osc} by
$3H(T_{\rm osc}) = m_A(T_{\rm osc})$

using T_{osc} calculate
the amount of dark matter

if it is too much/little iterate



Calibrated guess for dynamical with DIGA

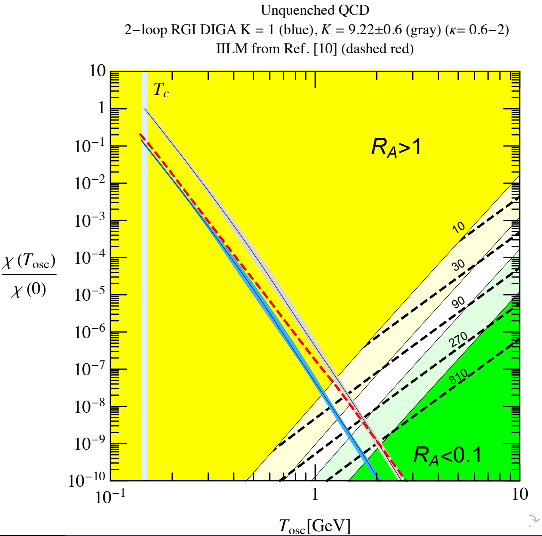
dynamic case with DIGA

quenched calibrated K-factor is $\mathcal{O}(10)$

cosmology can be used axionic dark matter & m_A can be determined

K-factor uncertainty means a factor two in m_A

dream: predict m_A
ADMX experiment: tune it
(eventually even find it)



About costs: quenched case

Cost of the conventional algorithm at relative error $\delta\chi_t$

$$\textit{costs} \propto \frac{1}{(\delta\chi_t)^2 \chi_t(T)}$$

relative cost $(4T_c)/(1T_c)$ (our highest T was $4T_c$: not enough)

from measured $\chi_t(T)$	$4^{7.1} \approx 2 \times 10^4$
from measured $\delta\chi_t$	$10^5 - 10^6$

- quenched $\chi_t(T=0)$ calculated $\sim$ 20 years ago
- Moores law leads to a factor of $\sim 10^5$ in 24 years

$\Rightarrow$ Just possible to do (dynamical case is probably hard)

About costs: dynamical QCD

Dynamic relative cost $\$(7T_c)/\$(1T_c)$ ($7T_c \sim$ 1200 MeV)

from estimated $\chi_t(T)$	$7^{7-8} \approx 10^6 - 10^7$
increasing τ_{int} with T	$10^7 - 10^9$

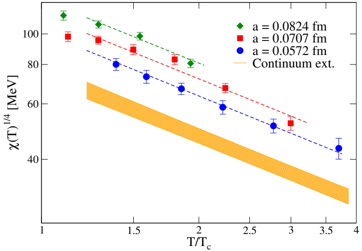
- dynamic $\chi_t(T=0)$ in 2010, Moore factor of ~ 10

$\Rightarrow$ conventional dynamical study not possible (needs 35 years)

Literature: full QCD

Interesting result: [Bonati:2015vqz]

- brute force fully dynamic in the continuum up to $\approx 4T_c$



Result: $b \sim 3$ unexpected (DIGA etc. $b \sim 8$)
one order of magnitude shift for the axion dark matter window

crosses quenched result at $4T_c$ (for quenched $\chi_t^{1/4}(4T_c)$=17 MeV)

$\Rightarrow$ further study is obviously necessary

What did we learn?

Only quenched result, but

- up to $T = 4T_c$ with full systematic errors
- use to calibrate, estimate the costs of full QCD

Finite volume effects in subvolume method

Brute force method

- this far, not further $\rightarrow$ need new ideas
- particularly, since orders of magnitudes needed for full QCD

Topological Charge

Integral

$$Q = \int_{\mathcal{M}} \mathrm{d}^4 x q(x)$$

over the topological charge density

$$q(x) = \frac{1}{4\pi^2} \epsilon_{\mu\nu\rho\sigma} \mathrm{tr}\left(F_{\mu\nu}(x) F_{\rho\sigma}(x)\right)$$

- discretized in finite volume on $\mathcal{M} = \mathbb{T}^4$
- sectors with different Q separated by infinite action barrier in continuum
- problem for ergodicity of MC algorithms with small "step" size in field space

Topological Susceptibility

Integral of qq correlator

$$\chi = \int_{\mathcal{M}} \mathrm{d}^4 x \langle q(0) q(x) \rangle$$

With global translation symmetry on $\mathcal{M} = \mathbb{T}^4$

$$\chi = \frac{1}{V_4} \langle Q^2 \rangle$$

- measurement must sample sectors with $Q \neq 0$
- difficult close to continuum
- difficult when $\chi V_4 = \langle Q^2 \rangle \ll 1$

Subvolume Trick [Brower:2014bqa]

Possible solution

- discretization of Q is finite volume effect
- continuous Q_{sub} on finite subvolumes of $\mathbb{R}^4$ and $\mathbb{T}^4$
- calculate $\chi_{sub} = \langle Q_{sub}^2 \rangle / V_{sub}$
- make infinite V_{sub} limit instead of infinite V_4 limit

Quenched and $T = 0$: large χ

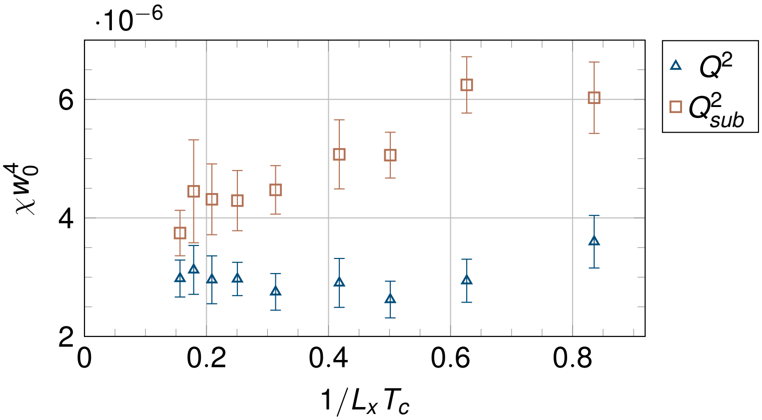
- plausible, works

Dynamic or $T \neq 0$: small χ

- finite volume corrections are T independent
- corrections are larger than χ for reasonable volumes

Subvolume Trick - Finite volume corrections

$T = 2T_c$, $N_t = 5$, $L_{sub} = L_z/2$ correction scales like $1/L$



Subvolume Trick 2

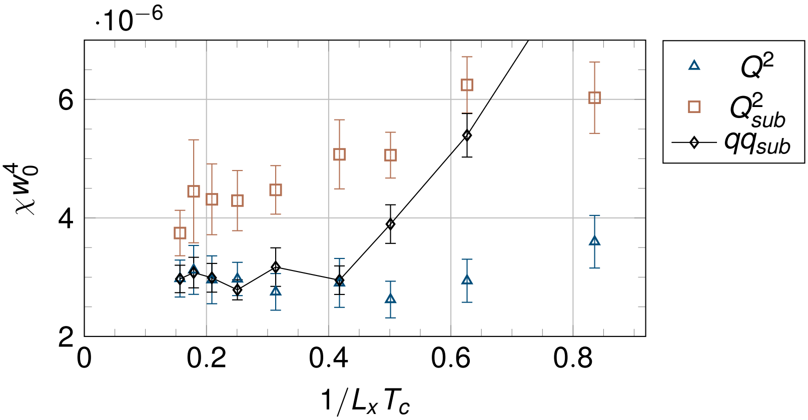
- step from $\chi = \int_{\mathcal{M}} \mathrm{d}^4 x \langle q(0) q(x) \rangle$ to $\chi = \langle Q^2 \rangle / V_4$ required translation invariance of $\mathcal{M} = \mathbb{T}^4$
- not valid for subvolume with boundary $\Rightarrow$ finite volume correction
- large cancellations in integral of correlator $\Rightarrow$ large finite volume error

Alternative:

- evaluate $\chi = \int_0^{L_{sub}} \mathrm{d}z \int \mathrm{d}^3 x \langle q(L_{sub}/2, \vec{0}) q(z, \vec{x}) \rangle$ directly
- correlator is only evaluated at distances in z smaller than $L_{sub}/2$ $\Rightarrow$ reduced finite volume corrections

Subvolume Trick 2 - Finite Volume corrections

$T = 2T_c$, $N_t = 5$, $L_{sub} = L_z/2$, identical configs   no $1/L$



Results - Full Volume versus Subvolume