Lattice QCD+QED

Towards a Quantitative Understanding of the Stability of Matter

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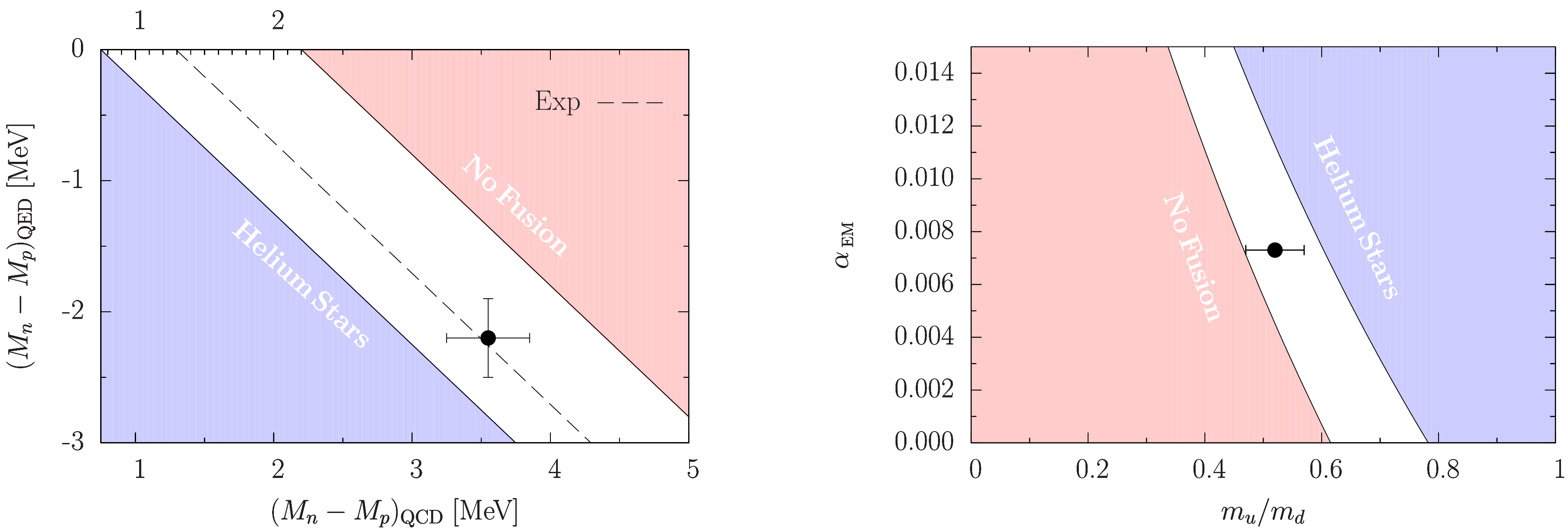
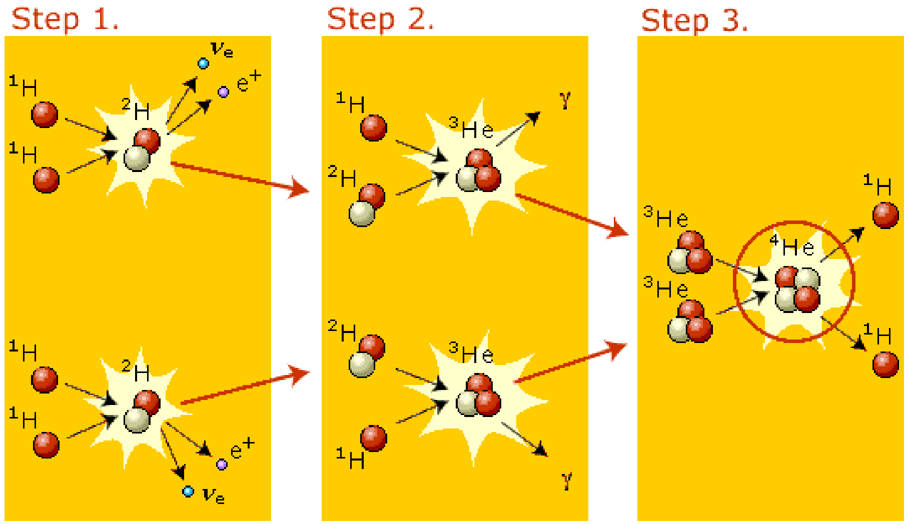
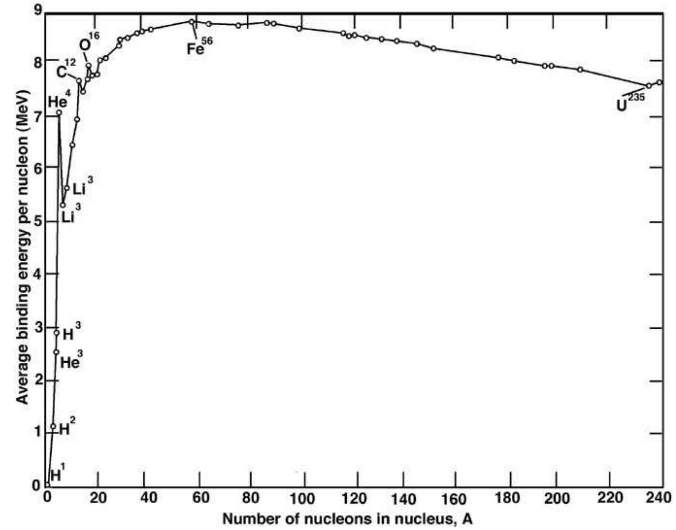
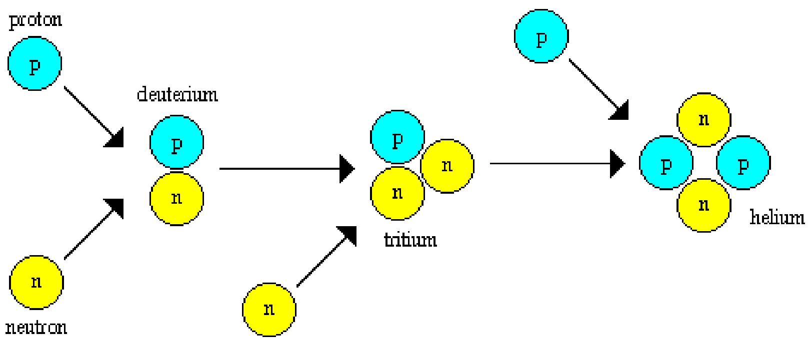
The Challenge



Having an analytic expression for the nucleon mass as a function of quark masses and α_{EM}, we can visualize the allowed region







If $m_u/m_d \gtrsim 0.8$ even, protons would decay spontaneously to neutrons

- The neutron – proton mass difference is one of the most consequential quantities of physics. It is extremely fine tuned for the stability of matter as we know it and the existence of our Universe. This calls for a calculation from first principles

- Lattice Gauge Theory is the method of choice. Lattice calculations are now reaching a level of precision, where it is possible to address isospin breaking effects

- These effects have two sources, the mass difference of u and d quarks, and electromagnetic interactions

- Both effects are of the same order of magnitude and cannot be separated unambiguously due to the nonperturbative nature of the strong interactions, which makes a direct calculation from QCD + QED necessary

Further issues

- A primary concern is to understand the pattern of flavor and isospin symmetry breaking in QCD + QED

- In particular, there is the prospect of making precise predictions for appropriate isospin violating processes

- Lattice calculations of hadronic processes are approaching O(1%) precision. At this level electromagnetic corrections must be included in the calculation

- Simulations of QCD + QED with electrically charged quarks allow to probe the effect of dynamical quarks on the confining properties of the QCD vacuum

Outline

Lattice QCD + QED

Vacuum Structure

Flavor Physics and Spectroscopy

Isospin Splittings

Conclusions

Lattice QCD + QED

BMW

arXiv:1406.4088

- Octet baryons

QCDSF

arXiv:1311.4554
arXiv:1508.06401
arXiv:1509.00799

- Vacuum structure
- Octet mesons
- Octet baryons
- Light quark masses

Underpinned by group theory

With

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R. Young (Adelaide), J. Zanotti (Adelaide)

Action

$$S = S_G + S_{QED} + S_F^u + S_F^d + S_F^s$$

$$S_G = \frac{6}{g^2} \sum_{x,\mu<\nu} \frac{1}{3} \mathrm{Tr} \left\{ c_0 \left[1 - U_{\mu\nu}(x)\right] + c_1 \left[1 - R_{\mu\nu}(x)\right] \right\}$$

$$S_{QED} = \frac{1}{2e^2} \sum_{x,\mu<\nu} \left(A_\mu(x) + A_\nu(x+\mu) - A_\mu(x+\nu) - A_\nu(x)\right)^2$$ noncompact

$$S_F^q = \sum_x \Big\{ \sum_\mu \Big[\overline{q}(x) \frac{\gamma_\mu - 1}{2} e^{-ie_q A_\mu(x)} \tilde{U}_\mu(x) q(x+\hat{\mu})$$

$$- \overline{q}(x) \frac{\gamma_\mu + 1}{2} e^{ie_q A_\mu(x-\hat{\mu})} \tilde{U}_\mu^\dagger(x-\hat{\mu}) q(x-\hat{\mu}) \Big]$$

$$+ \frac{1}{2\kappa_q} \overline{q}(x) q(x) - \frac{1}{4} c_{SW} \sum_{\mu\nu} \overline{q}(x) \sigma_{\mu\nu} F_{\mu\nu}(x) q(x) \Big\}$$

Lattice spacing a implicit

$$e_u = \frac{2}{3}\,,\ e_d = e_s = -\frac{1}{3}$$

Lattices

V	κ_u	κ_d	κ_s
$24^3 \times 48$	0.124362	0.121713	0.121713
$32^3 \times 64$	0.124362	0.121713	0.121713
$48^3 \times 96$	0.124362	0.121713	0.121713
$32^3 \times 64$	0.124440	0.121676	0.121676
$48^3 \times 96$	0.124440	0.121676	0.121676
$32^3 \times 64$	0.124508	0.121821	0.121466

} Symmetric point $\mu_u^D = \mu_d^D = \mu_s^D$ (first three rows)

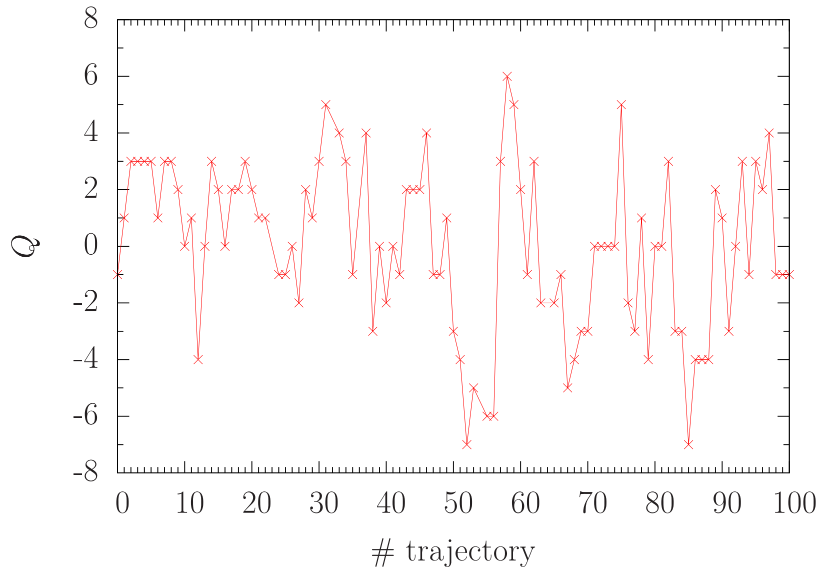
$$1/\kappa_u + 1/\kappa_d + 1/\kappa_s = \text{constant}$$

Couplings

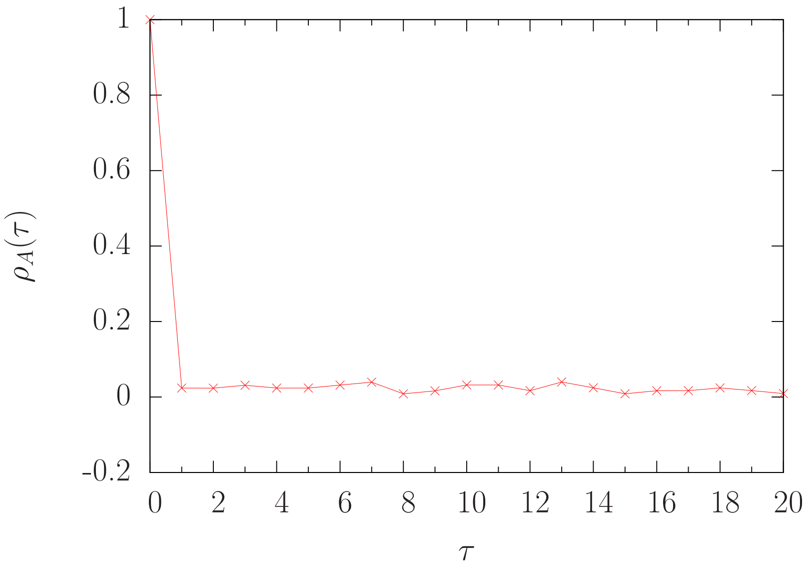
$$\beta \equiv \frac{6}{g^2} = 5.50\,, \quad \alpha_{\rm EM} \equiv \frac{e^2}{4\pi} = 0.10$$

corresponding to $a = 0.068(2)$ fermi. Later on we extrapolate the results to $\alpha_{\rm EM} = 1/137$

Efficiency



8
6
4
2
0
-2
-4
-6
-8
Q
0 10 20 30 40 50 60 70 80 90 100
trajectory



1
0.8
0.6
0.4
0.2
0
-0.2
$\rho_A(\tau)$
0 2 4 6 8 10 12 14 16 18 20
τ

Gauge fixing

The actions S_{QED} and S_F^q are invariant under U(1) gauge transformations

$$A_\mu(x) \to A_\mu(x) + \Delta_\mu\,\alpha(x)\,, \quad q(x) \to e^{ie_q\alpha(x)}\,q(x)$$

This is not the case for propagators of charged particles, which demands gauge fixing. We choose Landau gauge $\bar{\Delta}_\mu A_\mu(x) = 0$, which, however, does not eliminate all gauge degrees of freedom, but allows for shifts $\Delta_\mu\alpha(x)$ with $\Delta^2\alpha(x) = 0$. Maintaining (anti-)periodicity of the quark fields, this redundancy can be eliminated by adding multiples of $2\pi/e_q L_\mu$ to $A_\mu(x)$ such that

$$-\frac{\pi}{|e_q|L_\mu} < B_\mu \leq \frac{\pi}{|e_q|L_\mu}\,, \quad B_\mu = \frac{1}{V}\sum_x A_\mu(x)$$

The constant background field can be factored out from the link matrices and absorbed into the quark momenta p

$$q(x) \to e^{ie_qBx}\,q(x)\,,\ \bar{q}(x) \to \bar{q}(x)\,e^{-ie_qBx}$$

which amounts to a shift $p \to p + e_qB$. This leaves us with photon propagators that are devoid of zero modes

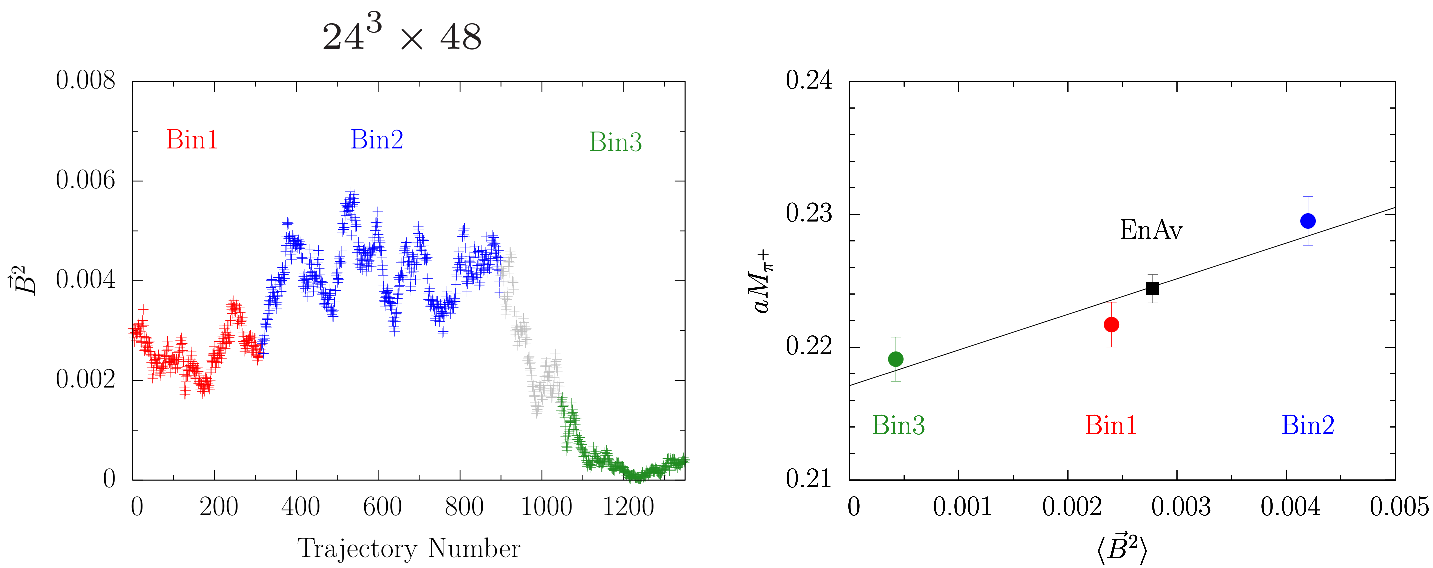
In the presence of a constant background field B_μ the correlator of a single hadron H then becomes

$$\langle 0|H(t)\bar{H}(0)|0\rangle \simeq |Z_H|^2 \, e^{-\sqrt{M_H^2+(\vec{p}+e_H\vec{B})^2}\,t}$$

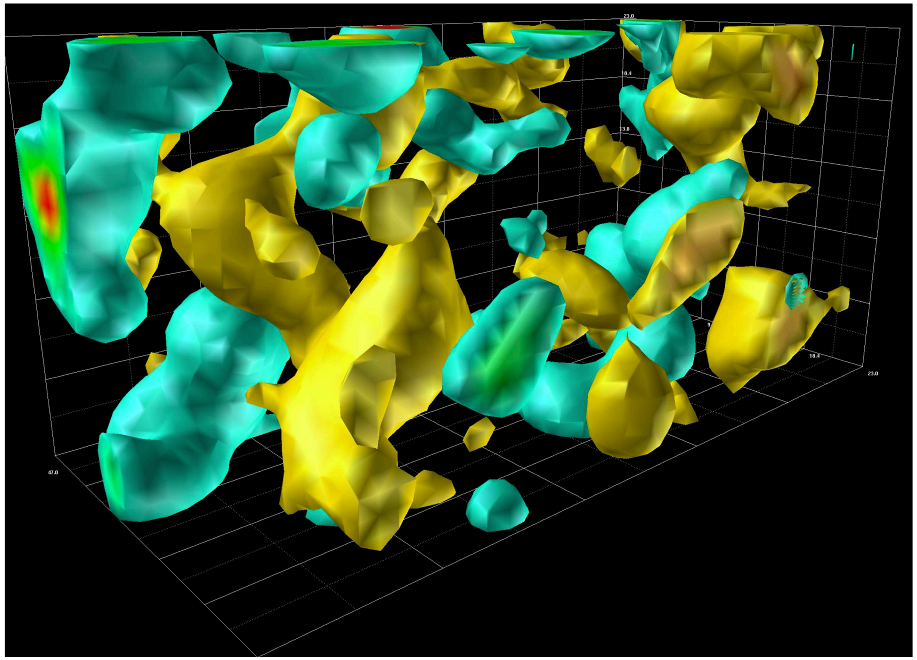
which amounts to a shift

$$M_H \to \sqrt{M_H^2 + e_H^2\,\vec{B}^2}$$

To extract masses we remove the influence of the background field by subtracting the associated kinetic energy from the ensemble averaged lattice energy



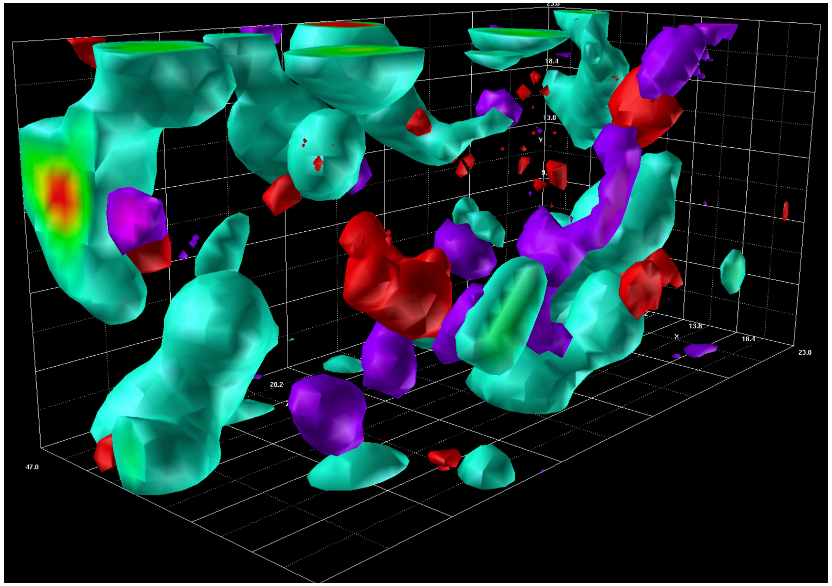
Vacuum Structure



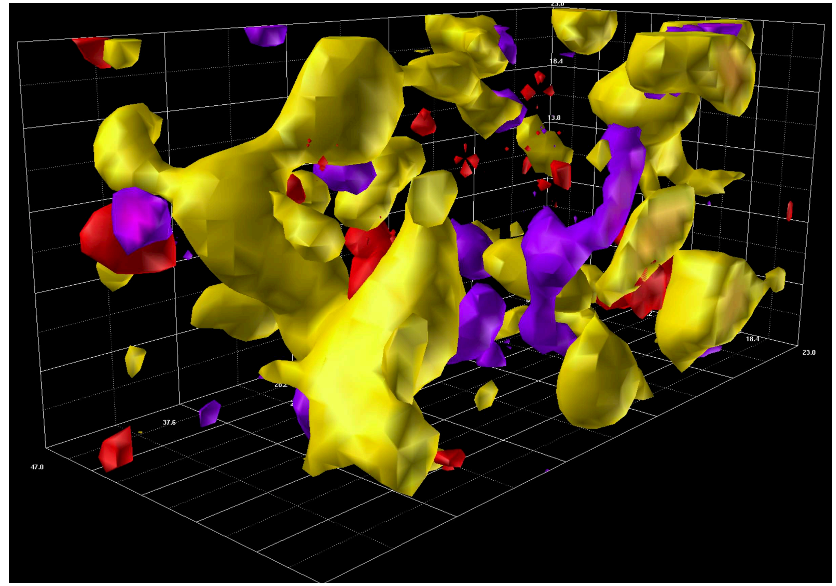
Density of QCD (aqua) and QED (yellow) actions

↑

Electromagnetic field strength
repelled by chromoelectric one



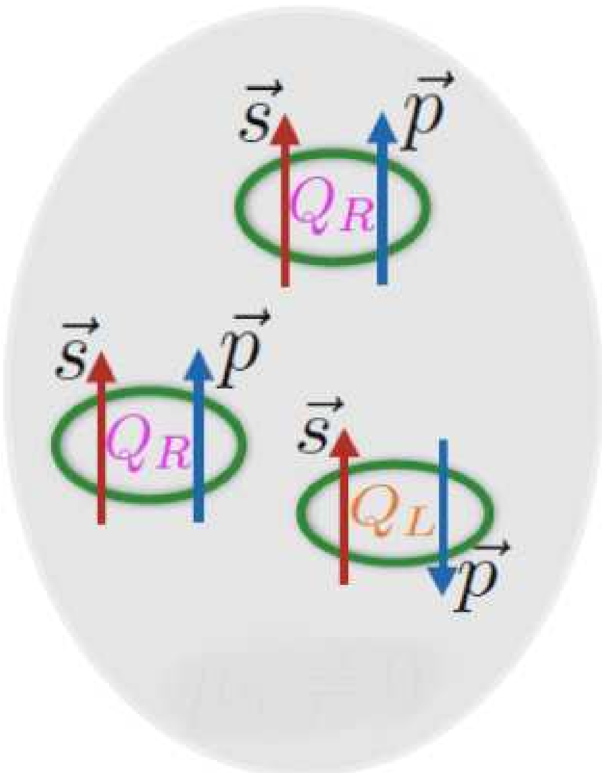
Density of positive (red) and negative (purple) charge compared with QCD action density



Density of positive (red) and negative (purple) charge compared with QED action density

(Charvetto)

Chiral Magnetic Effect



Instanton

$\Longrightarrow$

Excess of right-handed quarks due to chiral anomaly

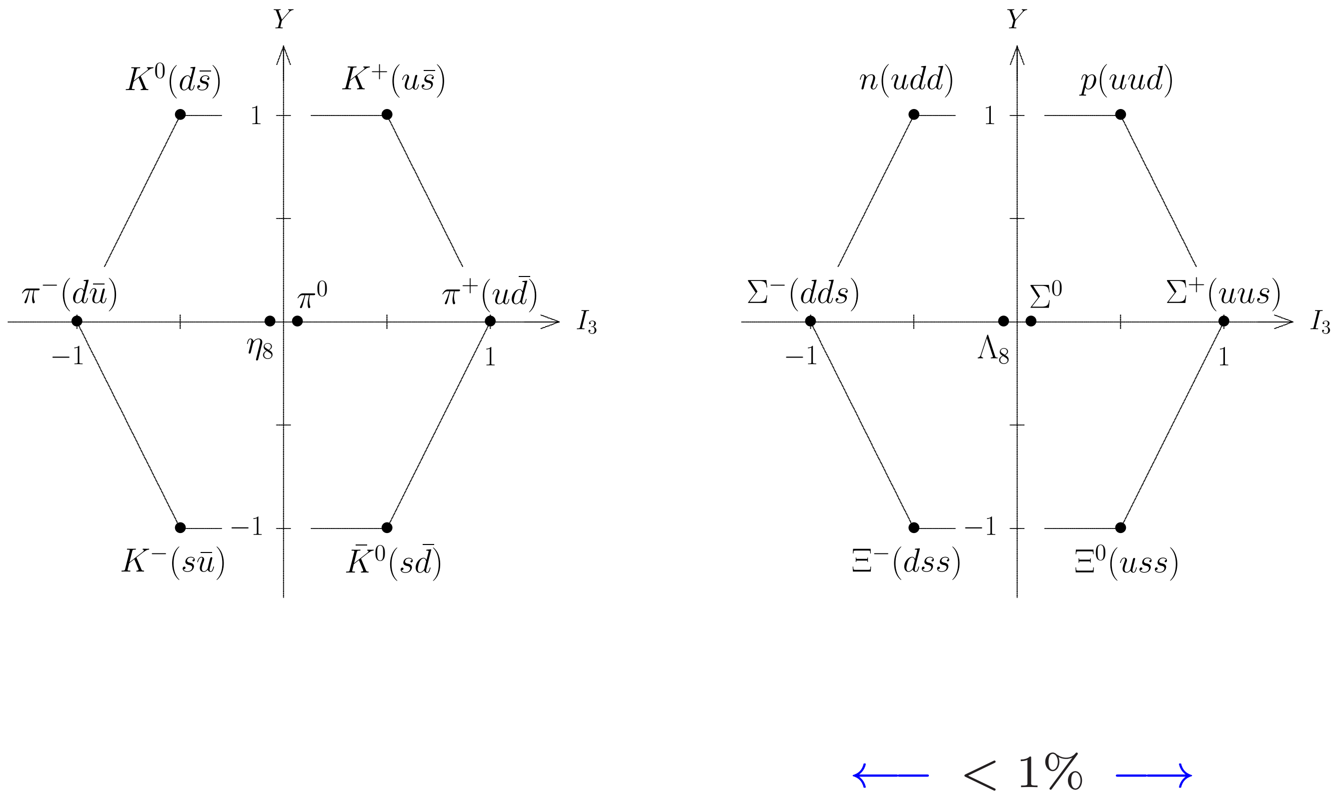
$$\downarrow$$

$$\vec{p} \,||\, \vec{s}$$

$$\vec{p} \propto \vec{J}\,,\ \vec{s} \propto \vec{B}$$

We find evidence for $\vec{J}\vec{B}$ to be correlated with position of instanton

Flavor Physics and Spectroscopy



Strategy

arXiv:1102.5300

- QCD interactions are flavor blind. The only difference between flavors comes from the quark mass matrix

- In lattice calculations one can vary the quark masses freely, which helps to illuminate the pattern of flavor symmetry breaking

- One has the best theoretical understanding when all quark masses are equal, because one can use the full power of flavor SU(3)

- We interpolate between the symmetric point $\mu_u = \mu_d = \mu_s$ and the physical point by keeping the sum of the quark masses $(\mu_u + \mu_d + \mu_s)/3 \equiv \bar{m}$ fixed at its physical value, which is particularly instructive

- The symmetry of the electromagnetic current is similar to the symmetry of the quark mass matrix

- The simplifications from keeping $\bar{m}$ = constant in the mass expansion are analog to the simplifications from the identity $e_u + e_d + e_s = 0$. We thus can read off the QED corrections from the mass expansion changing masses to charges

Benefit

Unitary

$$M^2(aab) = M_0^2 + \alpha_1 \left(2\delta m_a + \delta m_b\right) + \alpha_2 \left(\delta m_a - \delta m_b\right) \\ + \beta_0^{\rm EM} \left(e_u^2 + e_d^2 + e_s^2\right) + \beta_1^{\rm EM} \left(2e_a^2 + e_b^2\right) + \beta_2^{\rm EM} \left(e_a - e_b\right)^2 + \beta_3^{\rm EM} \left(e_a^2 - e_b^2\right)$$

$$\delta m_q = m_q - \bar{m} = 1/2\kappa_q^{\rm sea} - 1/\bar{\kappa}_q$$

Partially quenched

$$M^2(aab) = M_0^2 + \alpha_1 \left(2\delta \mu_a + \delta \mu_b\right) + \alpha_2 \left(\delta \mu_a - \delta \mu_b\right) \\ + \beta_0^{\rm EM} \left(e_u^2 + e_d^2 + e_s^2\right) + \beta_1^{\rm EM} \left(2e_a^2 + e_b^2\right) + \beta_2^{\rm EM} \left(e_a - e_b\right)^2 + \beta_3^{\rm EM} \left(e_a^2 - e_b^2\right)$$

$$\delta \mu_q = \mu_q - \bar{m} = 1/2\kappa_q^{\rm val} - 1/\bar{\kappa}_q$$

QCD

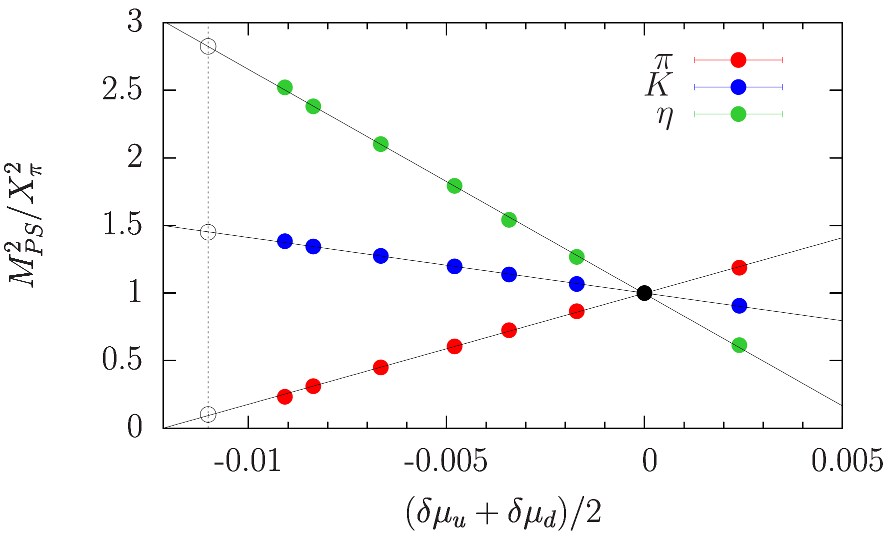
Gell-Mann–Okubo

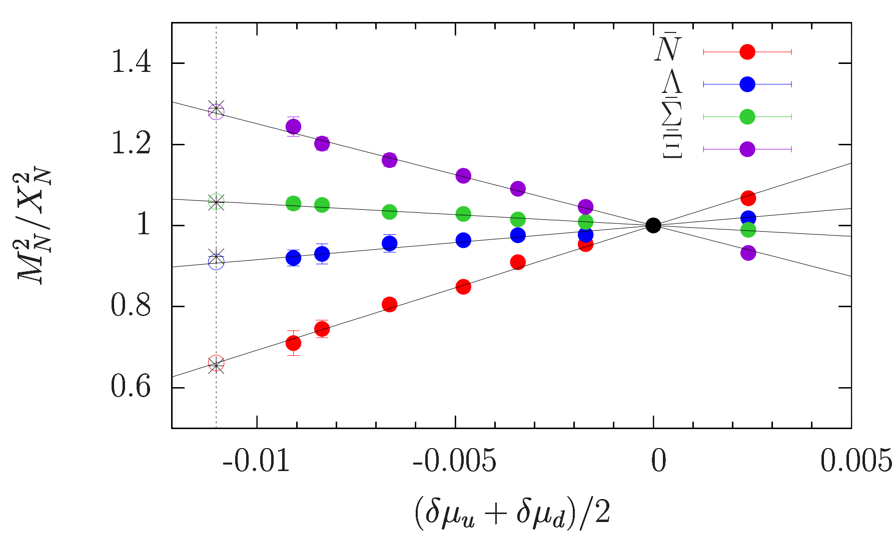
$\geq$ quadratic

1 **8** **10**, **27**

$$M^2(a\bar{b}) = M_0^2 + \alpha\,(\delta\mu_a + \delta\mu_b) + \cdots \qquad \delta\mu_q = \mu_q - \bar{m}$$

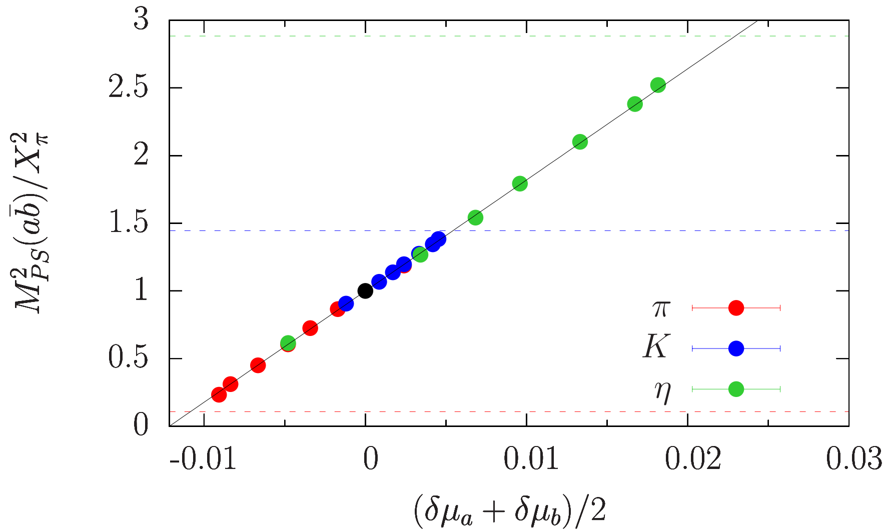
$$M^2(aab) = M_0^2 + \alpha_1(2\delta\mu_a + \delta\mu_b) + \alpha_2\,(\delta\mu_a - \delta\mu_b) + \cdots \qquad \delta\mu_u + \delta\mu_d + \delta\mu_s = 0$$





$$X_\pi^2 = (M^2_{K^0} + M^2_{K^+} + 2M^2_{\pi^0} - M^2_{\pi^+})/3$$

$$X_N^2 = (M^2_n + M^2_p + M^2_{\Sigma^-} + M^2_{\Sigma^+} + M^2_{\Xi^-} + M^2_{\Xi^0})/6$$



$$\delta\mu_q = \mu_q - \bar{m}$$

arXiv:1102.5300

Comparison with ChPT

$$M_0^2 = \bar{\chi}\left[1 - \frac{16\bar{\chi}}{f_0^2}\left(3L_4 + L_5 - 6L_6 - 2L_8\right) + \frac{\bar{\chi}}{24\pi^2 f_0^2}\ln\frac{\bar{\chi}}{\Lambda_\chi^2}\right]$$

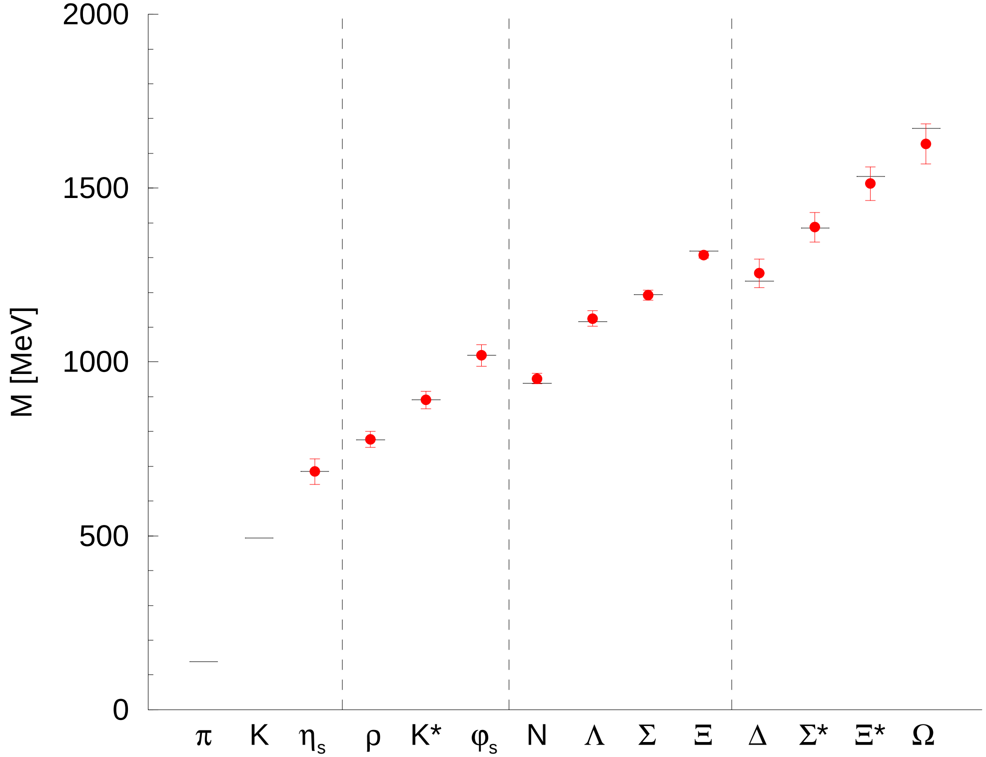
$$\alpha = Q_0\left[1 - \frac{16\bar{\chi}}{f_0^2}\left(3L_4 + 2L_5 - 6L_6 - 4L_8\right) + \frac{\bar{\chi}}{8\pi^2 f_0^2}\ln\frac{\bar{\chi}}{\Lambda_\chi^2}\right]$$

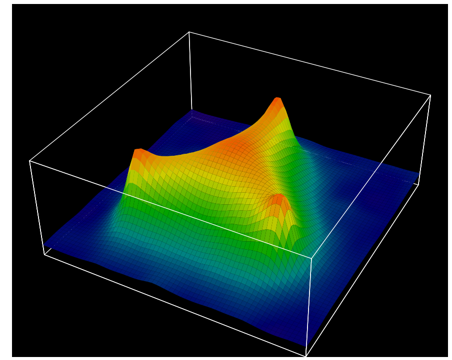
$$\bar{\chi} = 2\,Q_0\,\frac{Z_m^S}{Z_m^{NS}}\,\bar{m}$$

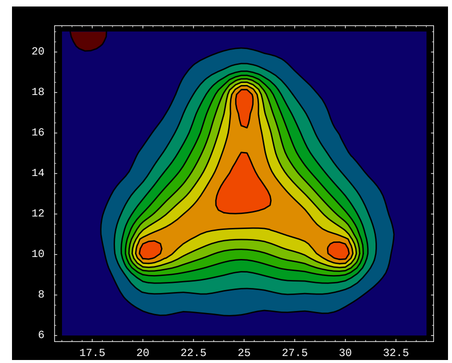
arXiv:1102.5300

Spectrum

80% of mass







Dual superconductor

QCD + QED

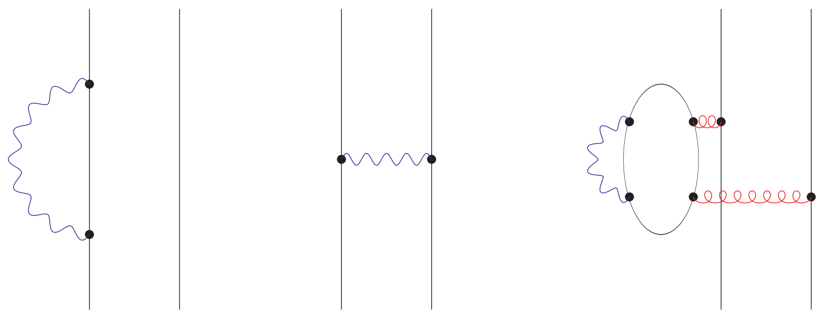
$$M^2(a\bar{b}) = M_0^2 + \alpha\,(\delta\mu_a + \delta\mu_b) + \beta_1^{\mathrm{EM}}\,(e_a - e_b)^2$$
$$+\,\gamma_1^{\mathrm{EM}}\,(e_a - e_b)^2\,(\delta\mu_a + \delta\mu_b) + \gamma_2^{\mathrm{EM}}\,(e_a^2 - e_b^2)\,(\delta\mu_a - \delta\mu_b)$$

Dashen scheme

$$M_{\pi^0}^2 = M_0^2 + \alpha\,(\delta\mu_u + \delta\mu_d)$$

$$M_{K^0}^2 = M_0^2 + \alpha\,(\delta\mu_d + \delta\mu_s)$$

Determine physical quark masses



Dashen scheme

- Define symmetric point by $M_{PS}(u\bar{u}) = M_{PS}(d\bar{d}) = M_{PS}(s\bar{s}) = M_{PS}(d\bar{s}) = M_{PS}(s\bar{d})$

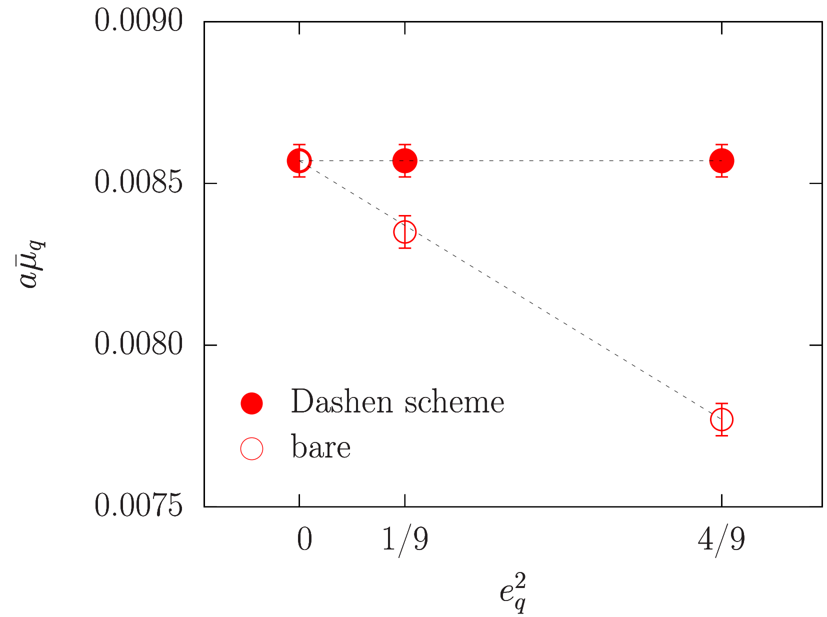
- Renormalize (rescale) quark masses

$$\delta\mu_q \to \delta\mu_q = \delta\mu_q(1 + Ke_q^2)$$

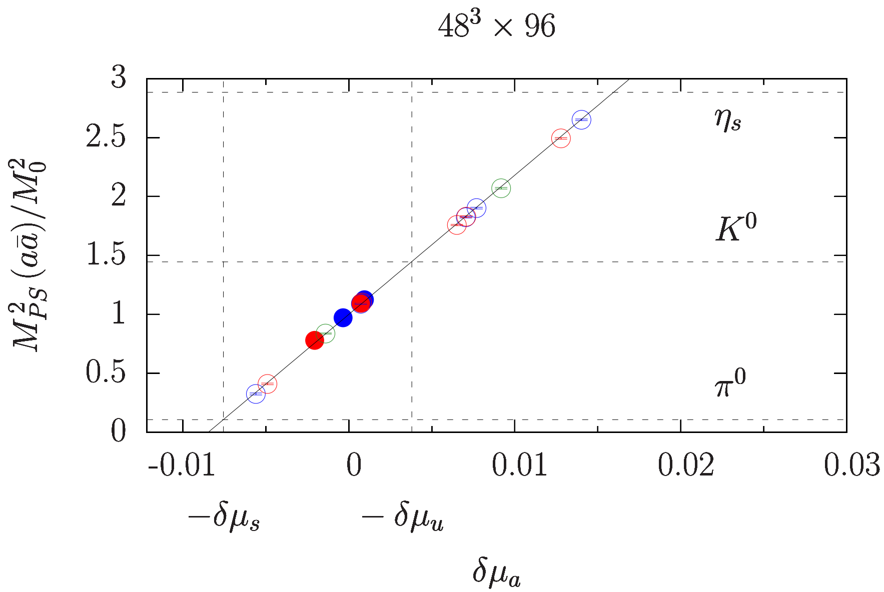
such that

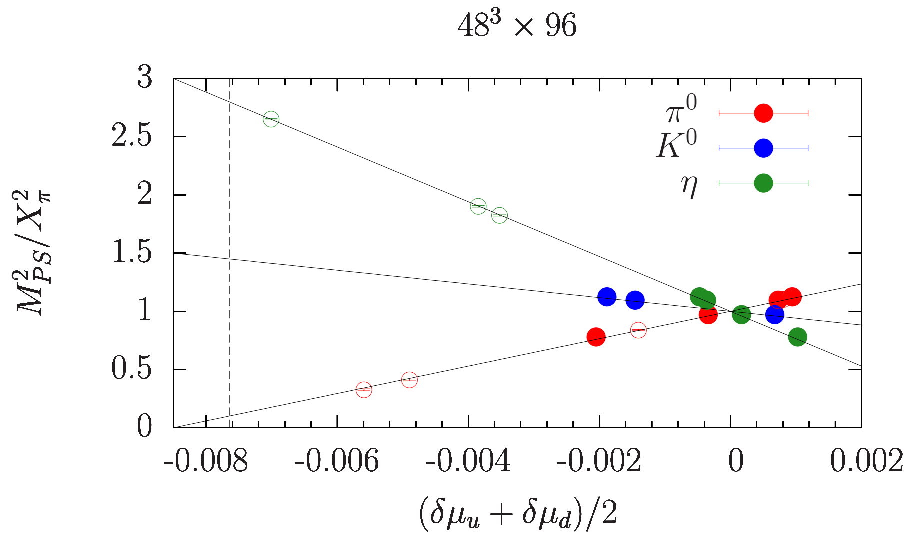
$$\bar{\mu}_u = \bar{\mu}_d = \bar{\mu}_s$$

at symmetric point



Neutral mesons

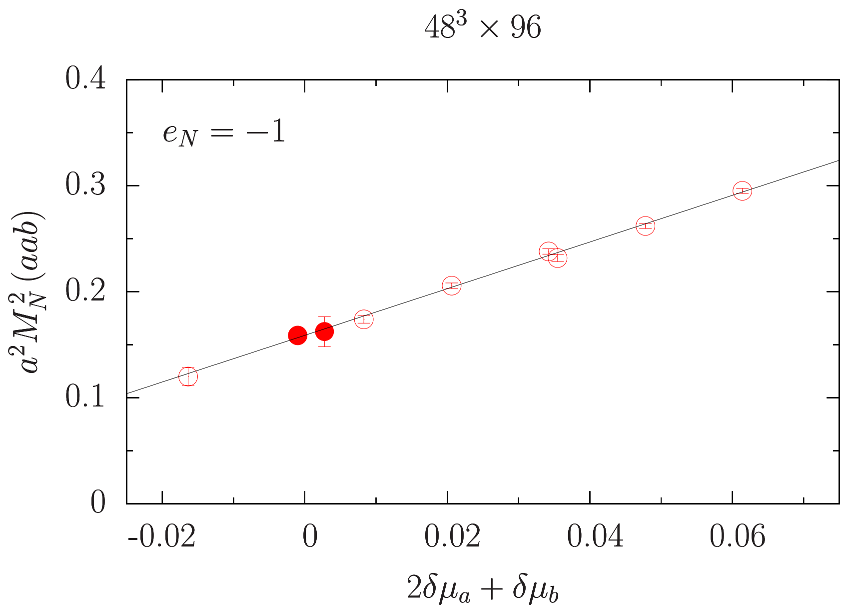




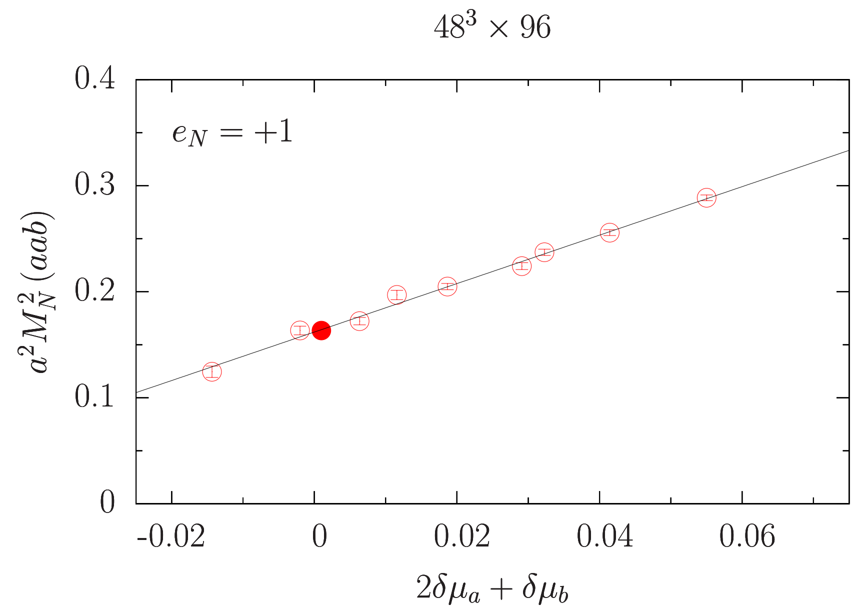
$\underline{\mathbf{1}} + \underline{\mathbf{8}}$

1 **8**

$$M^2(aab) = M_0^2 + \alpha_1\,(2\delta\mu_a + \delta\mu_b) + \alpha_2\,(\delta\mu_a - \delta\mu_b)$$
$$+\,\beta_1^{\mathrm{EM}}\,(2e_a^2 + e_b^2) + \beta_2^{\mathrm{EM}}\,(e_a - e_b)^2 + \beta_3\,(e_a^2 - e_b^2)$$



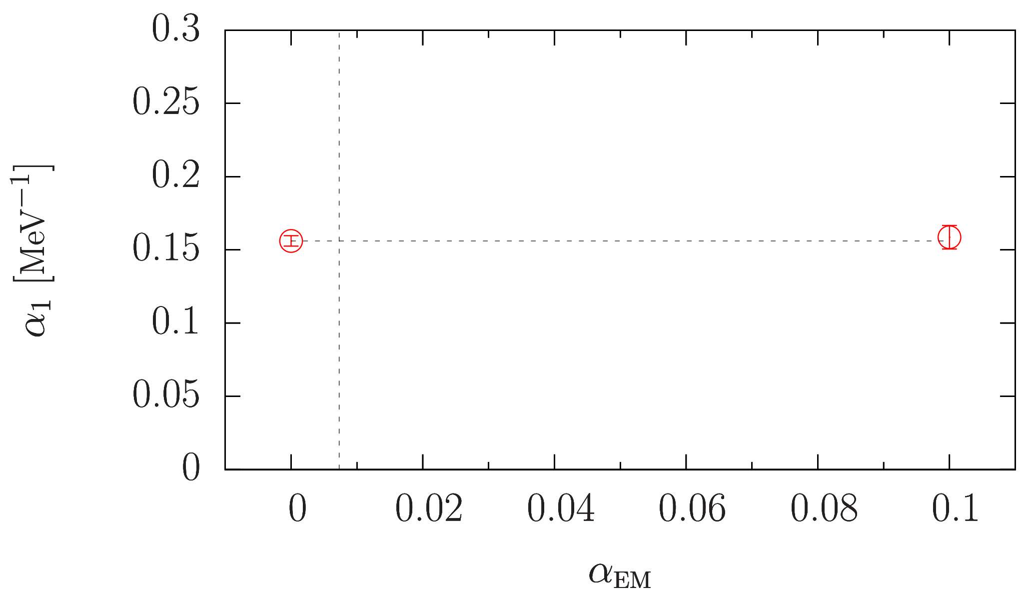
Σ^-, Ξ^-



p, Σ^+

Extrapolation

Extrapolate linearly from $\alpha_{\rm EM} = 0.1$ to $\alpha_{\rm EM} = 1/137$



Recover QCD results

Isospin Splittings

Quark Masses

$$M^2_{\pi^0} = M^2_0 + \alpha(\delta\mu_u + \delta\mu_d) = \alpha(\mu_u + \mu_d)$$

$$M^2_{K^0} = M^2_0 + \alpha(\delta\mu_d + \delta\mu_s) = \alpha(\mu_d + \mu_s)$$

$$\left.\right\} \quad \mu_u + \mu_d + \mu_s = \text{constant}$$

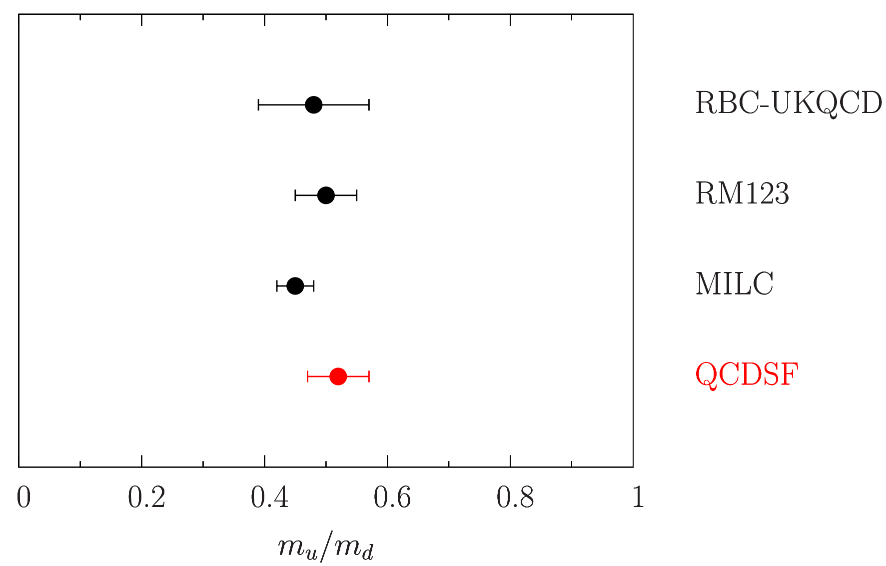
$$m_q = Z_m^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})\,\Delta Z_D^{\overline{\mathrm{MS}}}\mu_q$$

$$m_u = 2.49(14)\ \mathrm{MeV}$$

$$m_d = 4.80(27)\ \mathrm{MeV}$$

$$m_s = 94.5(52)\ \mathrm{MeV}$$

$$\frac{m_u}{m_d} = 0.52(2)\,, \quad \frac{m_s}{m_d} = 19.7(9)$$



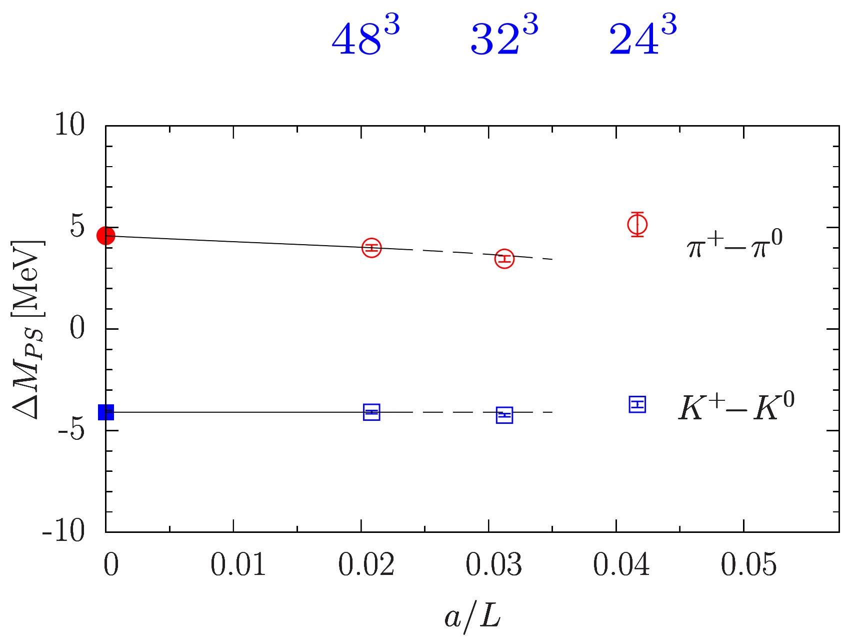
$$\mu_q^D = \Big(1 + \alpha_{\rm EM}\, e_q^2\, 2.20(9)\Big)\, \mu_q$$ numerically

Changing schemes

$$\mu_q^{\overline{MS}}(2\ \mathsf{GeV}) = \Big(1 + \alpha_{\rm EM}\, e_q^2\, 1.208 + O(\alpha_{\rm EM} g^2)\Big)\, \mu_q$$ one loop

$$\begin{aligned}
\mu_q^{\overline{MS}}(2\ \mathsf{GeV}) &= \Big(1 - \alpha_{\rm EM}\, e_q^2\, 1.0(5)\Big)\, \mu_q^D \\
&= \Delta Z_D^{\overline{MS}}(2\ \mathsf{GeV})\, \mu_q^D \\
&\approx \mu_q^D
\end{aligned}$$

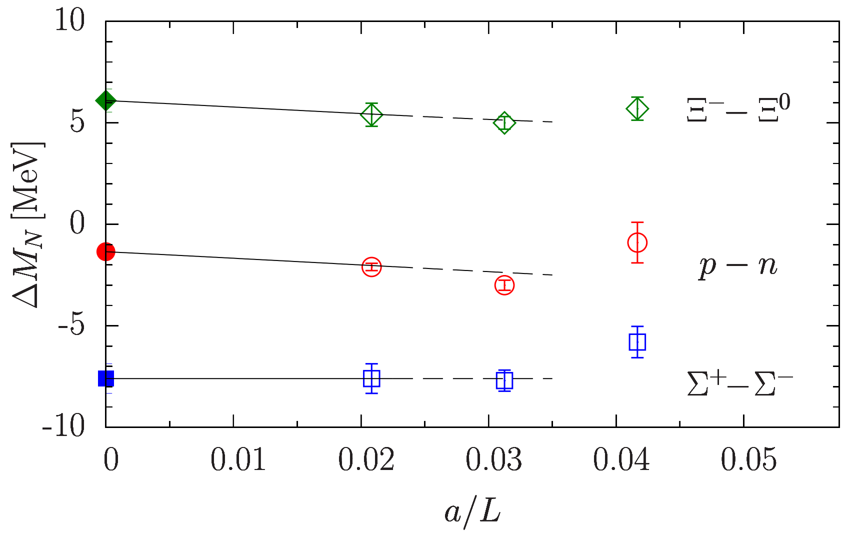
Meson Octet



$$\Delta M_{\pi^+} = \frac{\alpha_{\mathrm{EM}}}{2L} c_1 \left(1 + \frac{2}{M_{\pi^+} L}\right) + \frac{2\pi\alpha_{\mathrm{EM}}}{3L^3} \left(1 + \frac{4\pi}{M_{\pi^+} L} c_{-1}\right) \langle r^2 \rangle_{\pi^+} + \cdots$$

Davoudi & Savage

Baryon Octet

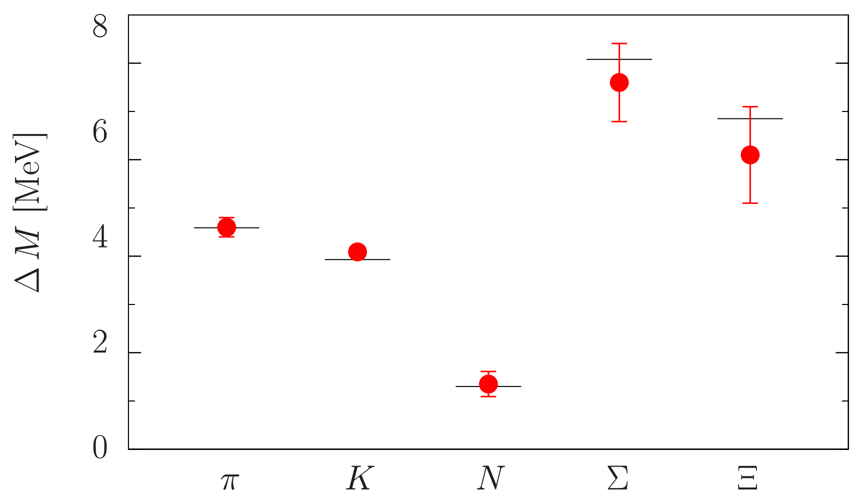


$$\Delta M_p = \frac{\alpha_{\rm EM}}{2L} c_1 \left(1 + \frac{2}{M_p L}\right) + \frac{2\pi\alpha_{\rm EM}}{3L^3} \left(1 + \frac{4\pi}{M_{\pi^+} L} c_{-1}\right) \langle r^2 \rangle_p + \cdots$$

Davoudi & Savage

Splittings

ΔM	QCD + QED	QED	Experiment
$M_{\pi^+} - M_{\pi^0}$		4.60(20)	4.59
$M_{K^0} - M_{K^+}$	4.09(10)	−1.66(6)	3.93
$M_n - M_p$	1.35(18)(8)	−2.20(28)(10)	1.30
$M_{\Sigma^-} - M_{\Sigma^+}$	7.60(73)(8)	−0.63(8)(6)	8.08
$M_{\Xi^-} - M_{\Xi^0}$	6.10(55)(45)	1.26(16)(13)	6.85



$$M_n - M_p + M_{\Sigma^+} - M_{\Sigma^-} + M_{\Xi^-} - M_{\Xi^0} = 0 \quad \propto \quad \underline{\mathbf{10}}$$

Coleman–Glashow

Changing schemes

$$[M^{D}_{\rm QED}]^2 = M^2(\alpha_{\rm EM}, \{\mu^D_q\}) - M^2(0, \{\mu^D_q\})$$

$$[M^{\overline{MS}}_{\rm QED}]^2 = M^2(\alpha_{\rm EM}, \{\mu^D_q\}) - M^2(0, \{\mu^{\overline{MS}}_q\})$$

Taking the difference gives

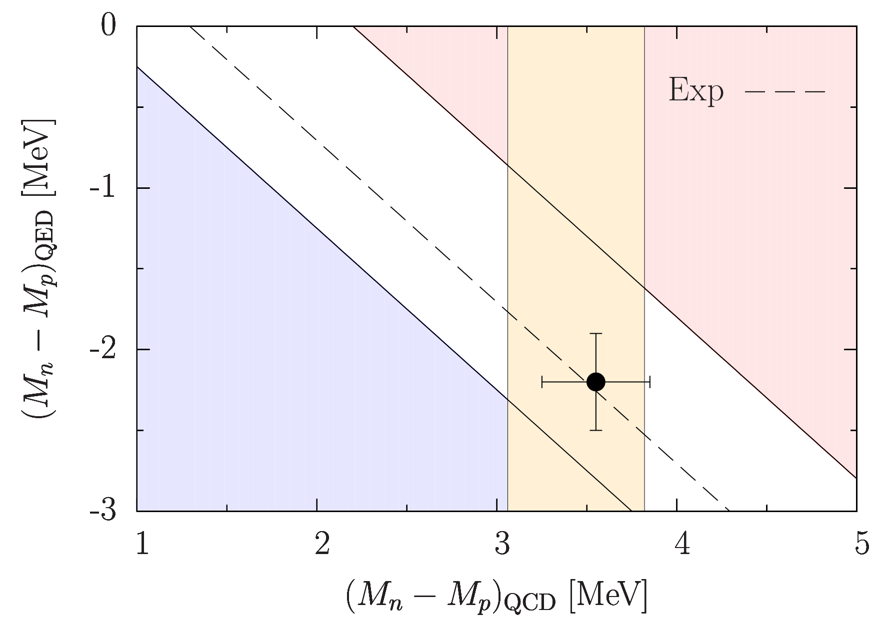
$$[M^{\overline{MS}}_{\rm QED}]^2 - [M^{D}_{\rm QED}]^2 = M^2(0, \{\mu^D_q\}) - M^2(0, \{\mu^{\overline{MS}}_q\})$$

In particular

$$[M^{\overline{MS}}_{\rm QED}]^2_p - [M^{D}_{\rm QED}]^2_n = \alpha_{\rm EM}\, 1.0(5)\ (\alpha_1 + 2\alpha_2)\left(\frac{4}{9}\delta\mu^D_u - \frac{1}{9}\delta\mu^D_d\right)$$

QCD vs QED

`arXiv:1508.05916`



Dashen scheme $\simeq \overline{\mathrm{MS}}$

Analytic Solution?

Renormalization Group — Tells us how the bare parameters of the theory must behave to keep the physics constant as the cut-off is varied

$$\frac{\partial \ln \mu_u/\mu_d}{\partial \ln \mu} = 0 - \frac{e^2}{8\pi^2} + \frac{e^2}{12\pi^2}\left(4\frac{\mu_u^2}{\mu^2} - \frac{\mu_d^2}{\mu^2}\right) \qquad \mu = 1/a$$

QCD QED

$\Uparrow$

$$\gamma_q^{\mathrm{EM}} = -\frac{3e_q^2}{8\pi^2}\left[1 - \frac{\mu_q^2}{\mu^2}\ln\left(1 + \frac{\mu^2}{\mu_q^2}\right)\right]$$

Solution

$$\frac{\mu_u}{\mu_d} \simeq \frac{1}{2}\,\mu^{-\frac{e^2}{8\pi^2}} = \frac{1}{2} - \frac{e^2}{16\pi^2}\ln \mu \quad \Longrightarrow \quad \boxed{\frac{m_u}{m_d} = \frac{1}{2}} \qquad m_q = Z^{\overline{\mathrm{MS}}}\mu_q$$

Conclusions

- Flavor and isospin symmetry breaking of hadron masses follow a very simple pattern, made visible by systematic lattice simulations of QCD + QED

- So far we have investigated isospin breaking of pseudoscalar meson and octet baryon masses. That allowed us to look simultaneously at both sources of isospin breaking, the quark mass differences and electromagnetic interactions, which are of comparable importance

- The stability of matter, and the existence of the Universe as we know it, largely hinges on the ratio of up to down quark mass

- From a broader perspective, we can look forward to a better understanding of the QCD vacuum and the mechanism of confinement and chiral symmetry breaking

- With increased computer power it will be possible to improve on the precision of the calculation, which is still limited