Hybrid Quarkonium with Non-Relativistic Effective Field Theories

Matthias Berwein

with: Nora Brambilla, Jaume Tarrús Castellà, Antonio Vairo

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Technische Universität München, Physik Department, T30f

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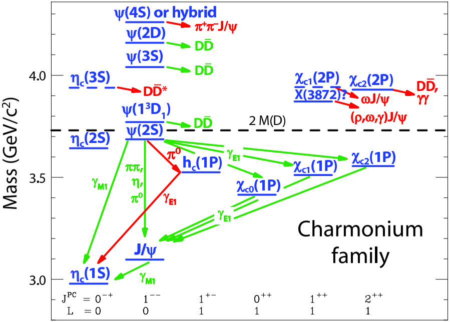
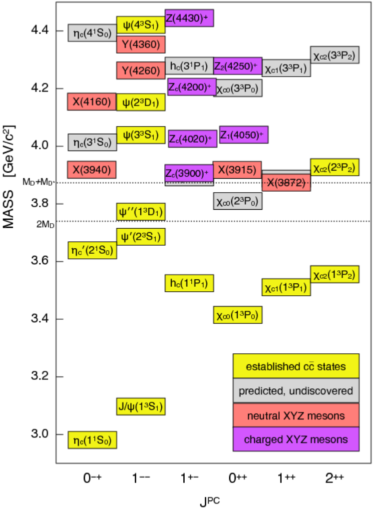
T30f
Theoretische Teilchen-
und Kernphysik

Introduction: Quarkonium and XYZ exotics

Eichten et al 2008



Olsen 2015



- quarkonium: $Q\bar{Q}$ bound state
- hybrids: gluonic excitation
- additional light quarks: tetraquark, meson molecule, diquarkonium, etc

Hybrids: theoretical approaches

Constituent gluon picture
Horn and Mandula 1978

- treat hybrids as a three-body system $Q\bar{Q}g$
- add J^{PC} quantum numbers of gluon and quarkonium

Fluxtube model
Isgur and Paton 1983

- gluons assumed to form string between heavy quarks
- hybrids correspond to vibrational excitations of string

Born-Oppenheimer (BO) approximation
Griffiths, Michael and Rakow 1983

- determine energy spectrum of static quarks
- solve Schrödinger equation with static energy as potential

Effective Field Theory (EFT) treatment

- exploit scale hierarchy $M \gg p_{\mathrm{rel}} \gg E_{\mathrm{kin}}$ in systematic expansion
- integrate out M (NRQCD) and p_{rel} (pNRCD)

Revision: QM perturbation theory

Split H into free and perturbation part, determine free spectrum:

$$H = H_0 + H_1\,, \qquad H_0|n\rangle_0 = E_n^{(0)}|n\rangle_0\,, \qquad H|n\rangle = E_n|n\rangle$$

Naive convergence criterion: ${}_0\langle n|H_1|n\rangle_0 \ll E_n^{(0)}$, but at higher orders:

$$E_n = E_n^{(0)} + {}_0\langle n|H_1|n\rangle_0 + \sum_{k\neq n} \frac{\left|{}_0\langle k|H_1|n\rangle_0\right|^2}{E_n^{(0)} - E_k^{(0)}} + \ldots$$

$\rightarrow$ NLO convergence criterion: $\left|{}_0\langle k|H_1|n\rangle_0\right| \ll E_n^{(0)} - E_k^{(0)}$.

Degenerate PT: new basis of states $|h\rangle_0 \in \mathrm{span}\left\{|n\rangle_0 \middle| E_n^{(0)} \approx E_{n'}^{(0)}\right\}$
such that $H_0 + H_1$ diagonal: ${}_0\langle h'|H_0 + H_1|h\rangle_0 = \delta_{h'h}E_h^{(1)}$

$$E_h = E_h^{(1)} + \sum_{k\notin\{h\}} \frac{\left|{}_0\langle k|H_1|h\rangle_0\right|^2}{E_h^{(1)} - E_k^{(0)}} + \ldots$$

QM perturbation theory for $1/M$ expansion

$1/M$ expanded Hamiltonian through EFT (NRQCD)

Caswell and Lepage 1986

Bodwin, Braaten and Lepage 1995

$$H_0 = \int d^3x \, \mathrm{Tr}\left[\boldsymbol{E}^2 + \boldsymbol{B}^2\right]$$

$$H_1 = \int d^3x \, \psi^\dagger \left(-\frac{\boldsymbol{D}^2}{2M_Q} - c_F \frac{g\boldsymbol{B}\cdot\boldsymbol{\sigma}}{2M_Q}\right)\psi + \int d^3x \, \chi^\dagger \left(\frac{\boldsymbol{D}^2}{2M_{\bar{Q}}} + c_F \frac{g\boldsymbol{B}\cdot\boldsymbol{\sigma}}{2M_{\bar{Q}}}\right)\chi$$

(light quarks and higher $1/M$ terms are neglected, H_0 is static case)

Outline:

- find eigenstates and eigenvalues of H_0
- check if eigenvalues are degenerate
- calculate corrections with (non-)degenerate perturbation theory

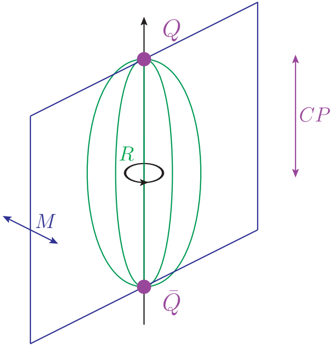
Note: Different situation depending on the relative quark-antiquark distance r

Symmetries of the static system

Static system has cylindric symmetry: $D_{\infty h}$
Elementary group transformations:

- Rotations R around Q-$\bar{Q}$ axis
- CP: Space inversion across center of Q-$\bar{Q}$ combined with charge conjugation
- Reflection M across plane with Q-$\bar{Q}$ axis

All other elements are combinations of these



Static States labeled with associated quantum numbers Λ_η^σ:

- Λ: rotational quantum number; labels Σ, Π, Δ correspond to $\Lambda = 0, 1, 2$
- η: eigenvalue of CP: $g \mathrel{\hat{=}} +1$ (gerade), $u \mathrel{\hat{=}} -1$ (ungerade)
- σ: sign of reflections M; σ only relevant for Σ representations
  for $\Lambda \geq 1$: $\sigma = \pm$ corresponds to different projections of angular momentum

Eigenvalues of H_0

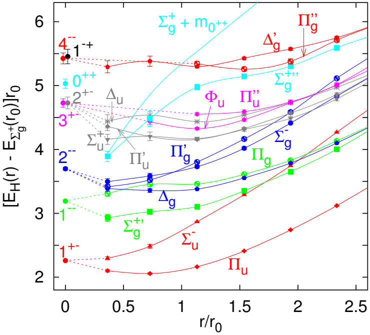
Eigenstates of H_0 are unknown, but lowest eigenvalues can be determined:

$$\langle X(T)|X(0)\rangle = \langle X|e^{-iH_0T}\sum_n |n\rangle_{00}\langle n|X\rangle = \sum_n \left|_0\langle n|X\rangle\right|^2 e^{-iE_n^{(0)}T}$$

If $|X_n\rangle$ has **same quantum numbers** as $|n\rangle_0$, then for those quantum numbers

$$E_n^{(0)} = \lim_{T\to\infty}\frac{i}{T}\ln\langle X_n(T)|X_n(0)\rangle$$

gives the **lowest eigenvalue**!



Bali and Pineda 2003

Lattice results:

- classification through quantum numbers (Λ_η^σ representations of $D_{\infty h}$ group)
- excited energies can be extracted with larger set of $|X_n\rangle$ states
- lowest states are Π_u and Σ_u^-
  $\rightarrow$ nearly degenerate at small r
  $\rightarrow$ well separated at large r

Study small r degeneracy with pNRQCD

Hybrids with pNRQCD

For $r \ll \Lambda_{\rm QCD}^{-1}$ multipole expanded EFT: pNRQCD

Bali and Pineda 1997
Brambilla, Pineda, Soto, Vairo 1999

$$H^{(0,0)} = \int d^3R\,\mathrm{Tr}\left[\boldsymbol{E}^2 + \boldsymbol{B}^2\right] + \int d^3r d^3R\,\mathrm{Tr}\left[S^\dagger V_s S + O^\dagger V_o O\right]$$

$$H^{(0,1)} = \int d^3r d^3R\,\mathrm{Tr}\left[V_A\left(S^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}O + O^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}S\right) + V_B O^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, O\}\right]$$

Convenient states: $|X_{\Pi_u}\rangle = \boldsymbol{r}\times\boldsymbol{B}^a O^{a\,\dagger}|\mathrm{vac}\rangle$ $|X_{\Sigma_u^-}\rangle = \boldsymbol{r}\cdot\boldsymbol{B}^a O^{a\,\dagger}|\mathrm{vac}\rangle$

Degenerate energies: $E^{(0)}_{\Pi_u/\Sigma_u^-}(r) = V_o(r) + \Lambda_B + \mathcal{O}\left(r^2\right)$

with gluelump mass $\Lambda_B = \lim_{T\to\infty}\frac{i}{T}\ln\langle\mathrm{vac}|\boldsymbol{B}^a(T)\cdot\boldsymbol{B}^a(0)|\mathrm{vac}\rangle$

Degeneracy is broken through $\mathcal{O}\left(r^2\right)$ terms given through 4-gluon correlators

$$\langle\mathrm{vac}|B_i^a(T)E_j^b(t)E_k^c(t')B_l^d(0)|\mathrm{vac}\rangle$$

Beyond the static limit

Lift short distance degeneracy with next term in expansion

$$H^{(1,-2)} = \int d^3r d^3R \, \mathrm{Tr} \left[-S^\dagger \frac{\boldsymbol{\nabla}_r^2}{2\mu} S - O^\dagger \frac{\boldsymbol{\nabla}_r^2}{2\mu} O \right]$$

In the pNRQCD power counting $H^{(1,-2)} \gg H^{(1,0)}$ and $H^{(1,-2)} \sim H^{(0,0)}$

In **degenerate PT**: find states that diagonalize $H^{(0,0)} + H^{(1,-2)}$

Write proper eigenstates of $H^{(0,0)}$ generically as $|n\rangle_0 = \hat{\boldsymbol{n}} \cdot \boldsymbol{G}^a O^{a\,\dagger} |\mathrm{vac}\rangle$

For Σ_u^- and Π_u: projectors $\hat{\boldsymbol{n}}_\Sigma \parallel \boldsymbol{r}$, $\hat{\boldsymbol{n}}_\Pi \perp \boldsymbol{r}$, and $\boldsymbol{G}$ some 1^{+-} gluon

Matrix elements given by:

$${}_0\langle n' | H^{(0,0)} + H^{(1,-2)} | n \rangle_0 = \delta_{n'n} E_n^{(0)}(r) + \hat{\boldsymbol{n}} \cdot \frac{\boldsymbol{\nabla}_r^2}{2\mu} \hat{\boldsymbol{n}}$$

Differential operator, so need **wave function** to diagonalize!

Coupled radial Schrödinger equations

Projection vectors in matrix elements allow for two different solutions (coupled or uncoupled) for the Σ_u^- and Π_u radial wave functions:

1st solution

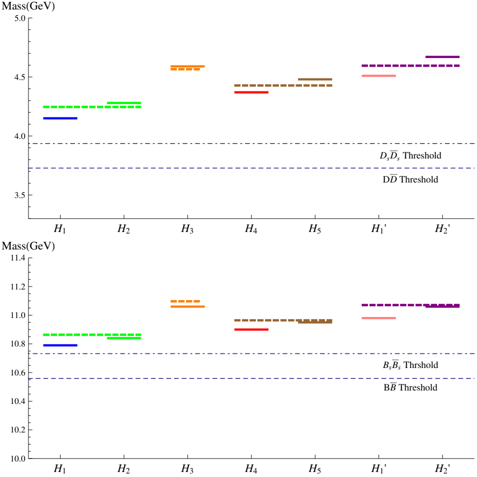
$$\left[-\frac{1}{2\mu r^2}\,\partial_r r^2 \partial_r + \frac{1}{2\mu r^2}\begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_\Sigma^{(0)} & 0 \\ 0 & E_\Pi^{(0)} \end{pmatrix}\right]\begin{pmatrix} \psi_\Sigma \\ \psi_\Pi \end{pmatrix} = \mathcal{E}\begin{pmatrix} \psi_\Sigma \\ \psi_\Pi \end{pmatrix}$$

2nd solution

$$\left[-\frac{1}{2\mu r^2}\,\partial_r\, r^2\, \partial_r + \frac{l(l+1)}{2\mu r^2} + E_\Pi^{(0)}\right]\psi_\Pi = \mathcal{E}\,\psi_\Pi$$

- energy eigenvalue $\mathcal{E}$ gives hybrid mass: $m_H = m_Q + m_{\bar{Q}} + \mathcal{E}$
- $l(l+1)$ is the eigenvalue of angular momentum $\boldsymbol{L}^2 = \left(\boldsymbol{L}_{Q\bar{Q}} + \boldsymbol{L}_g\right)^2$
- the two solutions correspond to **opposite parity** states: $(-1)^l$ and $(-1)^{l+1}$
- corresponding eigenvalues under charge conjugation: $(-1)^{l+s}$ and $(-1)^{l+s+1}$
- Schrödinger equations can be solved numerically

Results I: Comparison to BO approximation



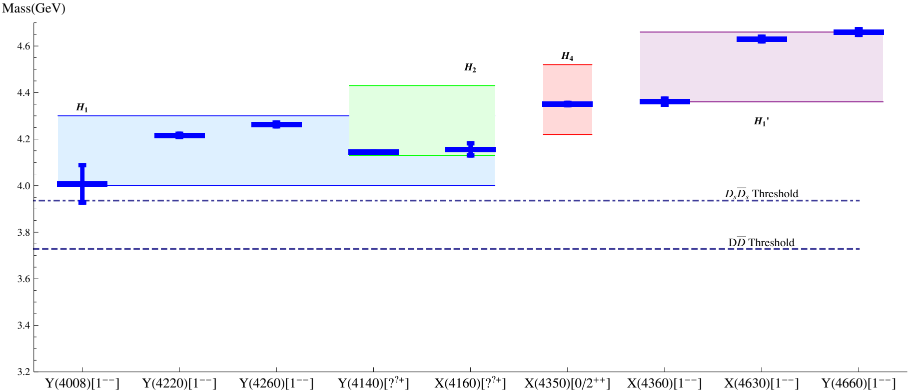
Braaten, Langmack and Smith 2014

For Hybrid multiplets:

	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

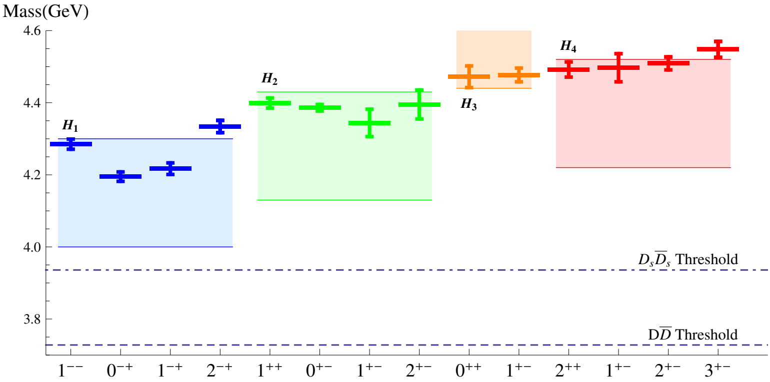
- no distinction between opposite parity states in BO
- mixed states lie lower than pure
- discrepancy for H_2, H_3 and H_5 due to different potential fits

Results II: Comparison to experiment



- comparison based on quantum numbers and mass
- error bands from uncertainty in gluelump mass Λ_B
- several candidates, but most violate heavy quark spin symmetry in decays
- only compatible state is $Y(4220)$, but not yet well established

Results III: Comparison to lattice



Hadron Spectrum Collaboration 2012

- in EFT distinction between different spins only at order $1/M^2$
- good agreement for relative distance between spin-averaged multiplets
- some overall shift for absolute values

Conclusions and outlook

Conclusions

- study of hybrids in EFT framework (NRQCD and pNRQCD)
- study of the spectrum of the static Hamiltonian with non-static corrections
- short distance degeneracy of static energies
- mixing of different static states
- breaking of degeneracy between opposite parity states

Outlook

- development of full EFT treatment
- inclusion of spin dependent corrections
- inclusion of light quark contributions $\rightarrow$ study of tetraquarks or decays

Thank you for your attention!