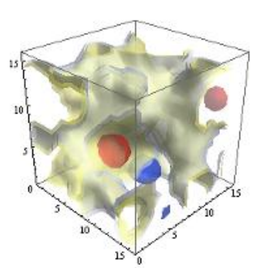
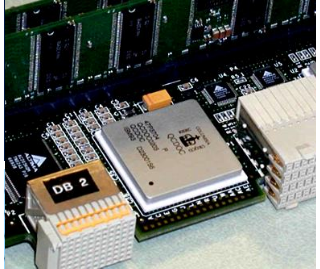
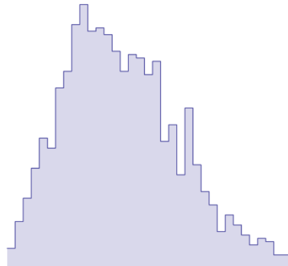
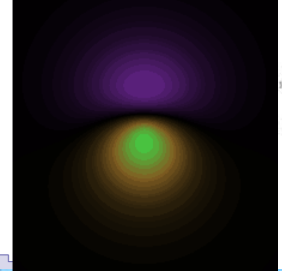
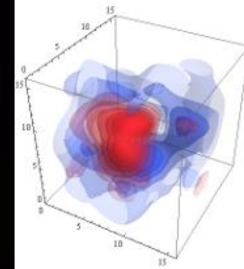
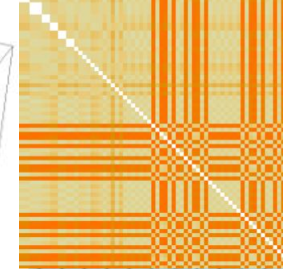
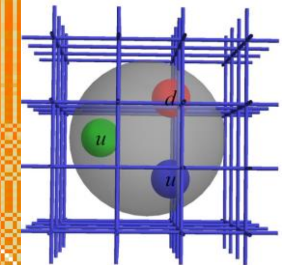



Bjorken-x Dependence of Nucleon Structure from LQCD

Huey-Wen Lin

This talk is based on

"Flavor Structure of the Nucleon Sea from Lattice QCD", PRD 91, 054510 [1402.1462]

and

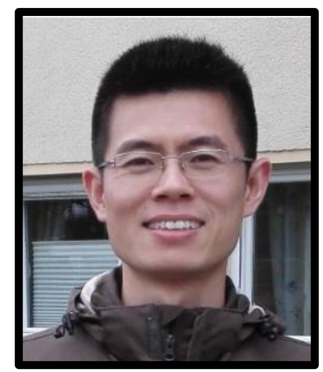
"Nucleon Helicity and Transversity Parton Distributions from Lattice QCD", 1603.06664 [hep-ph]

in collaboration with

Jiunn-Wei Chen (NTU)

Saul Cohen

Xiangdong Ji (UMD/SJTU/INPAC)

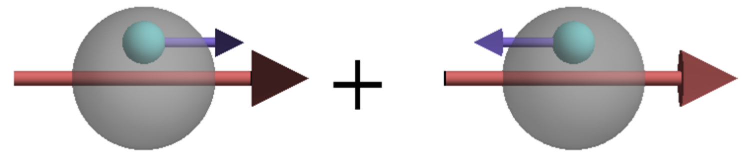


Jian-Hui Zhang (Regensburg)

Parton Distribution Functions

§ PDFs are universal quark/gluon distributions inside nucleon

§ Quark distribution $q(x)$



spin-averaged/unpolarized

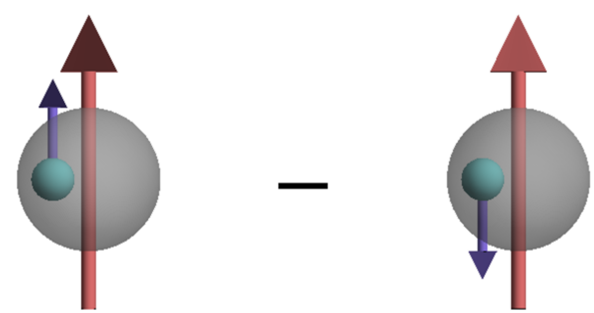
- Processes: DIS (F_2, σ), Drell-Yan, W-asymmetry, Z-rapidity, (γ+) jet, …
- Experiments: BCDMS, NMC, SLAC, JLab, HERA, E866, CDF, DØ,…

§ Helicity distribution $\Delta q(x)$

spin-dep./long. polarized

- Processes: polarized DIS, semi-inclusive DIS, photo- and electroproduction of hadrons and charm, pp collisions
- Experiments: EMC, HERMES, Hall A, CLAS, COMPASS, STAR, PHENIX, …

§ Transversity distribution $\delta q(x)$



transversely polarized

- Process: single-spin asymmetry in SIDIS, …
- Experiments: HERMES, COMPASS, Belle…

Parton Distribution Functions

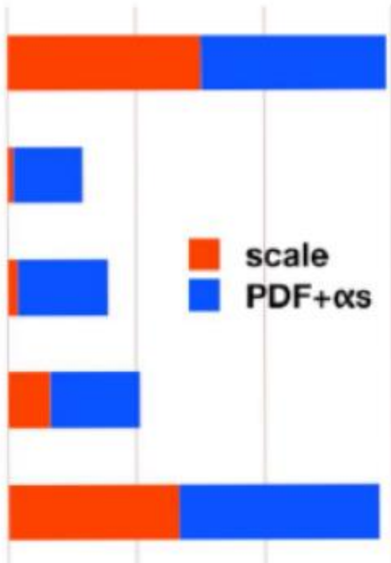
§ PDFs are universal quark/gluon distributions inside nucleon

§ Important inputs to discern new physics at LHC

- Currently dominate errors in Higgs production

HIGGS PRODUCTION

		σ (8 TeV)	uncertainty
NNLL QCD +NLO EW	gg→H	19.5 pb	14.7%
	VBF	1.56 pb	2.9%
NNLO QCD +NLO EW	WH	0.70 pb	3.9%
	ZH	0.39 pb	5.1%
NLO QCD	ttH	0.13 pb	14.4%

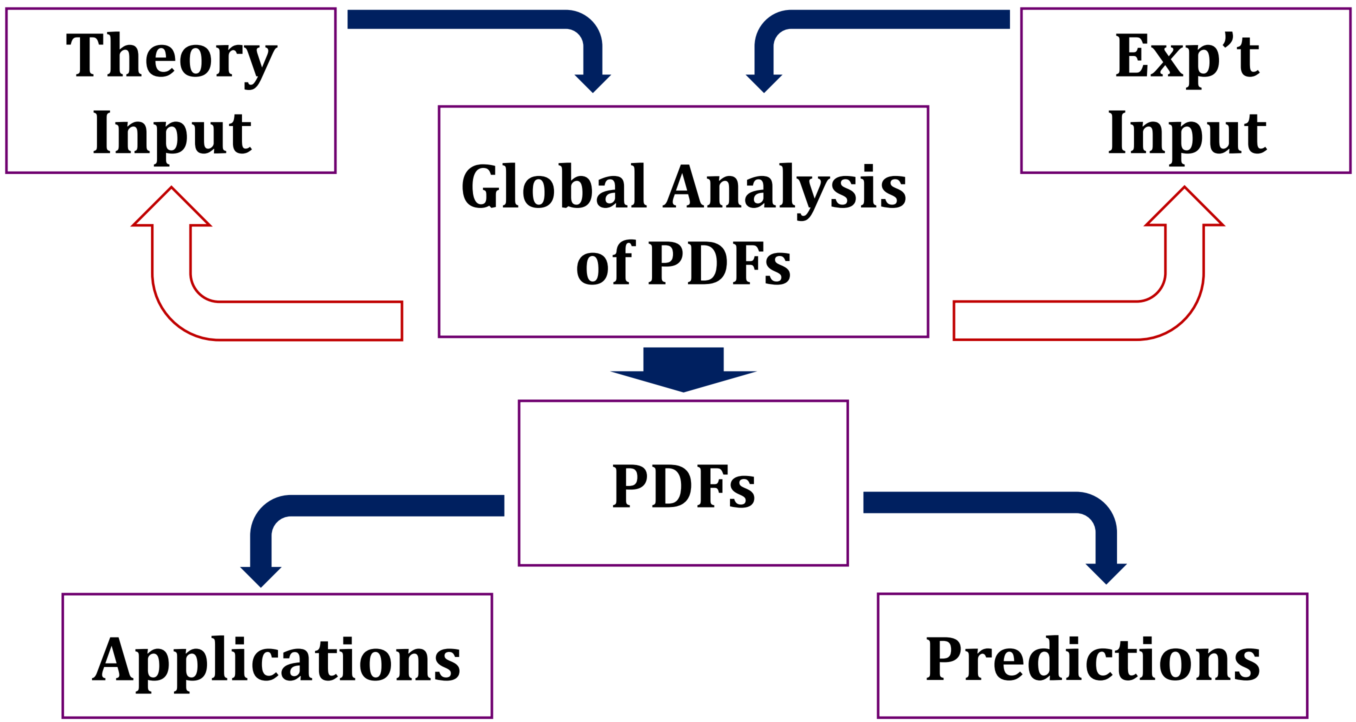


(J. Campbell, HCP2012)

Global Analysis

§ Experiments cover diverse kinematics of parton variables

- Global analysis takes advantage of all data sets



Global Analysis

Theory Input → **Global Analysis of PDFs** ← **Exp't Input**

§ Some choices made for the analysis

- Choice of data sets and kinematic cuts
- Strong coupling constant $\alpha_s(M_Z)$
- How to parametrize the distribution

$$f(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2} P(x)$$

$$P(x) = \begin{cases} 1 + a_3 x + a_4 x^2 \\ e^{a_3 x}(1+e^{a_4}x)^{a_5} \end{cases}$$

Global Analysis

Theory Input → **Global Analysis of PDFs** ← **Exp't Input**

§ Some choices made for the analysis

- Choice of data sets and kinematic cuts
- Strong coupling constant $\alpha_s(M_Z)$
- How to parametrize the distribution
- Assumptions imposed

  SU(3) flavor symmetry, charge symmetry, strange and sea distributions

For example, $s = \bar{s} = \kappa(\bar{u} + \bar{d})$
or symmetric sea in helicity

Global Analysis

Theory Input → **Global Analysis of PDFs** ← **Exp't Input**

§ Some choices made for the analysis

- Choice of data sets and kinematic cuts
- Strong coupling constant $\alpha_s(M_Z)$
- How to parametrize the distribution
- Assumptions imposed

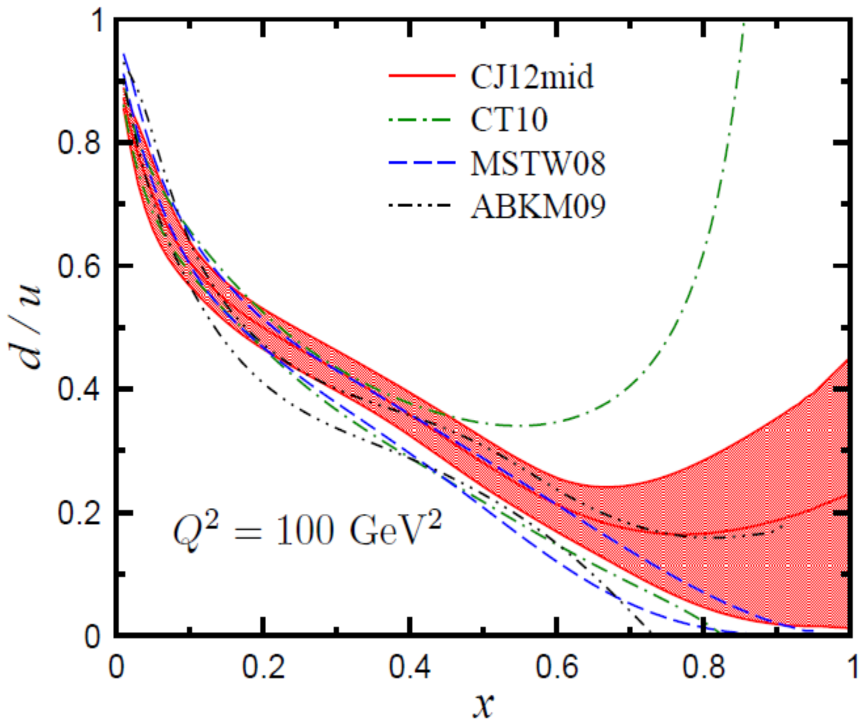
  SU(3) flavor symmetry, charge symmetry, strange and sea distributions

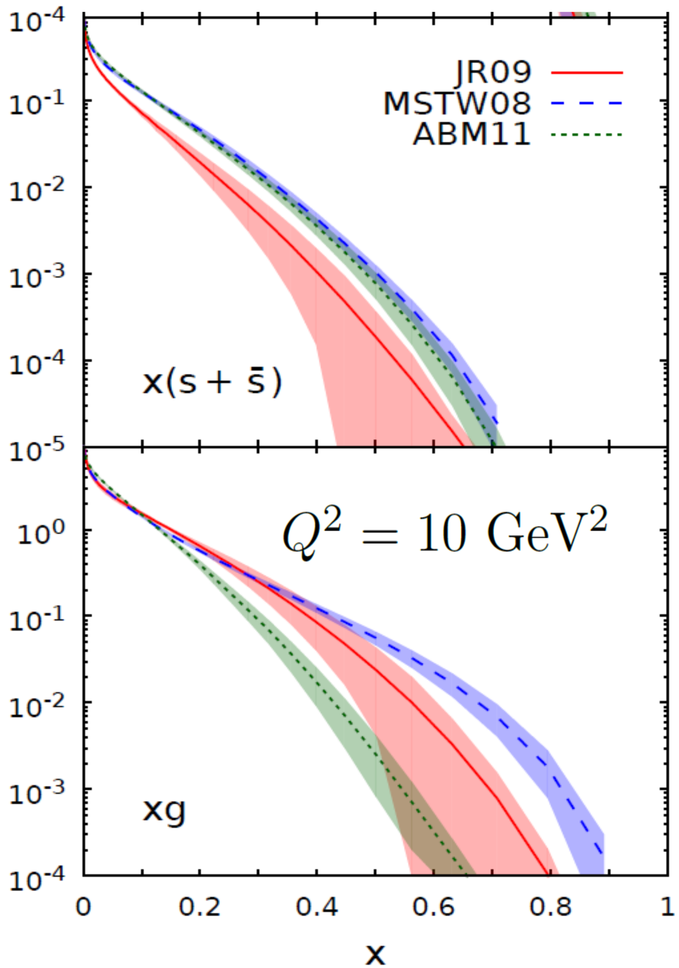
Discrepancies appear when data is scarce

Global Analysis

§ Many groups have tackled the analysis

- CTEQ, MSTW, ABM, JR, NNPDF, etc.





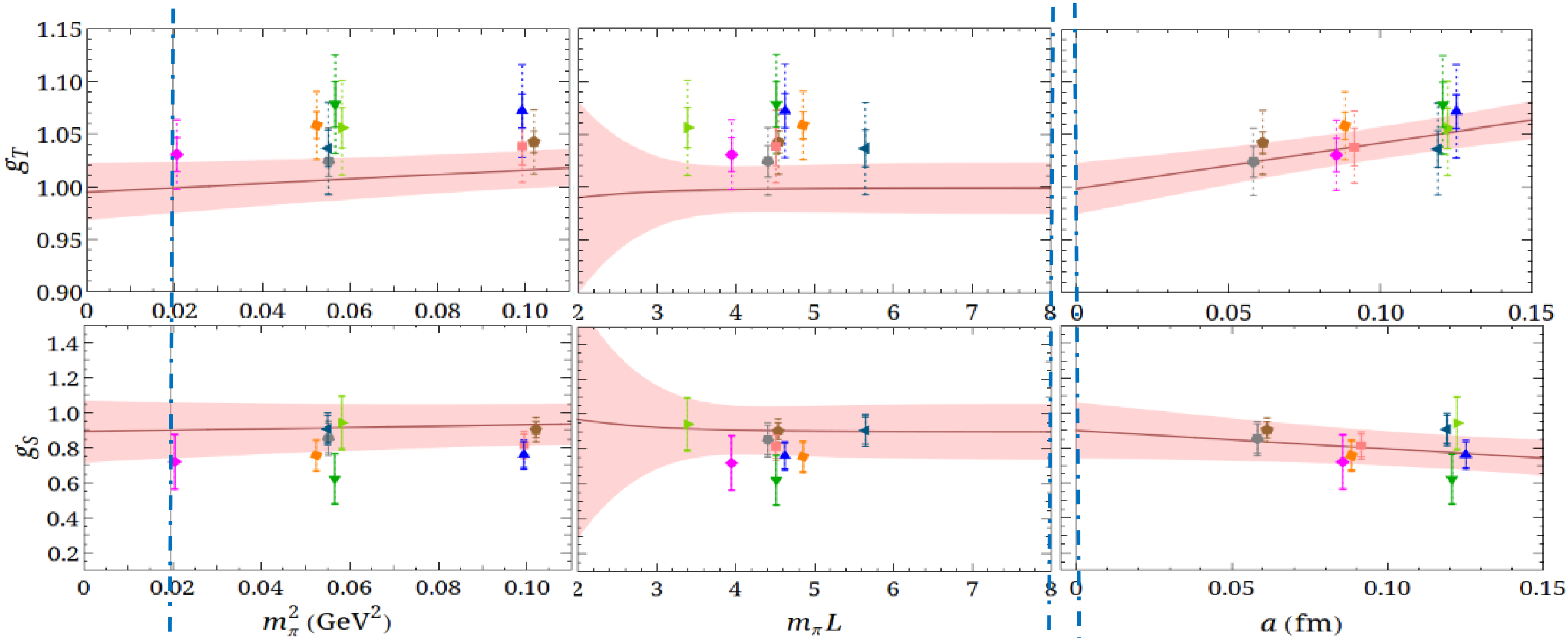
Jimenez-Delgado, Melnitchouk, Owens, J.Phys. G40 (2013) 09310

How Can LQCD Help?

- Lattice QCD is an ideal theoretical tool for investigating strong-coupling regime of quantum field theories
- We are beginning to do precision calculations in nucleons

How Can LQCD Help?

- Lattice QCD is an ideal theoretical tool for investigating strong-coupling regime of quantum field theories
- We are beginning to do precision calculations in nucleons
- PNDME's $g_{T,S}$ calculations PNDME, 1506.06411; 1506.04196; in prep.
  - Extrapolate to the physical limit $m_\pi \to m_\pi^{\text{phys}}$, $a \to 0$, $L \to \infty$



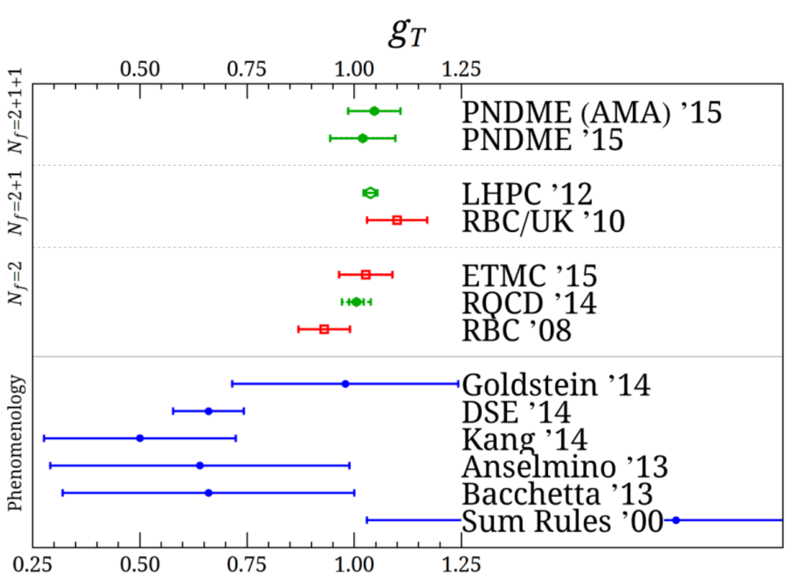
Tensor/Scalar Charges

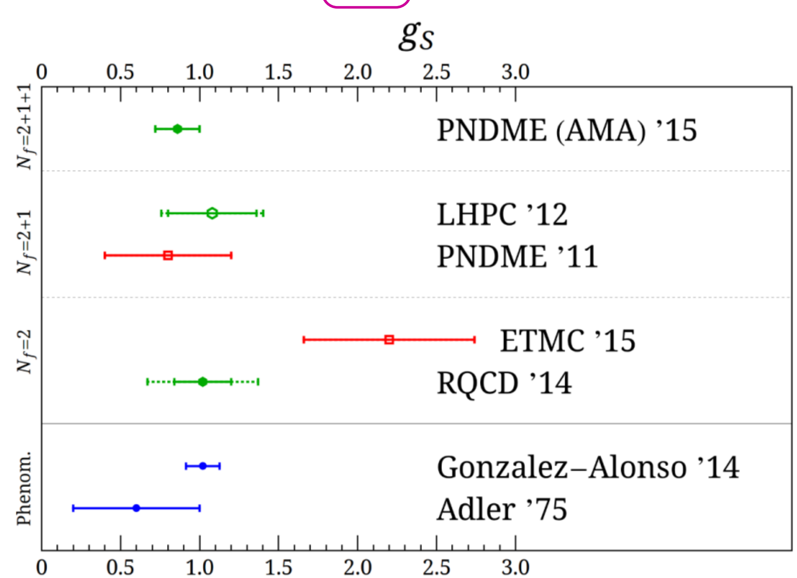
FLAG rating system

PNDME, 1506.06411; in preparation

New: excited-state rating

Collaboration	Ref.	publication status	N_f	chiral extrapolation	continuum extrapolation	finite volume	excited state	renormalization	g_T
PNDME'15	This work	P	2+1+1	★	★	★	★	★	1.020(76)[a]
ETMC'13	[30]	C	2+1+1	■	○	○	■	★	1.11(3)[b]
LHPC'12	[28]	A	2+1	★	○	★	○	★	1.037(20)[c]
RBC/UKQCD'10	[29]	A	2+1	○	■	★	★	★	1.10(7)[d]
RQCD'14	[31]	P	2	★	★	★	○	★	1.005(17)(29))[e]
ETMC'13	[30]	C	2	★	■	○	■	○	1.114(46)[f]
RBC'08	[32]	P	2	■	■	★	■	★	0.93(6)[g]

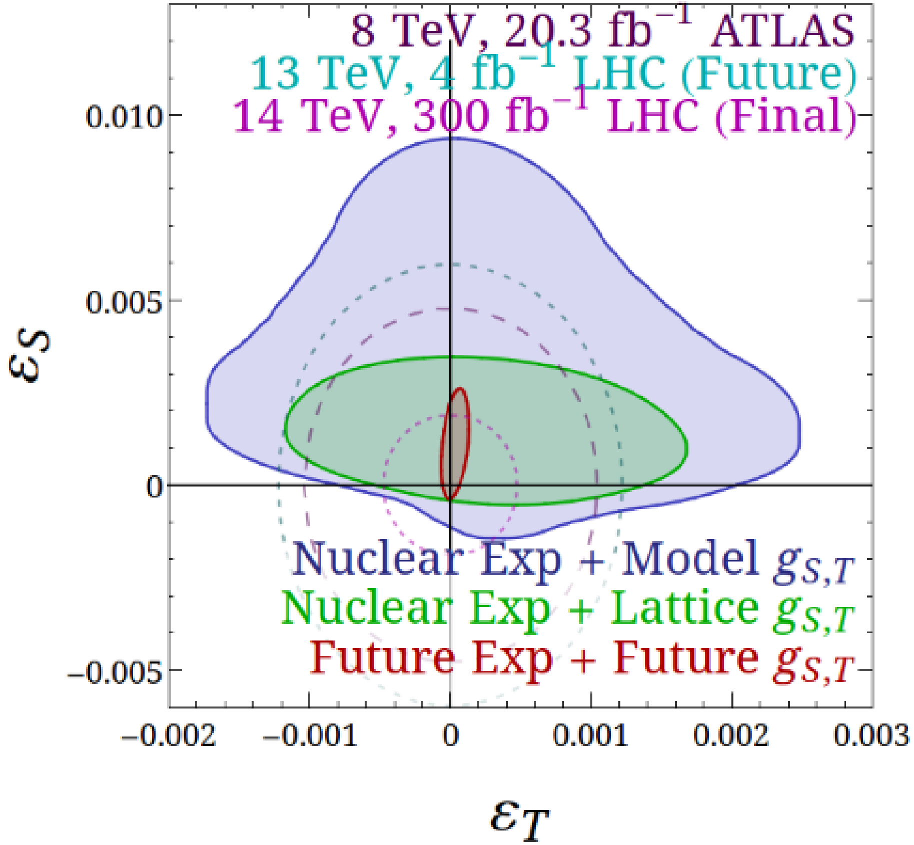




Beta Decays & BSM

§ Given precision $g_{S,T}$ and O_{BSM}, predict new-physics scales

Low-Energy Expt → $O_{\mathrm{BSM}} = f_O(\varepsilon_{S,T}\, g_{S,T})$ ← Precision LQCD input ($m_\pi \to 140$ MeV, $a \to 0$)



$\varepsilon_{S,T} \propto \Lambda_{S,T}^{-2}$

Upcoming precision low-energy experiments

LANL/ ORNL UCN neutron decay exp't

$|B_1 - b|_{\mathrm{BSM}} < 10^{-3}$

$|b|_{\mathrm{BSM}} < 10^{-3}$

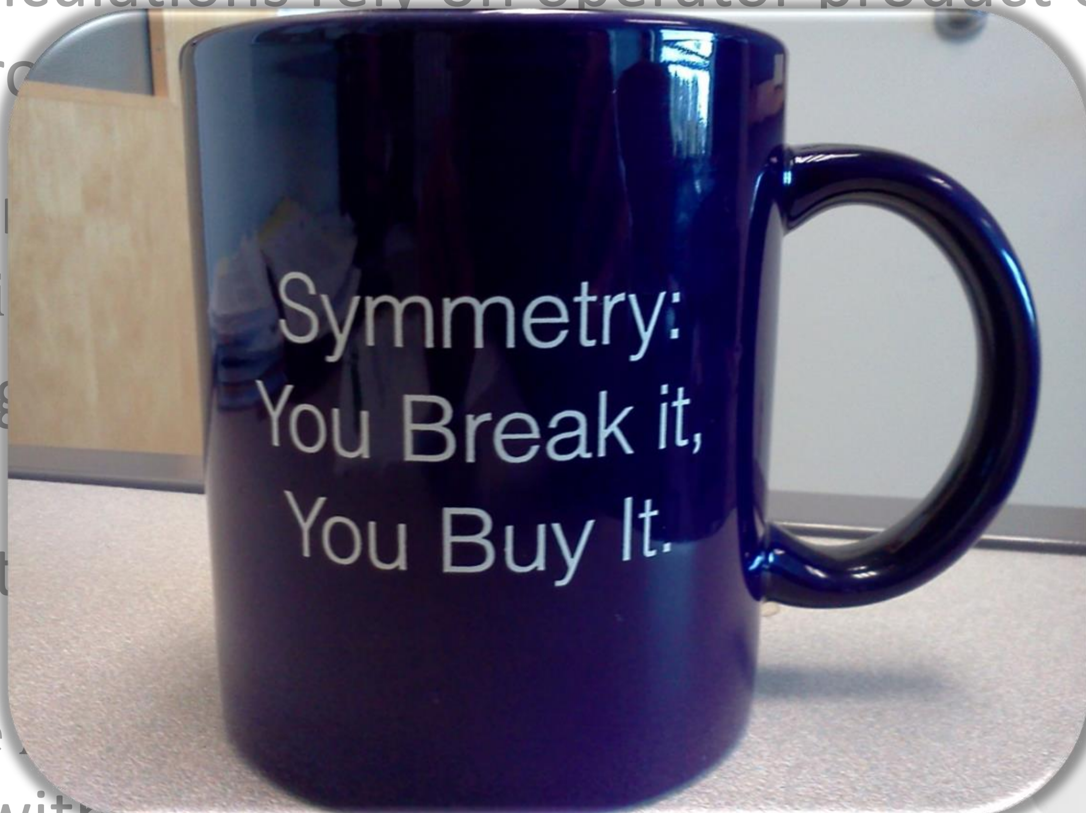
CENPA: $^6\mathrm{He}(b_{\mathrm{GT}})$ at 10^{-3}

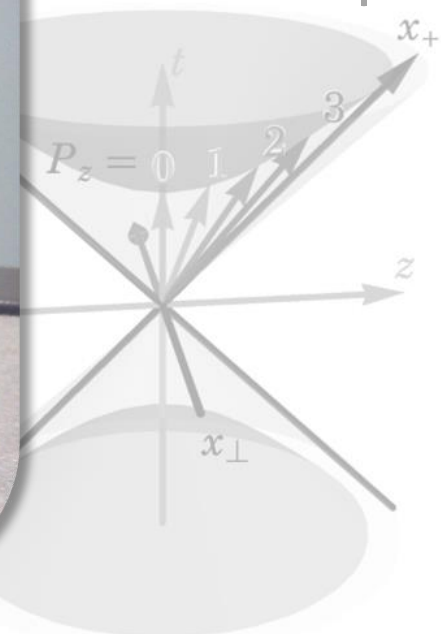
PNDME, PRD85 054512 (2012); 1306.5435; in preparation

PDFs on the Lattice

Long existing obstacles!

- Lattice calculations rely on operator product expansion,
  only pr $\int_{-1}^{1} dx\, x^{n-1} q(x)$
- For higher imension ops
  - No practi
- New Strate
- Calculate
  quark dist
  - In $P_z \to \infty$
  - For finite
- Feasible with today's resources!





Xiangdong Ji, Phys. Rev. Lett. 111, 039103 (2013)

PDFs on the Lattice

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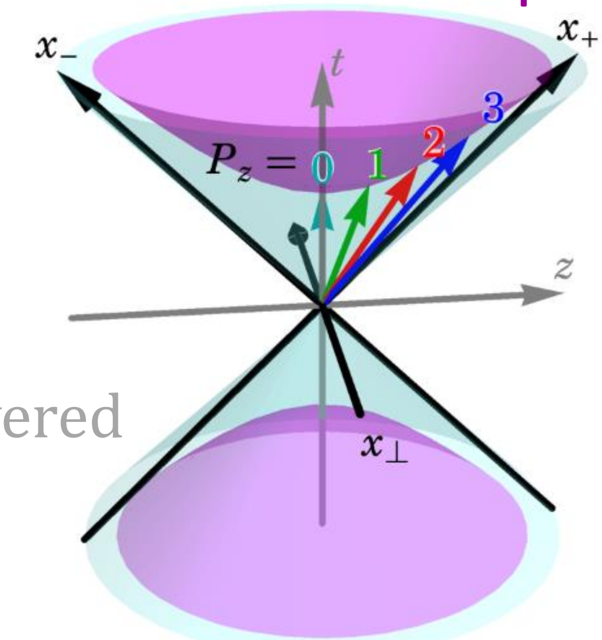
§ Lattice calculations rely on operator product expansion, only provide moments $\langle x^n \rangle$

$$\langle x^{n-1} \rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

§ For higher moments, all ops mix with lower-dimension ops

- No practical proposal to overcome this



New Strategy (LaMET):

§ Calculate finite-momentum boosted quark distribution

- In $P_z \to \infty$ limit, parton distribution is recovered
- For finite P_z, corrections are needed

§ Feasible with today's resources!

Xiangdong Ji, Phys. Rev. Lett. 111, 039103 (2013)

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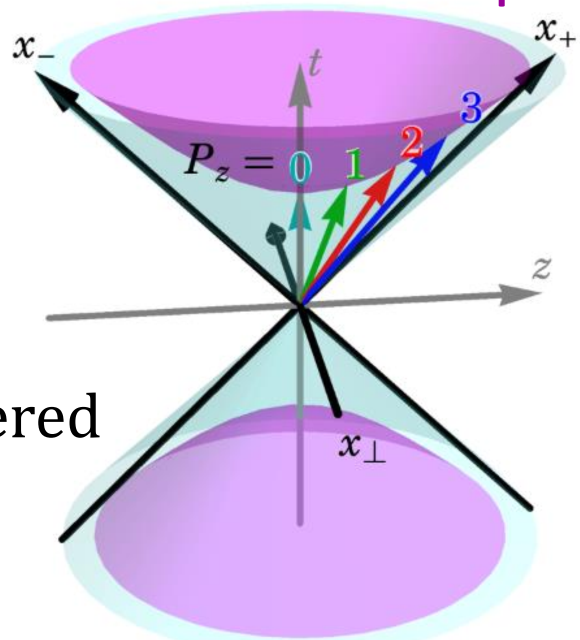

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Xiangdong Ji, Phys. Rev. Lett. 111, 039103 (2013)

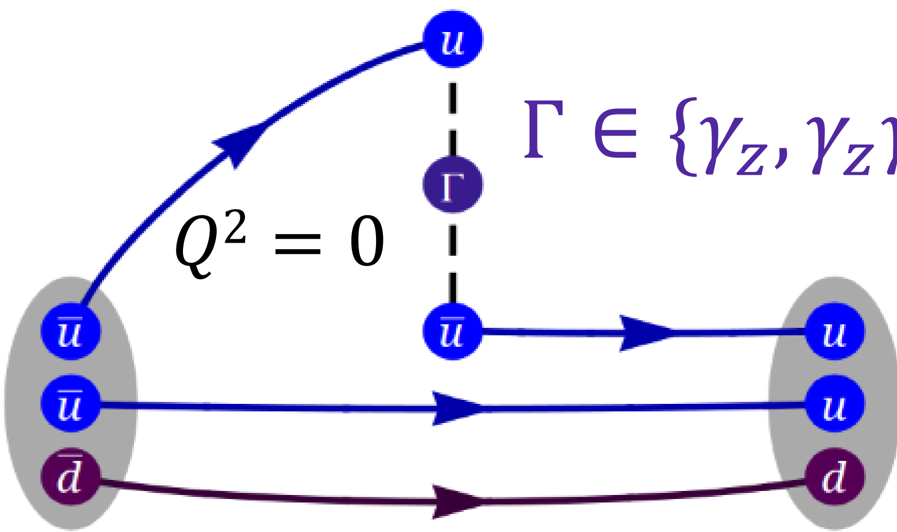
Parton Distribution Functions

Large-Momentum Effective Theory for PDFs

$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \overline{\psi}(z)\, \Gamma \exp\!\left(-ig\int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$

$\Gamma \in \{\gamma_z, \gamma_z\gamma_5, \gamma_z\gamma_\perp\gamma_5\}$

$p(P_z)$



$p(P_z)$

$P_z \to \infty$ limit

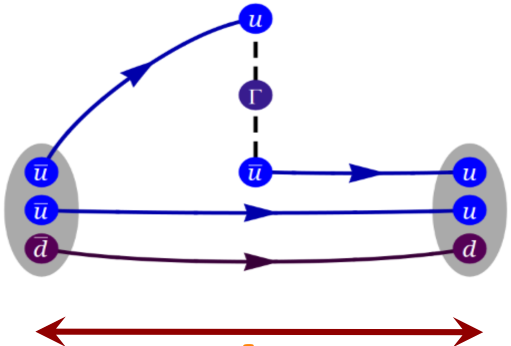
$$q(x,\mu) = \tilde{q}(x,\mu,P_z) + \mathcal{O}(\alpha_s) + \mathcal{O}(M_N^2/P_z^2) + \mathcal{O}\left(\Lambda_{\text{QCD}}^2/P_z^2\right)$$

X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

Some Lattice Details

§ Exploratory study

- N_f = 2+1+1 clover/HISQ lattices (MILC)
  $M_\pi \approx$ **310 MeV**, $a \approx$ **0.12 fm** ($L \approx$ 2.88 fm)
- Isovector only ("disconnected" suppressed)
  gives us flavor asymmetry between up and down quark
- 2 source-sink separations ($t_{sep} \approx$ 0.96 and 1.2 fm) used



§ Properties known on these lattices

- Lattice Z_Γ for bilinear operator ~ 1
  (with HYP-smearing)
- $M_\pi L \approx 4.6$ large enough to avoid finite-volume effects

Hyak @ UW

§ Feasible with today's computational resources!

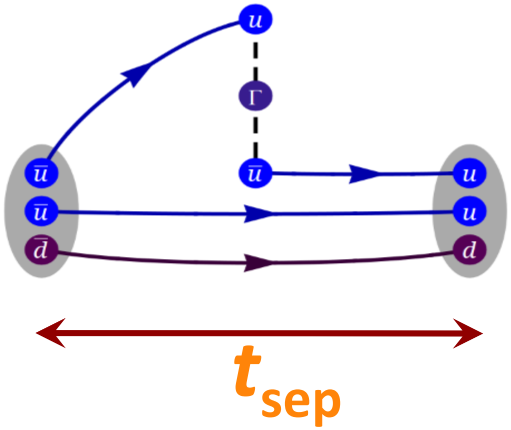
- 256/512 cores on UW Hyak cluster **(NSF grant PHY-09227700)**

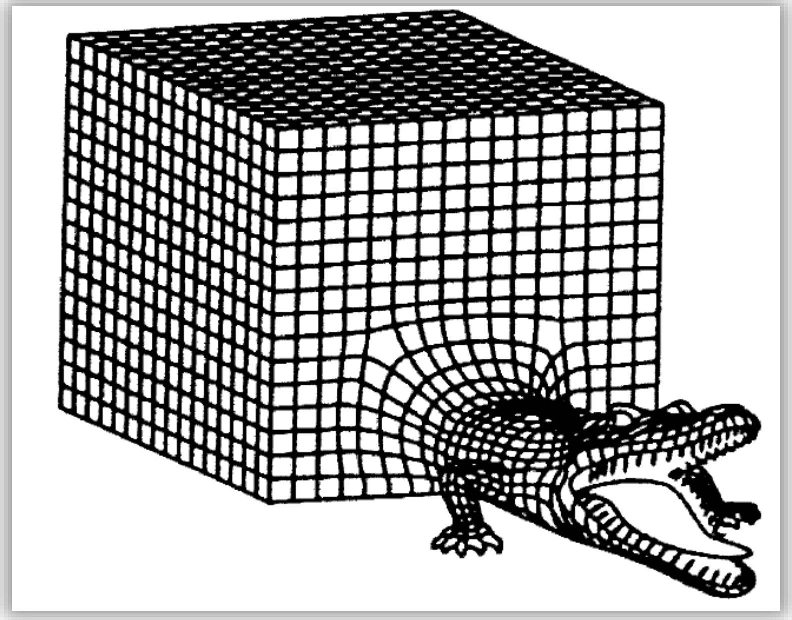
HWL, Jiunn-Wei Chen, Saul D. Cohen, Xiangdong Ji, 1402.1462 [hep-ph]

Warning!

§ Exploratory study

- N_f = 2+1+1 clover/HISQ lattices (MILC)
  $M_\pi \approx$ **310 MeV**, $a \approx 0.12$ fm ($M_\pi L \approx 4.5$)





NO SYSTEMATICS YET!

§ Demonstration that the method works!

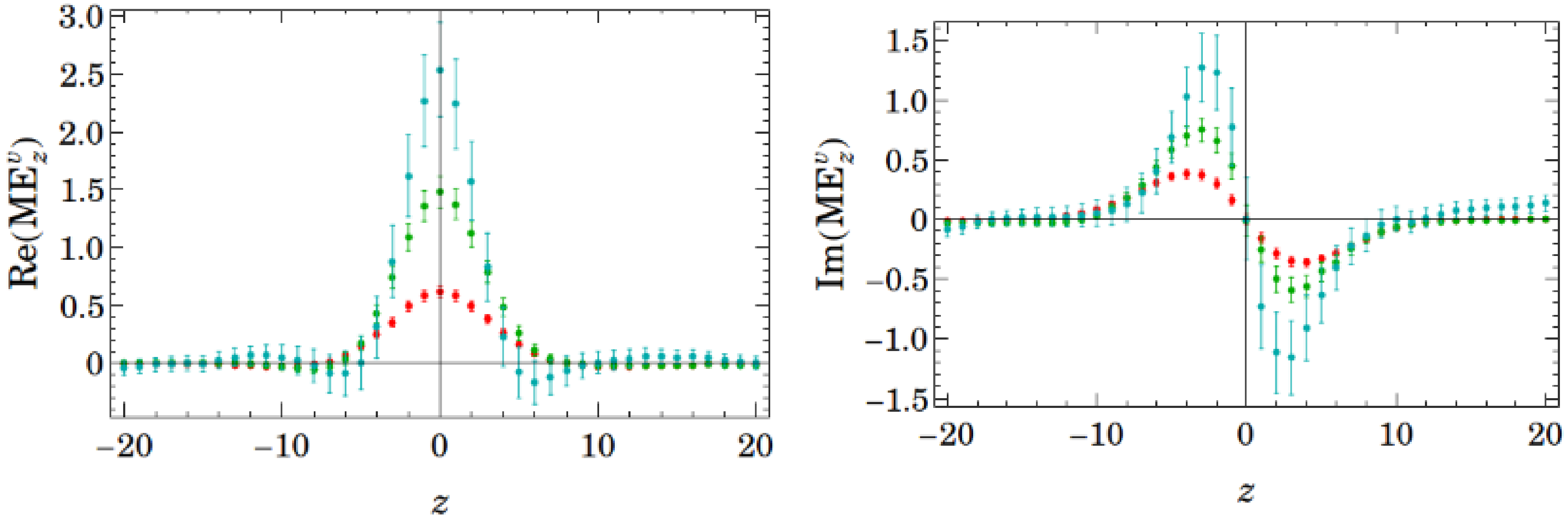
- Intend to motivate future LQCD work on many quantities

Quark Distribution

§ Exploratory study

$$\left\langle P \,\middle|\, \overline{\psi}(z)\,\gamma_z \exp\!\left(-ig\int_0^z dz'\, A_z(z')\right)\psi(0) \,\middle|\, P\right\rangle$$

- How many links are needed?



- Lattice momenta discretized by finite size of volume

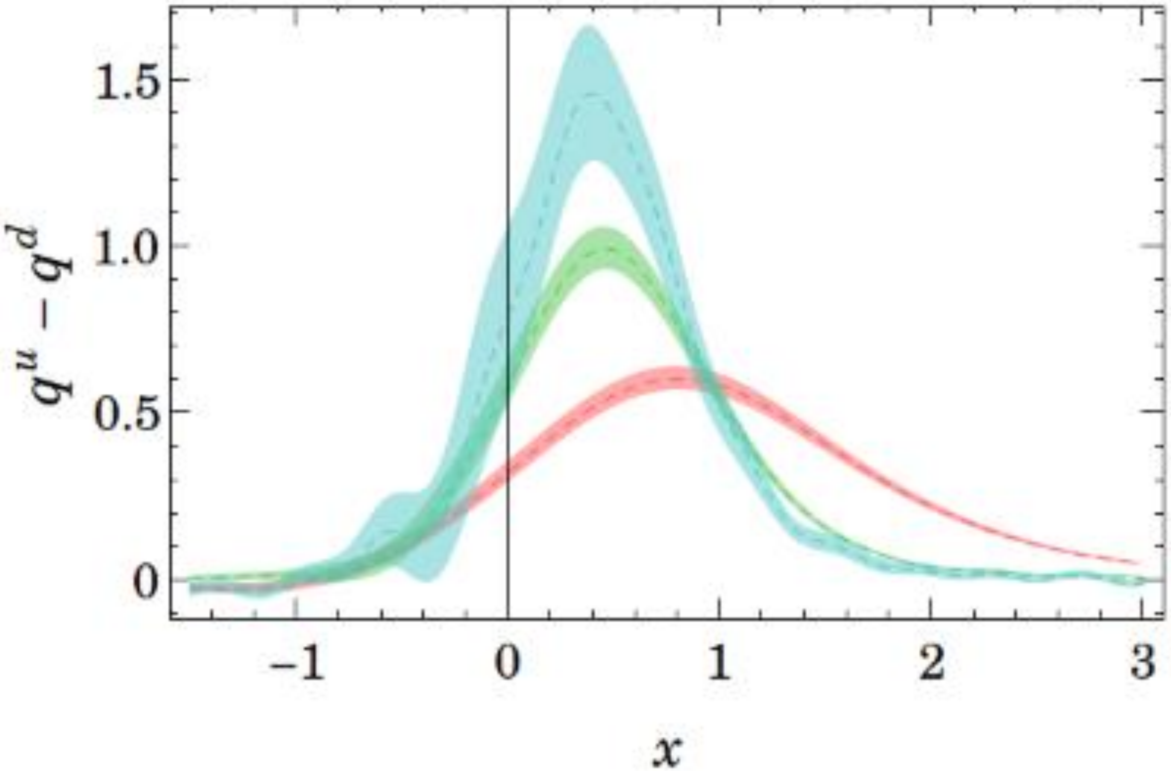
$P_z \in \{1, 2, 3\}\, {}^{2\pi}/_{L}$

Quark Distribution

§ Exploratory study

$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \overline{\psi}(z)\, \gamma_z \exp\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$

$$P_z \in \{1, 2, 3\}\, {}^{2\pi}/_L$$



Uncorrected bare lattice results

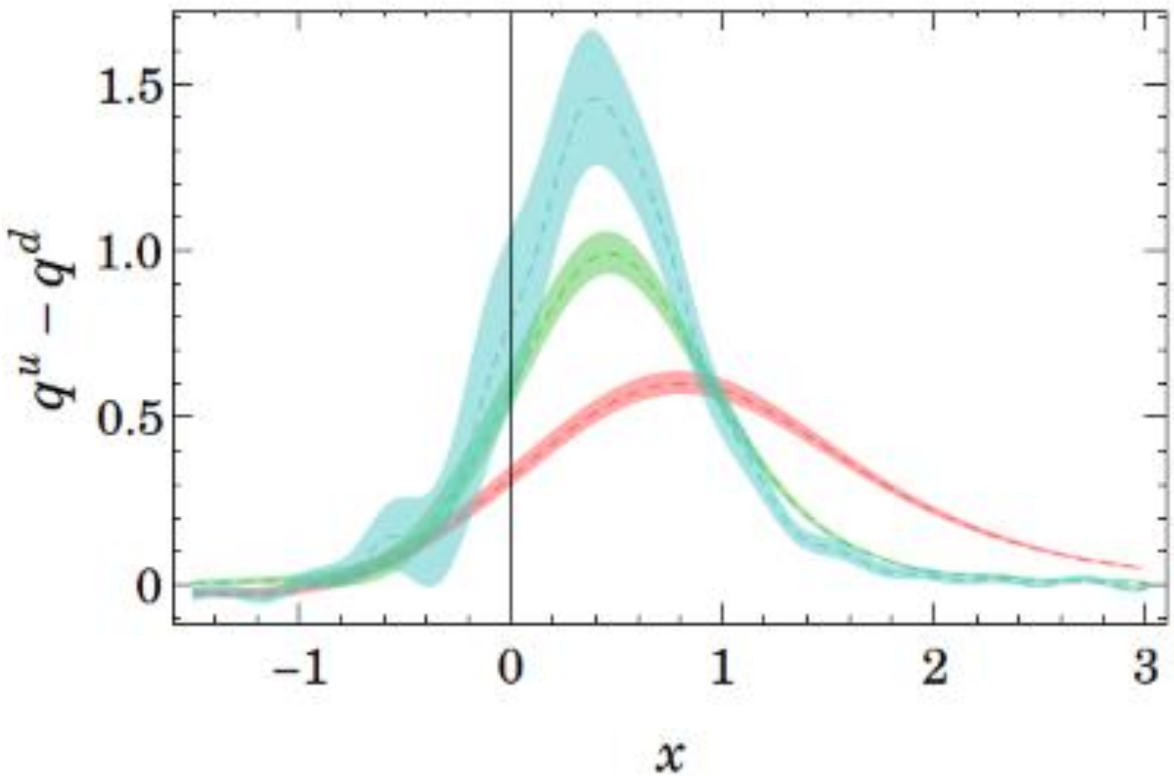
$x = k_z/P_z$

Quark Distribution

§ Exploratory study

$$\int \frac{dz}{4\pi}\, e^{-izk_z} \left\langle P \,\middle|\, \overline{\psi}(z)\, \gamma_z \exp\!\left(-ig\int_0^z dz'\, A_z(z')\right) \psi(0) \,\middle|\, P\right\rangle$$

$$P_z \in \{1, 2, 3\}\, {}^{2\pi}/_{L}$$



Distribution gets sharper as P_z increases

Artifacts due to finite P_z on the lattice

Improvement?

Work out leading-P_z corrections

Quark Distribution

§ Finite-P_z corrections

Xiangdong Ji, Phys. Rev. Lett. 111, 039103 (2013)

$$q(x,\mu) = \tilde{q}(x,\mu,P_z) + \mathcal{O}(M_N^2/P_z^2) + \mathcal{O}(\alpha_s) + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$$

What we want

Quark Distribution

§ Finite-P_z corrections

Xiangdong Ji, Phys. Rev. Lett. 111, 039103 (2013)

$$q(x,\mu) = \tilde{q}(x,\mu,P_z) + \mathcal{O}(M_N^2/P_z^2) + \mathcal{O}(\alpha_s) + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$$

Quasi-distribution
What we calculate
on the lattice
$P_z \in \{1, 2, 3\}\, {}^{2\pi}/_{L}$

Quark Distribution

§ Finite-P_z corrections

Xiangdong Ji, Phys. Rev. Lett. 111, 039103 (2013)

$$q(x,\mu) = \tilde{q}(x,\mu,P_z) + \mathcal{O}(M_N^2/P_z^2) + \mathcal{O}(\alpha_s) + \mathcal{O}\left(\Lambda_{\text{QCD}}^2/P_z^2\right)$$

Dominant correction (for nucleon); known scaling form

HWL et al. 1402.1462

J.-W. Chen et al, 1603.06664

Quark Distribution

§ Finite-P_z corrections

Xiangdong Ji, Phys. Rev. Lett. 111, 039103 (2013)

$$q(x,\mu) = \tilde{q}(x,\mu,P_z) + \mathcal{O}(M_N^2/P_z^2) + \mathcal{O}(\alpha_s) + \mathcal{O}\left(\Lambda_{\text{QCD}}^2/P_z^2\right)$$

Finite $P_z \leftrightarrow \infty$ perturbative matching

$$q(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) \tilde{q}(y,\mu)$$

$$Z(x,\mu/P_z) = C\delta(x-1) - \frac{\alpha_s}{2\pi} Z^{(1)}(x,\mu/P_z)$$

Non-singlet case only

X. Xiong, X. Ji, J. Zhang, Y. Zhao, 1310.7471;

Ma and Qiu, 1404.6860

Quark Distribution

§ Finite-P_z corrections

Xiangdong Ji, Phys. Rev. Lett. 111, 039103 (2013)

$$q(x,\mu) = \tilde{q}(x,\mu,P_z) + \mathcal{O}(M_N^2/P_z^2) + \mathcal{O}(\alpha_s) + \mathcal{O}(\Lambda_{\mathrm{QCD}}^2/P_z^2)$$

Smaller P_z correction
complicated higher-twist operator
J.-W. Chen et al, 1603.06664

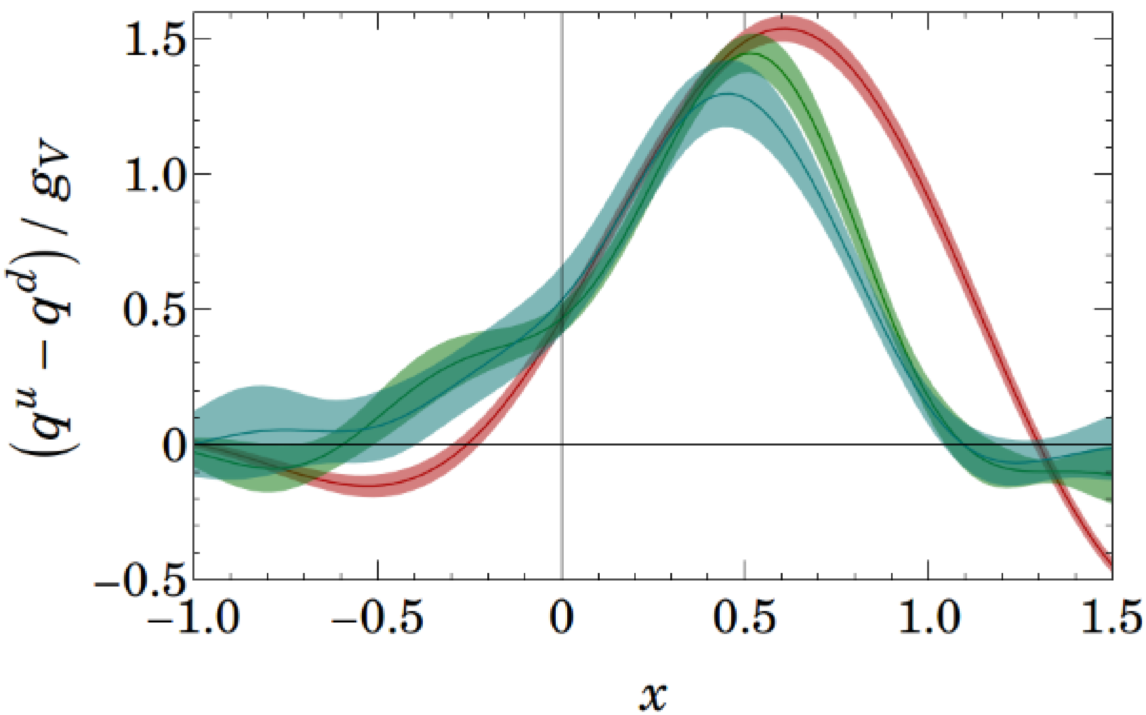
(extrapolate it away)

Quark Distribution

§ Exploratory study

- Take ratios (partially cancel statistical and systematic errors)

$$q_{\text{norm}}(x,\mu,P_z) = \frac{q(x,\mu,P_z)}{\int dx\, q(x,\mu,P_z)} \times g_V^{\overline{\text{MS}}}(2\text{ GeV})$$



Removing $O(M_N^n/P_z^n)$ errors + $O(\alpha_s)$

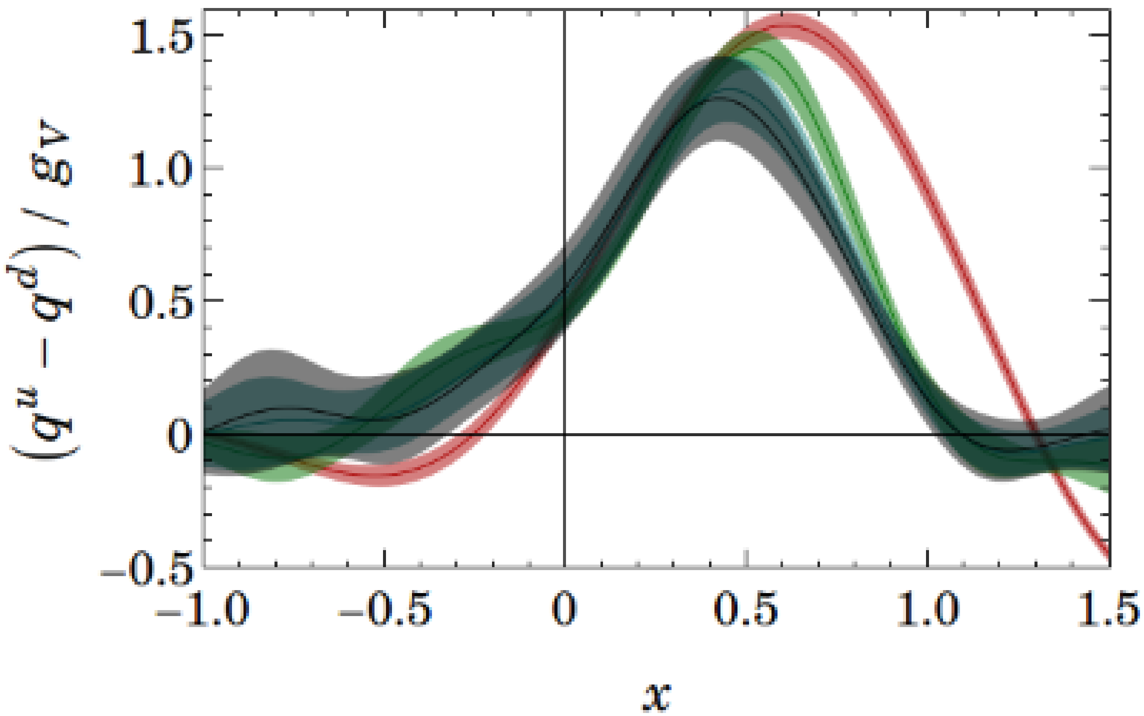
No significant finite-momentum effect seen for $P_z>1$

Quark Distribution

§ Exploratory study

- Take ratios (partially cancel statistical and systematic errors)

$$q_{\text{norm}}(x,\mu,P_z) = \frac{q(x,\mu,P_z)}{\int dx\, q(x,\mu,P_z)} \times g_V^{\overline{\text{MS}}}(2\text{ GeV})$$



Removing $O(M_N^n/P_z^n)$ errors + $O(\alpha_s)$

No significant finite-momentum effect seen for $P_z > 1$

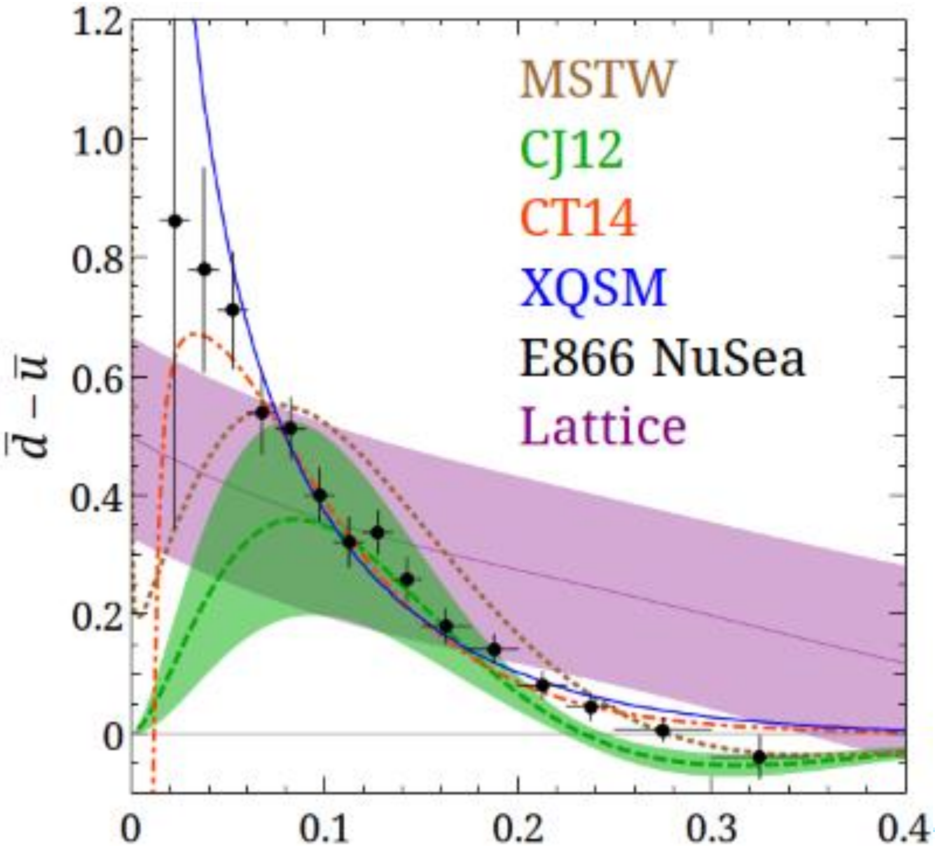
Further removing $O(\Lambda_{\text{QCD}}^2/P_z^2)$ errors

Sea Flavor Asymmetry

§ First time in LQCD history to study antiquark distribution!

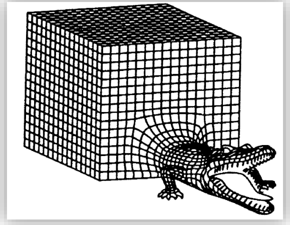
- $M_\pi \approx 310$ MeV

HWL et al. 1402.1462



x

$$\bar{q}(x) = -q(-x)$$



Lost resolution in small-*x* region

Future improvement:
larger lattice volume

$$\int dx\,(\bar{u}(x)-\bar{d}(x)) \approx -0.16(7)$$

Experiment	x range	$\int_0^1 [\bar{d}(x)-\bar{u}(x)]dx$
E866	$0.015<x<0.35$	0.118 ± 0.012
NMC	$0.004<x<0.80$	0.148 ± 0.039
HERMES	$0.020<x<0.30$	0.16 ± 0.03

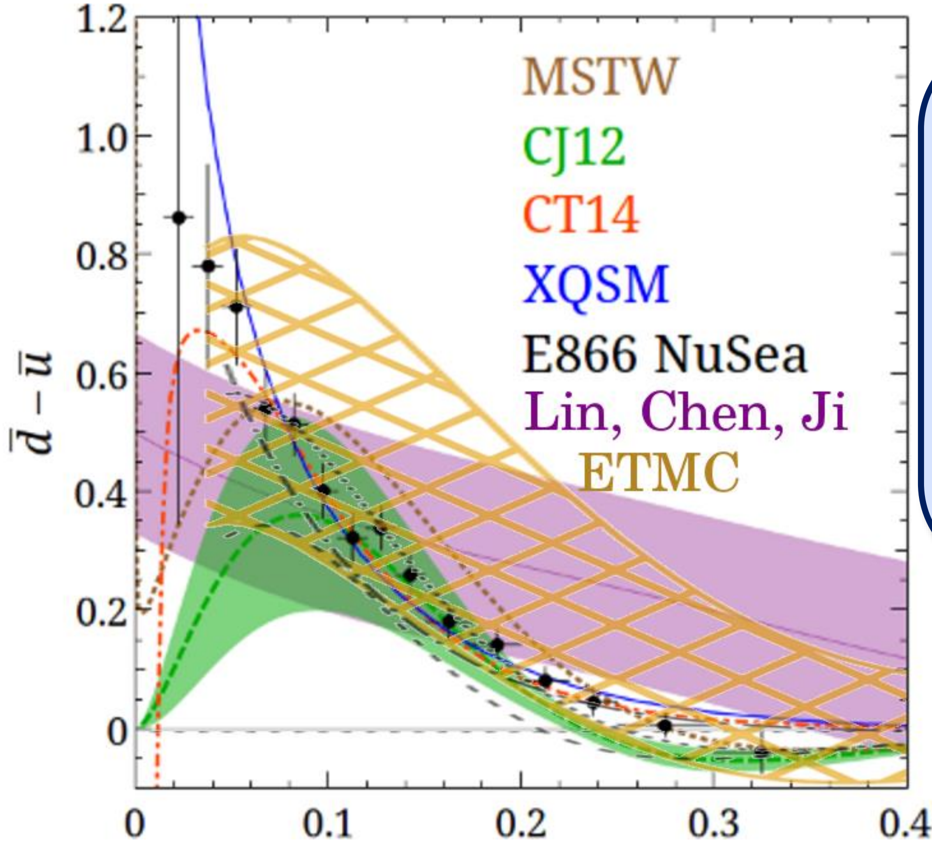
R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

Sea Flavor Asymmetry

§ Lattice exploratory study

- $M_\pi \approx 310$ MeV

HWL et al 1402.1462



Compared with E866

Too good to be true?

One year later,
this calculation is
repeated by ETMC,
at $M_\pi \approx 373$ MeV
Found similar results.

ETMC, 1504.07455

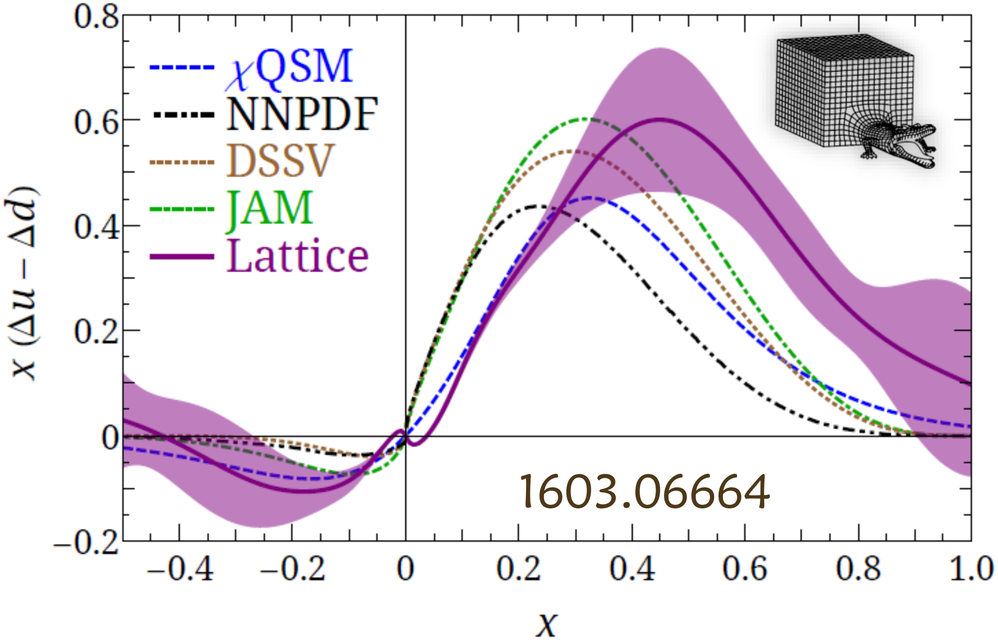
6(7)

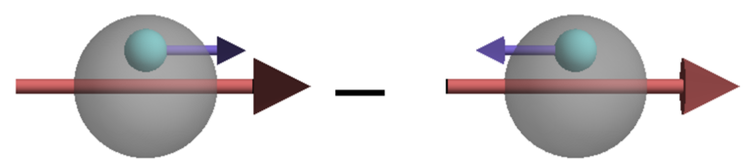
Experiment	x range	$\int_0^1[\bar{d}(x)-\bar{u}(x)]dx$
E866	$0.015<x<0.35$	0.118 ± 0.012
NMC	$0.004<x<0.80$	0.148 ± 0.039
HERMES	$0.020<x<0.30$	0.16 ± 0.03

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

Helicity Distribution

§ Exploratory study ☙ $M_\pi \approx 310$ MeV





Removing
$O(M_N^n/P_z^n)$ errors + $O(\alpha_s)$
+ $O(\Lambda_{\mathrm{QCD}}^2/P_z^2)$

☙ We see polarized sea asymmetry $\int dx\,\left(\Delta\bar{u}(x) - \Delta\bar{d}(x)\right) \approx 0.14(9)$

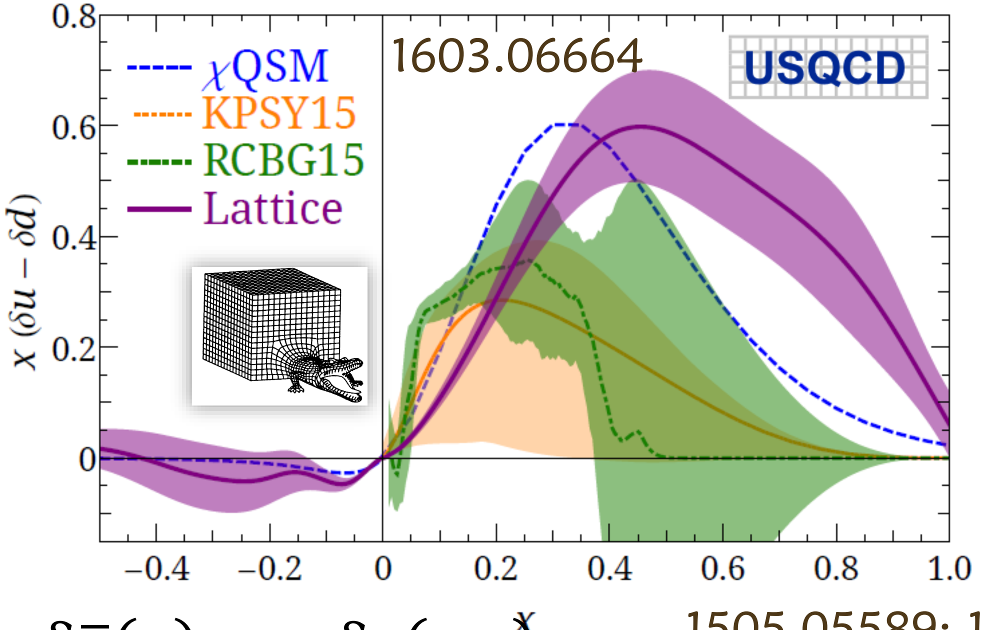
☙ Both STAR and PHENIX at RHIC see $\Delta\bar{u} > \Delta\bar{d}$

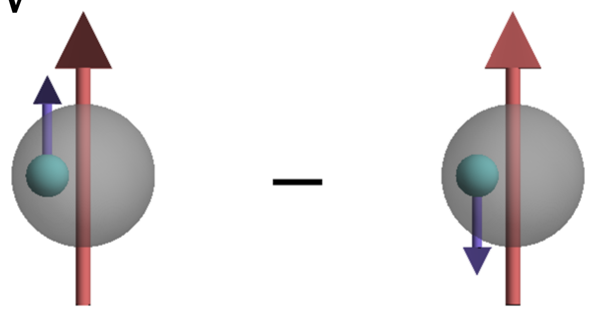
1404.6880 and 1504.07451

☙ Other experiments, Fermilab DY exp'ts (E1027/E1039), future EIC

Transversity Distribution

§ Exploratory study ☙ $M_\pi \approx 310$ MeV





Removing

$O(M_N^n/P_z^n)$ errors + $O(\alpha_s)$

+ $O(\Lambda_{QCD}^2/P_z^2)$

$\delta\bar{q}(x) = -\delta q(-x)$ 1505.05589; 1503.03495

☙ We found sea asymmetry of $\int dx\,(\delta\bar{u}(x) - \delta\bar{d}(x)) \approx -0.10(8)$

☙ Chiral quark-soliton model $\int dx\,(\delta\bar{u}(x) - \delta\bar{d}(x)) \approx -0.082$

P. Schweitzer et al., PRD 64, 034013 (2001)

☙ SoLID at JLab, Drell-Yan exp't at FNAL (E1027+E1039), EIC, ...

A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

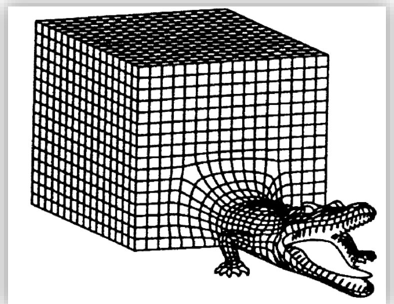
The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful

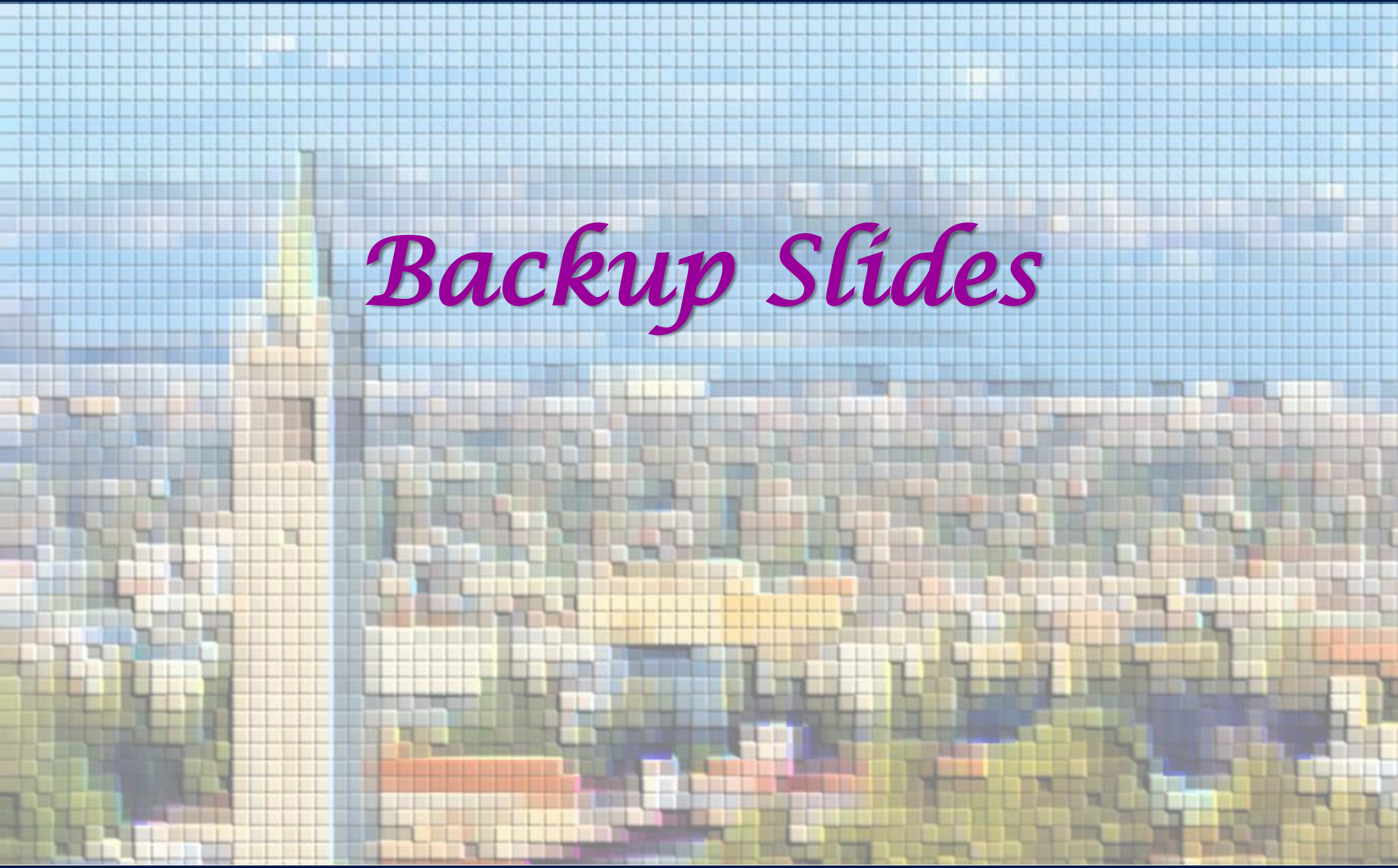
Summary and Outlook

Exciting time for studying structure on the lattice

- § Overcoming longstanding obstacle to full *x*-distribution
  - Most importantly, this can be done with today's computer
- § First lattice approach to study sea asymmetry
  - Promising results on unpolarized and polarized sea asymmetry
  - Prediction of transversity sea asymmetry
- § Caveats
  - Need to improve large-momentum signal: large-q form factors, heavy-flavor physics, …
  - Not a precision calculation *yet*; needs better statistics, proper renormalization, ...



- § Opens doors to much future lattice-QCD work
  - GPDs, gluons, TMDs…



Backup Slides

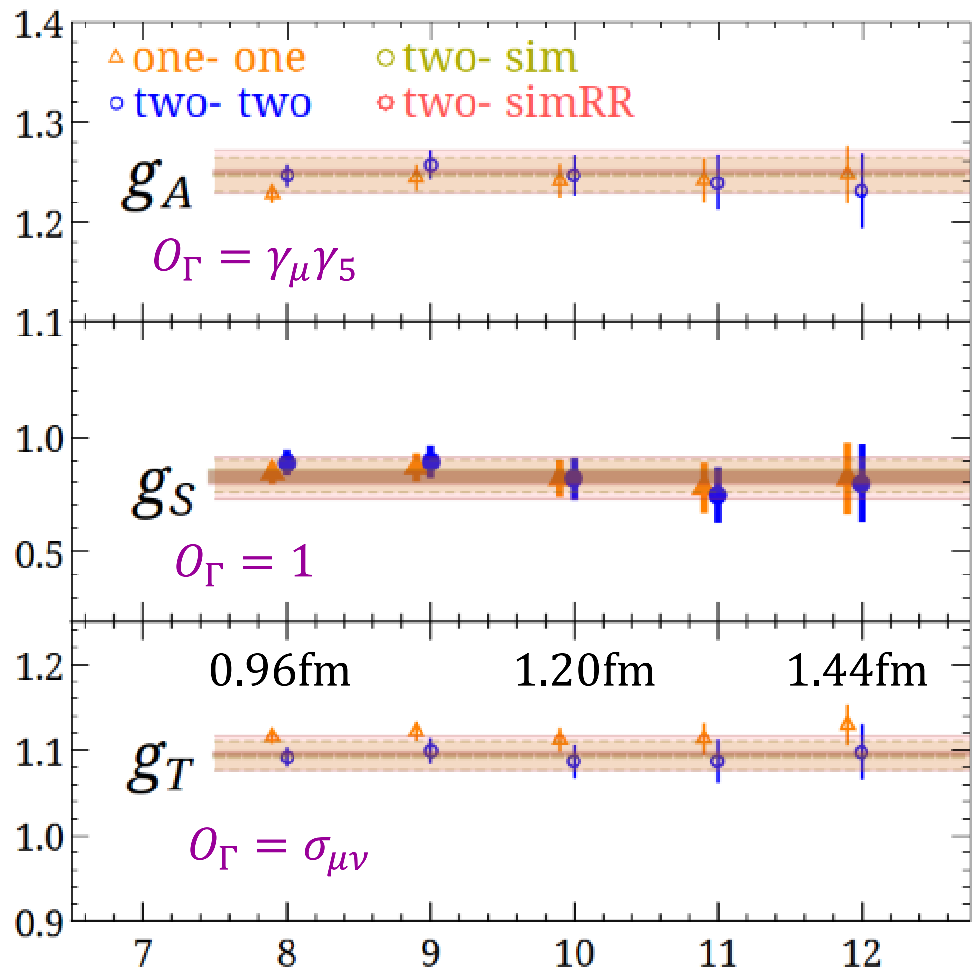
Systematic Control

- Much effort has been devoted to controlling systematics
- A state-of-the art calculation (PNDME) $a = 0.12$ **fm**, 310-MeV pion

❧ Move the **excited-state systematic** into the statistical error

$$C^{3\text{pt}}(t_f,t,t_i) = |\mathcal{A}_0|^2\langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f-t_i)}$$
$$+\mathcal{A}_0\mathcal{A}_1^*\langle 0|\mathcal{O}_\Gamma|1\rangle e^{-M_0(t-t_i)}e^{-M_1(t_f-t)}$$
$$+\mathcal{A}_0^*\mathcal{A}_1\langle 1|\mathcal{O}_\Gamma|0\rangle e^{-M_1(t-t_i)}e^{-M_0(t_f-t)}$$
$$+|\mathcal{A}_1|^2\langle 1|\mathcal{O}_\Gamma|1\rangle e^{-M_1(t_f-t_i)}$$

❧ No obvious contamination between 0.96 and 1.44 fm separation



Systematic Control

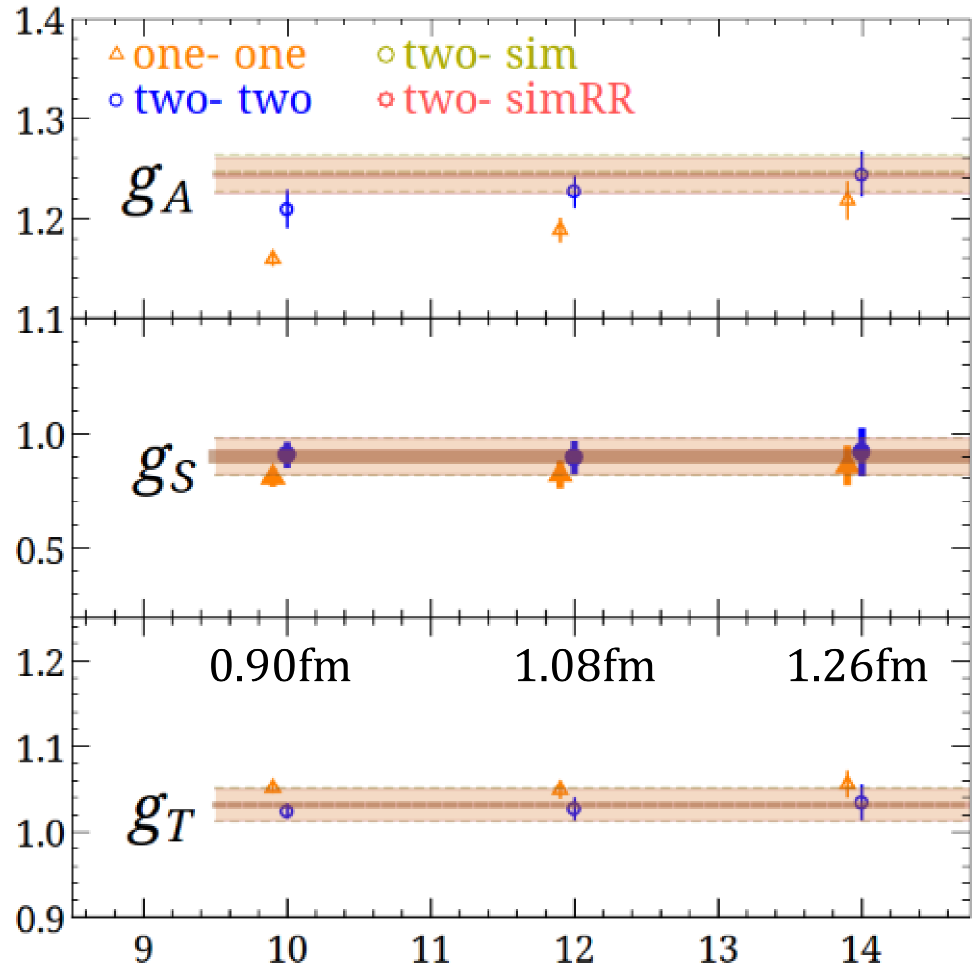
§ Much effort has been devoted to controlling systematics

§ A state-of-the art calculation (PNDME) $a = 0.09$ fm, 310-MeV pion

- Move the **excited-state systematic** into the statistical error

$$C^{3\text{pt}}(t_f, t, t_i) = |\mathcal{A}_0|^2 \langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f-t_i)}$$
$$+\mathcal{A}_0\mathcal{A}_1^*\langle 0|\mathcal{O}_\Gamma|1\rangle e^{-M_0(t-t_i)} e^{-M_1(t_f-t)}$$
$$+\mathcal{A}_0^*\mathcal{A}_1\langle 1|\mathcal{O}_\Gamma|0\rangle e^{-M_1(t-t_i)} e^{-M_0(t_f-t)}$$
$$+|\mathcal{A}_1|^2\langle 1|\mathcal{O}_\Gamma|1\rangle e^{-M_1(t_f-t_i)}$$

- Much stronger effect at finer lattice spacing!
- Needs to be studied case by case



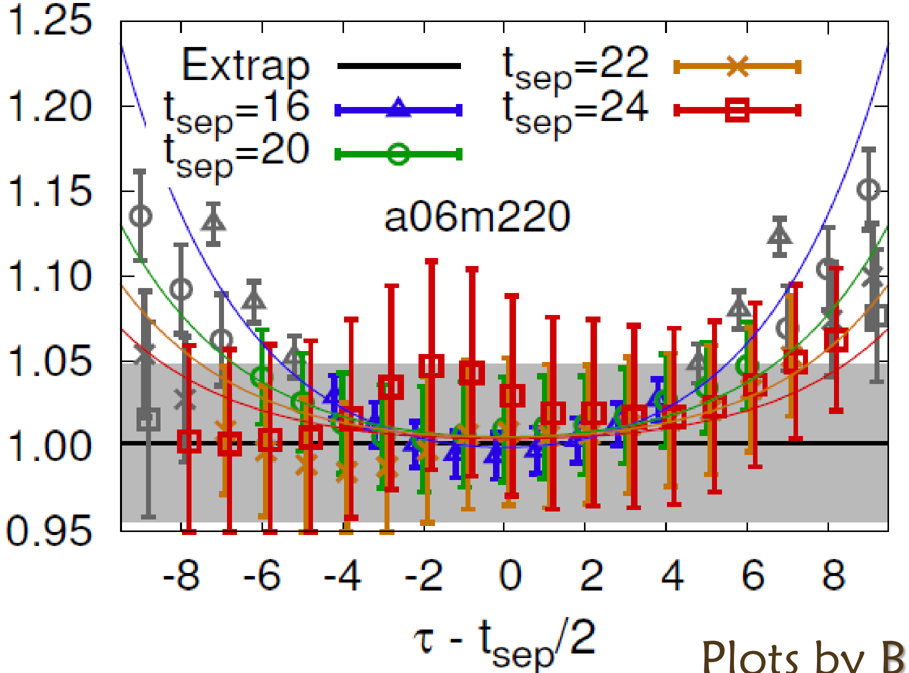
Systematic Control

- Much effort has been devoted to controlling systematics
- A state-of-the art calculation (PNDME)

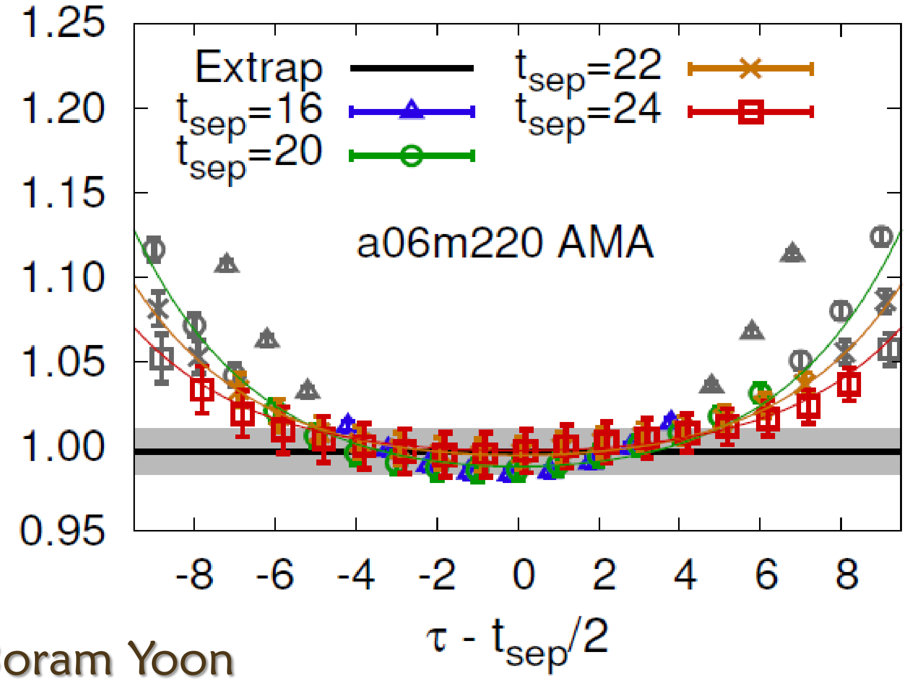
❧ Statistical effect $a = 0.06$ **fm**, 220-MeV pion

g_T

2.6k



41.6k



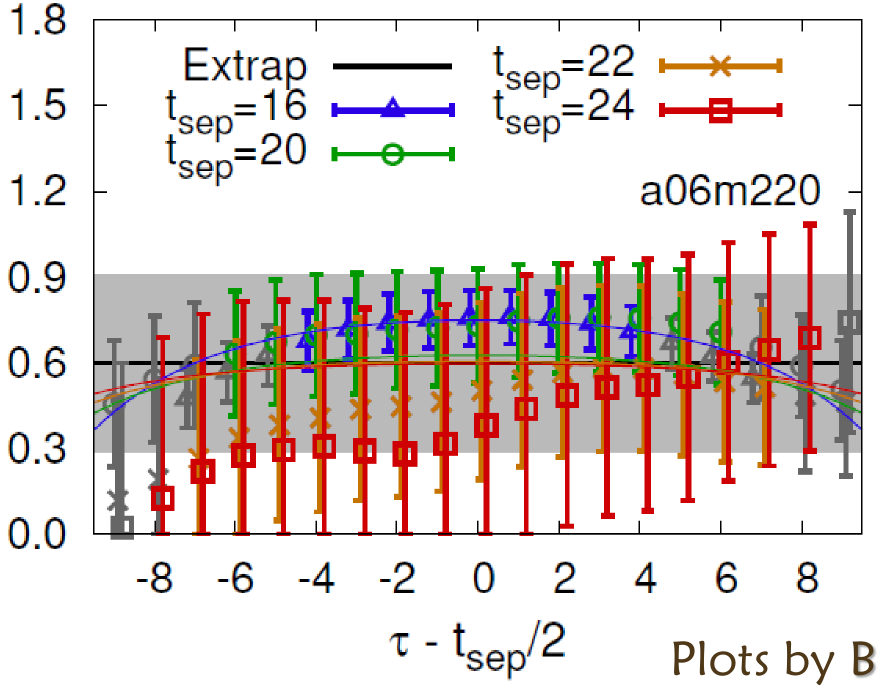
Plots by Boram Yoon

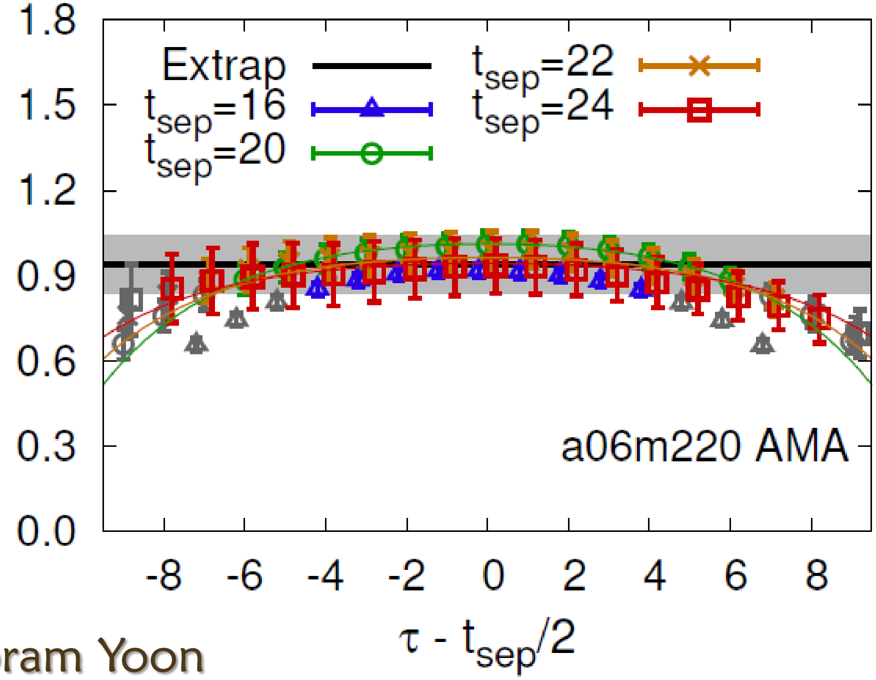
Systematic Control

- Much effort has been devoted to controlling systematics
- A state-of-the art calculation (PNDME)

Statistical effect $\quad$ $a = 0.06$ fm, 220-MeV pion

2.6k $\quad g_S \quad$ 41.6k





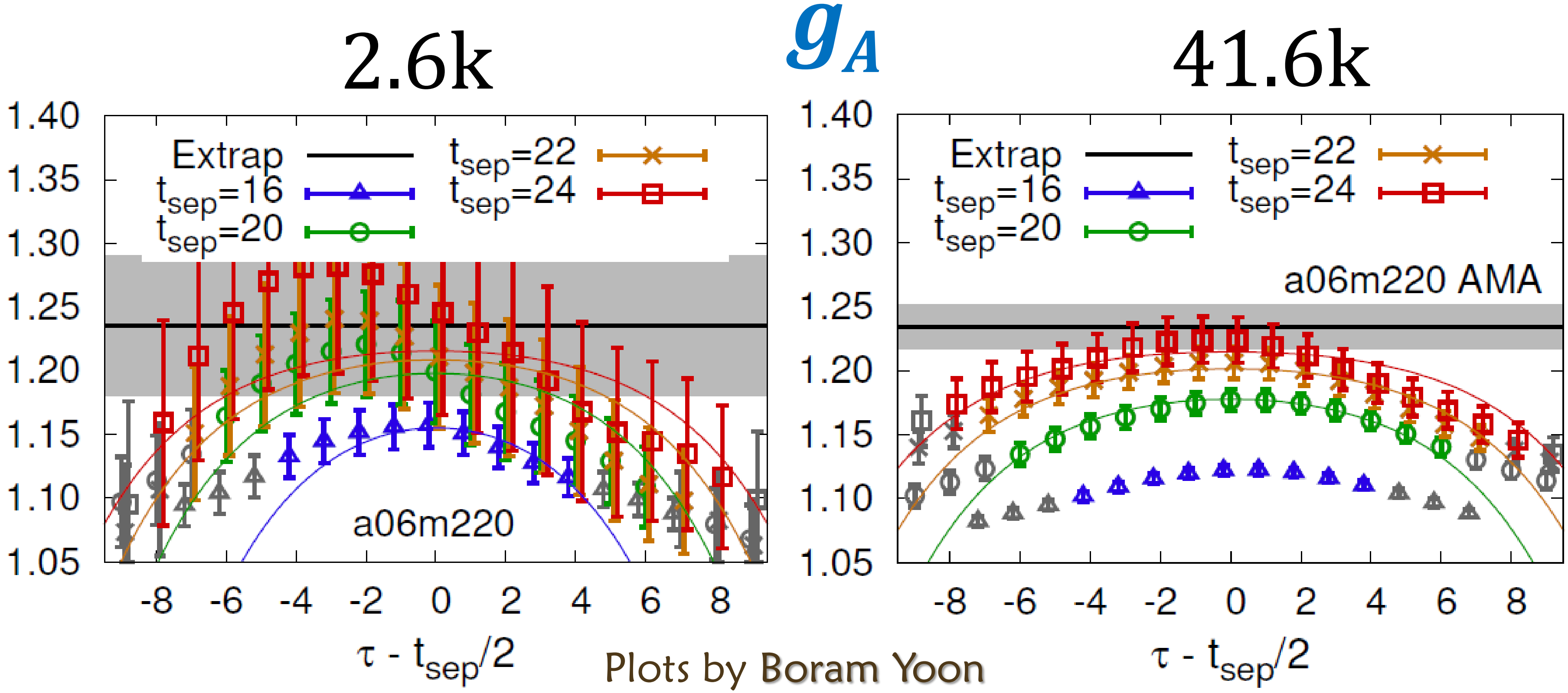
Plots by Boram Yoon

Systematic Control

§ Much effort has been devoted to controlling systematics

§ A state-of-the art calculation (PNDME)

- Statistical effect (worst case) **a = 0.06 fm**, 220-MeV pion



Plots by Boram Yoon

Systematic Control

§ Much effort has been devoted to controlling systematics

§ For

Lessons Learned

- g_A is *not* a gold-plated quantity

  Early impressions that g_A would be easy underestimated systematics

- No review of g_A in this talk

  Stay tuned for a better controlled g_A in the future

- You can still trust lattice g_A

  ...from groups who do due diligence for *every* ensemble and carefully study systematics

- Getting g_A to subpercent precision will be very hard

  Sitting back and letting the computer do the work won't get us there

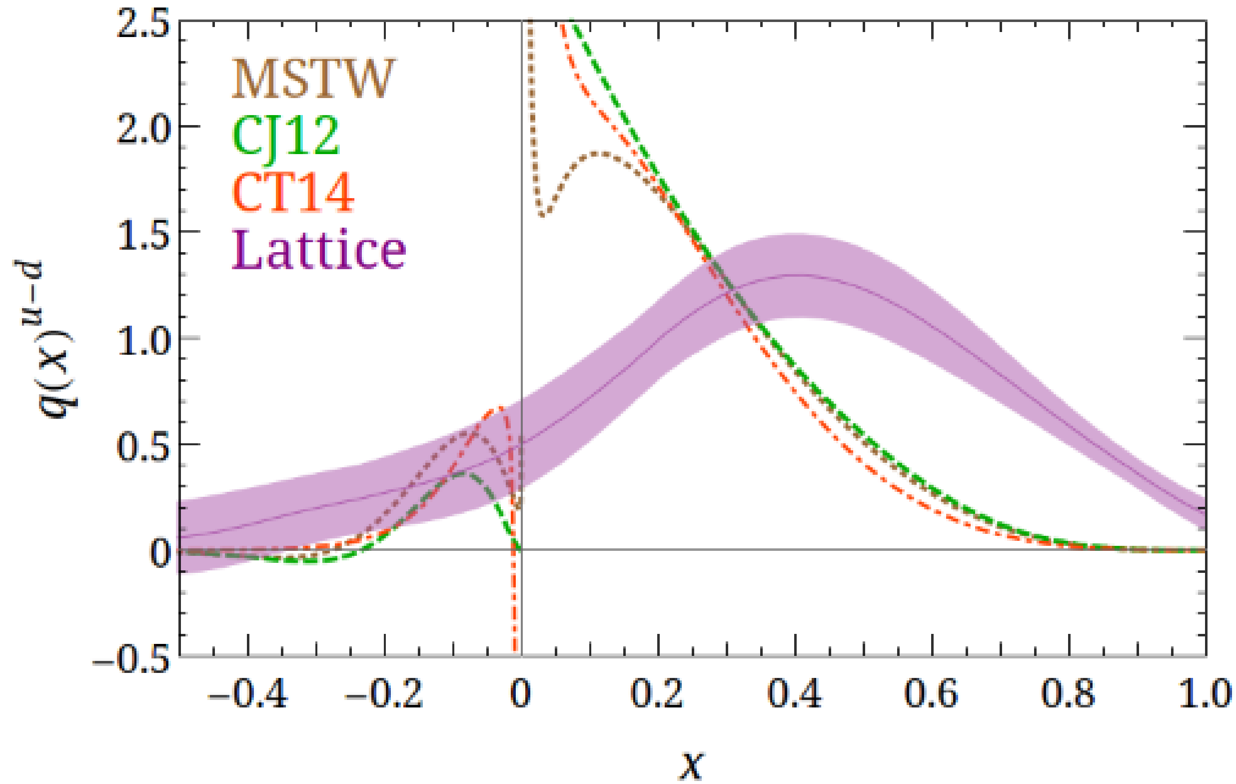
- Perhaps g_T is the real gold-plated quantity

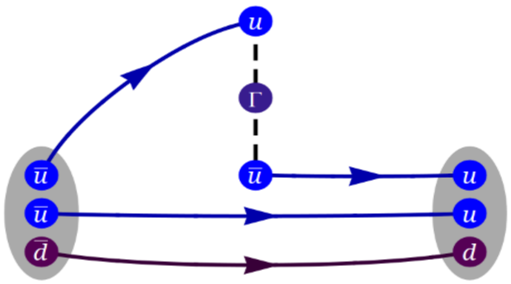
Quark Distribution

§ A first exploratory study

HWL et al. 1402.1462

- N_f = 2+1+1 clover/HISQ lattices (MILC)
  $M_\pi \approx$ **310 MeV**, $a \approx 0.12$ fm ($M_\pi L \approx 4.5$), O(10^3) measurements





A.D. Martin et al. Eur.Phys.J. C63, 189 (2009)

J.F. Owens et al. PRD 87, 094012 (2012)

S. Dulat et al. arXiv:1506.07443