Free energy at $T > 0$

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work done with
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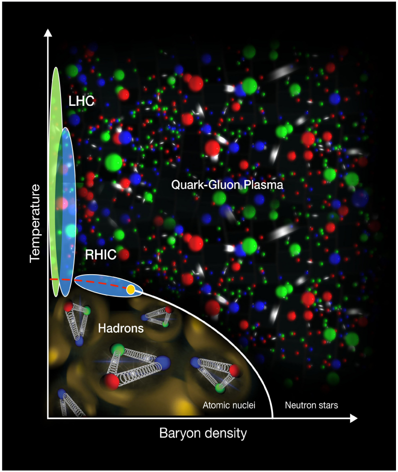
- Overview & introduction
- Static energy at $T > 0$
- Free energy at $T > 0$
  - Polyakov loop correlator
  - Singlet free energy
  - Polyakov loop
- Summary

QCD phase diagram

Time evolution since Big Bang



Plasma phase:

quark-gluon-plasma
$T > T_c \approx 160$ **MeV**
deconfinement,
color screening,
iso-vector chiral symmetry,
. . .

Hadronic phase:

dilute hadron gas
$T_c \gg T \approx 0$ **MeV**
confinement,
hidden chiral symmetry,
center symmetry (YM),
. . .

Why free energy?

Direct extraction of the **static energy** for $T > 0$ involves **ill-posed problem**.

$\Rightarrow$ workarounds to study thermal effects in the interaction of static quarks:

- **Free energy** of $Q\bar{Q}$ pair in terms of the Polyakov loop correlator
- **Singlet free energy** in terms of the Wilson line correlator
- **Free energy** of single quark in terms of the Polyakov loop

HISQ/Tree ($\mathcal{O}(\alpha_s a^2, a^4)$), $m_l = m_s/20$, $N_\tau = 4, 6, 8, 10, 12$, $N_\sigma = 4N_\tau$

Four different regimes in thermal EFTs at **weak-coupling**:

$$g^2T \ll gT \ll T \ll 1/r, \qquad g^2T \ll gT \ll T \sim 1/r,$$
$$g^2T \ll gT \sim 1/r \ll T, \qquad g^2T \sim 1/r \ll gT \ll T.$$

Only two (?) different regimes are accessible in **thermal LGT**:

$$g^2T \ll gT \ll T \ll 1/r, \qquad g^2T \sim gT \sim T \sim 1/r$$

Compare to weak-coupling EFTs at **short distances or high temperatures**.

Color singlet-octet structure

Vacuum for $T \sim 0$: $Q\bar{Q}$ pairs in color-octet configuration disfavored

Medium for $T > T_c$: $Q\bar{Q}$ pairs in color-octet configuration relevant

Thermal **singlet-octet transitions** – no separation of color singlet or octet

Only two well-defined (non-mixing) combinations of singlet and octet

$$\exp[-\frac{F_{Q\bar{Q}}}{T}] = \frac{1}{9}\exp[-\frac{F_S}{T}] + \frac{8}{9}\exp[-\frac{F_O}{T}]$$

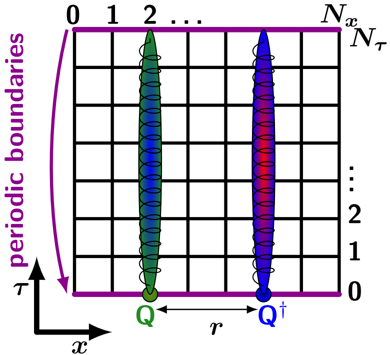
Color averaged **free energy** $F_{Q\bar{Q}}$ with **singlet-octet** decomposition

$$\frac{1}{9}(F_S - F_O) = \frac{1}{8}\left(F_S - F_{Q\bar{Q}}\right) = \left(F_O - F_{Q\bar{Q}}\right)$$

Additive renormalization constant $(2C_Q)$ for $F_{Q\bar{Q}}$, F_S, F_O same as for V_s.

Any definition of color singlet and octet involves some scheme dependence

Polyakov loop correlator



Use the **Polyakov loop correlator**

$$C_P(\tau, \boldsymbol{r}) = \frac{1}{9}\mathrm{Tr}\, W(\tau, 0)\mathrm{Tr}\, W^\dagger(\tau, \boldsymbol{r})$$

to study color **screening radii**.

(Color averaged) Free energy:

$$F^b_{Q\bar{Q}}(\boldsymbol{r}, T) = -T \log C_P(N_\tau, \boldsymbol{r}, T).$$

Asymptotic behavior for large r:

$$\lim_{r\to\infty} F_{Q\bar{Q}}(r, T) = F_\infty(T) = 2F_Q(T)$$

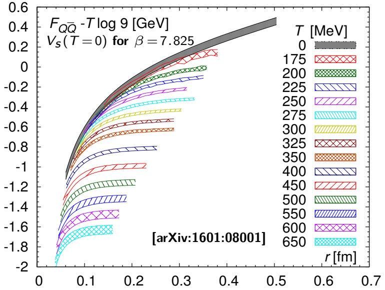
$\Leftrightarrow$ Q and $\bar{Q}$ are fully screened.

Singlet-octet decomposition of $F_{Q\bar{Q}}$:

$$e^{-\frac{F_{Q\bar{Q}}}{T}} = \frac{1}{9}e^{-\frac{F_S}{T}} + \frac{8}{9}e^{-\frac{F_O}{T}}$$

$F_{Q\bar{Q}}$, F_S, F_O share same asymptotic behavior $(r \gg 1/T, 1/gT, 1/g^2T)$.

Color-averaged free energy $F_{Q\bar{Q}}$



Hybrid potentials are repulsive $\Leftrightarrow$ $V_o \gg V_s$ and $F_O \gg 0$ for small r

Short distances dominated by F_S:

$$F_{Q\bar{Q}} \lesssim F_S + T\log 9 + \dots$$

Octet contribution to $F_{Q\bar{Q}}$ negative for small r ($V_o > 0$).

EQCD: $F_{Q\bar{Q}}$ is screened with **Debye mass** for $gT \gg r^{-1} \gg g^2T$ as

$$F_{Q\bar{Q}}(r,T) \sim e^{-2m_Dr}/r^2$$

or for $g^2T \ll r^{-1}$ as

$$F_{Q\bar{Q}}(r,T) \sim e^{-(2m_D+\#_1 g^2T)r}/r.$$

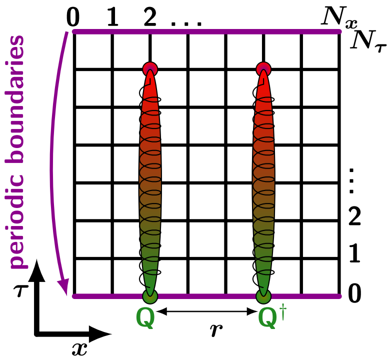
Extract **Debye mass** for large r?

STN ratio falls too fast for reliable determination of the Debye mass.

Smeared Polyakov loops, *plane-averaged* Polyakov loops, combination of both?

$\Rightarrow$ remains **work in progress**. . .

Singlet free energy F_S (I)



Color singlet Wilson line correlator

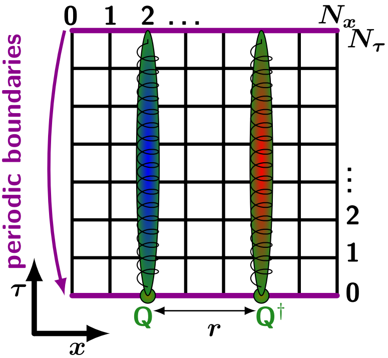
$$C_S(\tau,\boldsymbol{r}) = \frac{1}{3}\left\langle \mathrm{Tr}\, W(\tau,0) W^\dagger(\tau,\boldsymbol{r})\right\rangle$$

Coulomb gauge fixing is necessary.

Static energy: $\tau < N_\tau$, s small

$$V^b(\tau,\boldsymbol{r},T) = -\frac{1}{as}\log\frac{C_S(\tau,\boldsymbol{r},T)}{C_S(\tau-s,\boldsymbol{r},T)}$$

Singlet free energy F_S (I)



Color singlet Wilson line correlator

$$C_S(\tau, \boldsymbol{r}) = \frac{1}{3}\left\langle \mathrm{Tr}\, W(\tau, 0) W^\dagger(\tau, \boldsymbol{r}) \right\rangle$$

Coulomb gauge fixing is necessary.

Singlet Free energy: $\tau = s = N_\tau$

$$F_S^b(\boldsymbol{r}, T) = -\frac{1}{aN_\tau} \log \frac{C_S(N_\tau, \boldsymbol{r}, T)}{C_S(0, \boldsymbol{r}, T)}$$

w. $C_S(0, \boldsymbol{r}, T) = 1$ and $aN_\tau = T^{-1}$.

Asymptotic freedom: vacuum-like regime in F_S and V_s for small $\boldsymbol{r}$

$\Rightarrow$ (p)NRQCD relates F_S and V_s:

$$F_S(\boldsymbol{r}, T) = V_s(\boldsymbol{r}, T) + \mathcal{O}(\alpha_s^2\, rT)$$

F_S as estimate for Re V_s at $T > 0$.

Color screening: screening regime in both F_S and V_s for large $\boldsymbol{r}$.

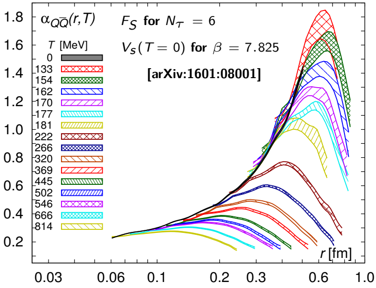
$\Rightarrow$ Quantitative description of the change between regimes (in r).

Effective coupling $\alpha_{Q\bar{Q}}$

Effective coupling constant $\alpha_{Q\bar{Q}}$

$$\alpha_{Q\bar{Q}}(r,T) = \frac{r^2}{C_F}\frac{\partial E(r,T)}{\partial r}$$

w. $E = \{F_S, V_s(T=0), V_s(T>0)\}$



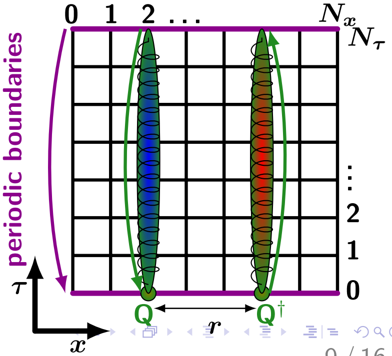
r^2 rise of $\alpha_{Q\bar{Q}}(r,T) \Leftrightarrow$ vacuum-like

e^{-mr} fall of $\alpha_{Q\bar{Q}}(r,T) \Leftrightarrow$ screening

No discontinuous change at the QCD crossover ($T_c(6) \sim 170$ MeV)

Maximum of $\alpha_{Q\bar{Q}}$: QGP @ LHC is **strongly-coupled** for $T \lesssim 300$ MeV

Fall of $\alpha_{Q\bar{Q}}$ indicates string breaking (formation of **static D mesons**)



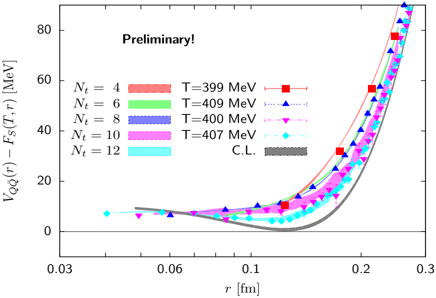
Small distance asymptotic behavior: $V_s - F_S$

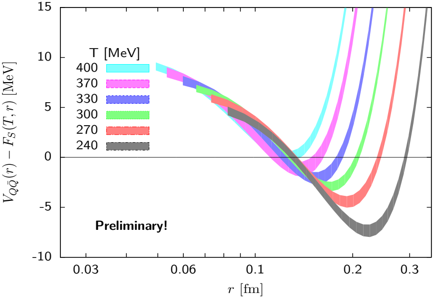
pNRQCD: $F_S = V_s + \mathcal{O}[\alpha_s^2(rt)]$:

$$V_s(r) - F_S(r)$$

is a *renormalization invariant* quantity and *weakly-coupled* for small r.

N.b. $V_s(r, T = 0)$ and $V_s(r, T > 0)$ are both admissible.

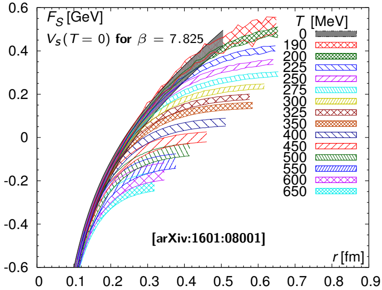




$V_s - F_S$ for $rT \lesssim 1/6$ has only very few underlying data points.

Control systematic uncertainty of the continuum extrapolation by adding finer lattices ($N_\tau = 16, 24$).

Singlet free energy F_S (II)



small r: $F_S \sim V_s$ within errors

$r \lesssim 1/(2T)$: $dF_S/dr > dV_s/dr$

$r \gtrsim 1/(2T)$: $dF_S/dr < dV_s/dr$

$r \gtrsim 1/T$: $dF_S/dr \sim 0$

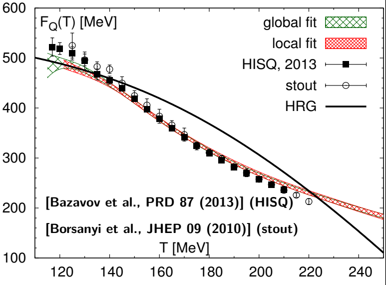
The errors are dominated by the renormalization constant $2C_Q$: $\Delta C_Q \sim 15\,\mathrm{MeV}$ for $\beta = 7.825$.

Asymptotic behavior for large r:

$$\lim_{r\to\infty} F_S(r,T) = F_\infty(T) = 2F_Q(T)$$

$\Leftrightarrow$ Q and $\bar{Q}$ are fully screened.

[arXiv:1603.06637 (to appear in PRD)]



Free energy of a single quark F_Q.

Large distance asymptotic behavior: single quark entropy S_Q for low T

Bare **free energy** F_Q^b also defined in terms of (bare) **Polyakov loop** $\langle L \rangle^b$

$$f_Q^b(T) = F_Q^b(T)/T = -\log \langle L \rangle^b$$

renormalize $f_Q = f_Q^b + N_\tau c_Q$.

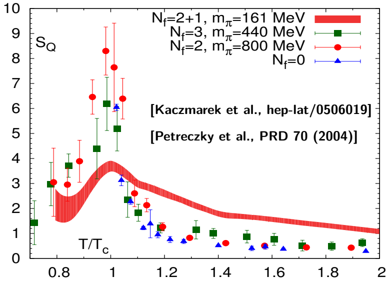
$aC_Q = c_Q = c + ba + \mathcal{O}(a^3) \Rightarrow$ **scheme dependence** of $\langle L \rangle$ and F_Q.

The **entropy** S_Q is given through a temperature derivative of F_Q

$$S_Q(T) = -\frac{dF_Q(T)}{dT}$$

N.b. the pressure vanishes: $p_Q = 0$.

S_Q: no scheme dependence left after T derivative & continuum limit.



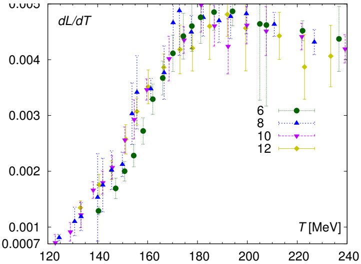
S_Q peaks in the **crossover region**:

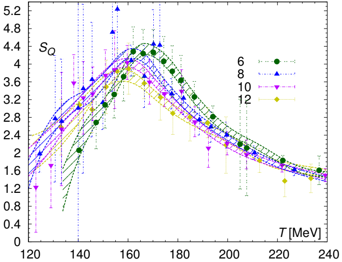
$$T_S = 153^{+6.5}_{-5}\ \text{MeV} \qquad \left[\frac{\partial S_Q}{\partial T}\Big|_{T_S} = 0 \right].$$

S_Q indicates whether the lightest dof is a D meson or a single quark.

Inflection point of the Polyakov loop

Inflection point of F_Q (via S_Q) at $T_S \sim 148\text{-}160\,\mathrm{MeV}$ **[Our result (TUMQCD)]**
Inflection point of Polyakov loop at $T_L \sim 170\text{-}175\,\mathrm{MeV}$ **[Aoki et al., PLB 643 (2006)]**

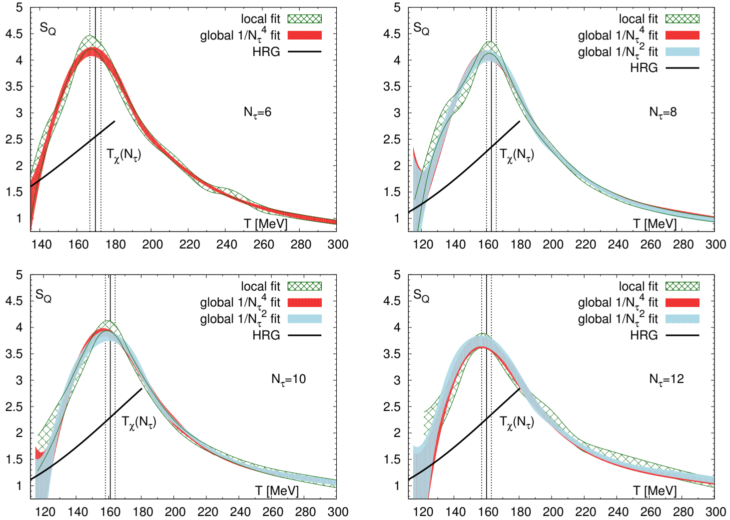




$$0 \stackrel{!}{=} \frac{1}{L^{\mathrm{ren}}} \frac{\partial^2 L^{\mathrm{ren}}}{\partial T^2}\Bigg|_{T_L} = \frac{1}{T}\left(\underbrace{\frac{1}{T}\frac{(F_Q-2T)F_Q}{T^2}}_{\stackrel{!}{=}\mathcal{O}(1)} + \underbrace{\frac{S_Q^2-2S_Q}{T} + \left(\frac{\partial S_Q}{\partial T}\right)}_{\text{scheme independent, } \mathcal{O}(\frac{1}{T})} \right)\Bigg|_{T_L}$$

T_L is scheme dependent, T_S is scheme-independent.

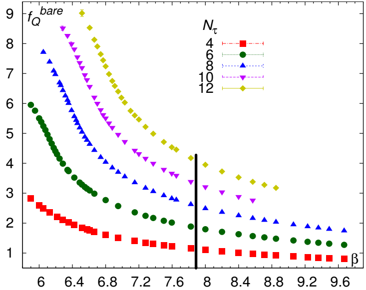
Relation to chiral crossover



T_χ is from $O(2)$ scaling fits to chiral susceptibilities **[Bazavov et al., PRD 85 (2012)]**

Large distance asymptotic behavior: single quark entropy S_Q for high T

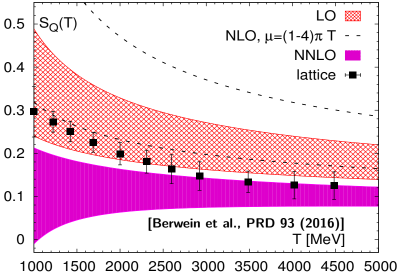
Renormalization constant C_Q from $T = 0$: only available for $\beta \lesssim 7.825$



Direct renormalization scheme

[Gupta et al., PRD 77 (2008)]

$$c_Q(\beta) = \frac{1}{N_\tau}\left\{N_\tau^{\mathrm{ref}} c_Q(\beta^{\mathrm{ref}}) + \Delta_{N_\tau, N_\tau^{\mathrm{ref}}} + f_Q^{\mathrm{b}}(\beta^{\mathrm{ref}}, N_\tau^{\mathrm{ref}}) - f_Q^{\mathrm{b}}(\beta, N_\tau)\right\}$$



nnlo @ pNRQCD vs LGT @ $N_\tau = 4$ consistency reached for $T \sim 3\,\mathrm{GeV}$

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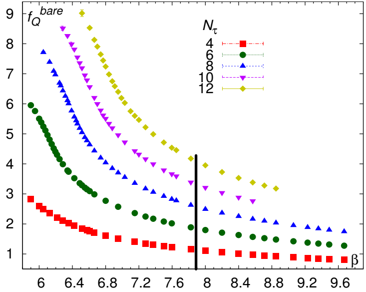
Renormalization constant C_Q from $T = 0$: only available for $\beta \lesssim 7.825$



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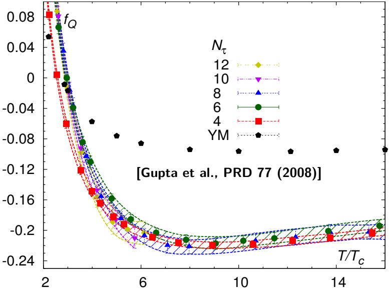


nnlo @ pNRQCD vs LGT @ $N_\tau = 4$ consistency reached for $T \sim 3\,\mathrm{GeV}$

2+1 flavor contribution fo free energy for $T > 4\,T_c \sim 60\%$

$\Rightarrow$ consider **charm** contribution?

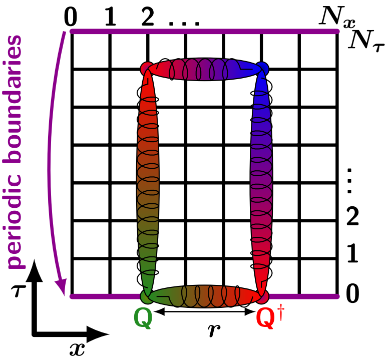
Summary

Free energy at $T > 0$:

- Polyakov loop correlator, multiplicatively renormalizable
  $\Rightarrow$ obtain screening radius and Debye mass (EQCD) (work in progress)
- Color singlet Wilson line correlator in Coulomb gauge:
  $\Rightarrow$ vacuum-like regime, $r \ll T$: $V_s - F_S$, onset of screening (pNRQCD)
  $\Rightarrow$ screening regime, $rm_D \gtrsim 1$: electric screening mass, $F_S \sim e^{-m_E r}/r$
- Polyakov loop, determines asymptotics of any $Q\bar{Q}$ free energies
  $\Rightarrow$ single quark entropy S_Q, deconfinement at $T_S = 153^{+6.5}_{-5}$ MeV,
  $\Rightarrow$ T_S is scheme independent and consistent with chiral susceptibilities
  $\Rightarrow$ high T: weak-coupling for $T \sim 3\,\text{GeV}$, quark contribution: $\sim 60\%$

Thank You for Your attention!

Static energy at $T = 0$



Effective potential for large τ, e.g.

$$V^b(\tau, \boldsymbol{r}) = -\frac{1}{s} \log \frac{W_S(\tau, \boldsymbol{r})}{W_S(\tau - s, \boldsymbol{r})}$$

Renormalization: $V = V^b + 2C_Q$

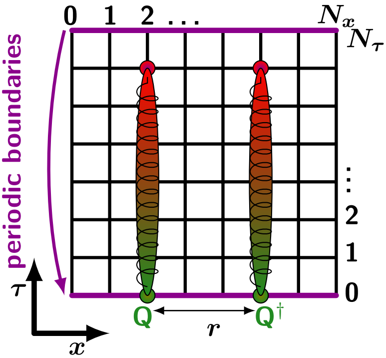
Cusp and linear (in $\boldsymbol{r}$) *divergences cancel in the ratio*, but very large statistical samples are required.

Cyclic Wilson loop

$$W_S(\tau, \boldsymbol{r}) = \left\langle \square (\tau, \boldsymbol{r}) \right\rangle = \frac{1}{3} \left\langle \mathrm{Tr} \left[W_\tau(\tau, 0) W_\sigma(\tau, \boldsymbol{r}) W_\tau{}^\dagger(\tau, \boldsymbol{r}) W_\sigma{}^\dagger(0, \boldsymbol{r}) \right] \right\rangle$$

Divergent quantity: cusp divergences, linear divergences in r, more cusp divergences introduced by corners of $W_\sigma^{(\dagger)}(., \boldsymbol{r})$ for off-axis correlations

Static energy at $T = 0$



Color singlet Wilson line correlator

$$C_S(\tau, \boldsymbol{r}) = \frac{1}{3} \left\langle \mathrm{Tr}\, W(\tau, 0) W^\dagger(\tau, \boldsymbol{r}) \right\rangle$$

Coulomb gauge fixing is necessary.

Effective potential for large τ, e.g.

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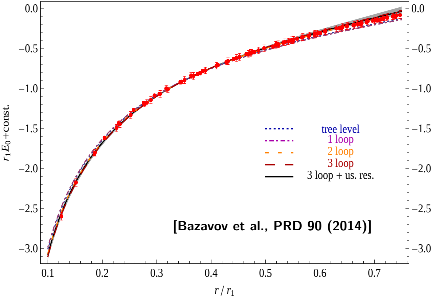
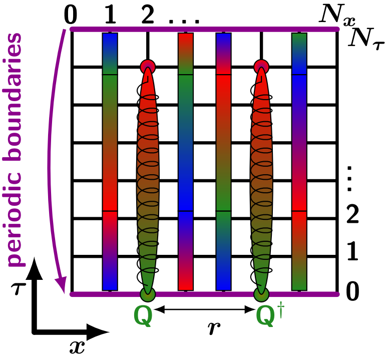


Consistency of EFT and Lattice ✓

Static energy at $T > 0$



Color singlet Wilson line correlator

$$C_S(\tau, \boldsymbol{r}) = \frac{1}{3} \sum_{a=1}^{3} W_a(\tau, 0) W_a^\dagger(\tau, \boldsymbol{r})$$

Coulomb gauge fixing is necessary.

$N_x \gg N_\tau \geq \tau$: is τ **large enough?**

Im $V \neq 0$ due to a **thermal width**.

Extract Im V from real correlator?

EFT definition of **static potential**:

$$V_s^b(r, T) = \lim_{\tau \to \infty} \left\langle \boxed{} (\tau, \boldsymbol{r}, T) \right\rangle$$

Limit $\tau \to \infty$ not possible in LGT.

Construct **spectral function** ρ from the imaginary-time correlator

$$C_s(\tau, r, T) = \int_0^\infty d\omega \; e^{-\omega\tau} \rho(\omega, r, T)$$

Obtain the real-time correlator w. Fourier transform of ρ, extract V.

Spectral function in thermal medium

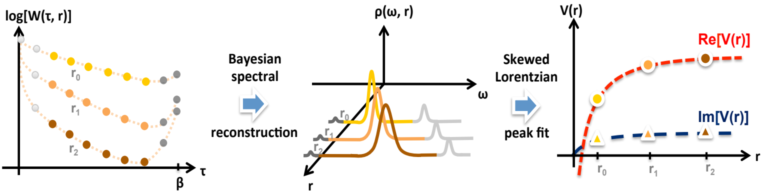
Lowest peak for Wilson loop (Wilson line correlator) is a **skewed Lorentzian**

$$\rho(\omega, r) \propto \frac{[\mathrm{Im}\, V(r)] \cos[\mathrm{Re}\, \sigma_\infty(r)] - (\mathrm{Re}\, V(r) - \omega) \sin[\mathrm{Re}\, \sigma_\infty(r)]}{[\mathrm{Im}\, V(r)]^2 + (\mathrm{Re}\, V(r) - \omega)^2}$$
$$+ c_0(r) + c_1(r)\,(\mathrm{Re}\, V(r) - \omega) + \ldots$$

[Burnier et al., PRD 86 (2012)]

HTL perturbation theory or pNRQCD produce $Q\bar{Q}$ spectra of this type

[Laine et al., JHEP 0703 (2008); Brambilla et al., PRD 78 (2008)]



Reconstruction of the real-time potential

Laplace transform: $$C_s(\tau, r, T) = \int_0^\infty d\omega \; e^{-\omega\tau} \rho(\omega, r, T)$$

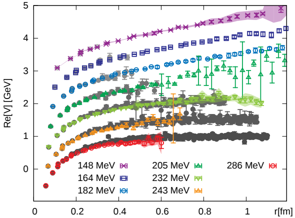
$\mathcal{O}(10)$ data: $C_S(\tau, r, T)$ or $W_S(\tau, r, T)$ $\Rightarrow$ $\rho(\omega, r, T)$ for $\mathcal{O}(10^3)$ needed

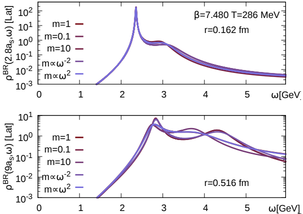
Severely ill-posed problem, must make use of **any additional knowledge**

Bayesian inference of spectral function using data (D) and information (I)

$$P[\rho|D, I]) = \frac{P[D|\rho, I] P[\rho, I]}{P[D|I]}$$

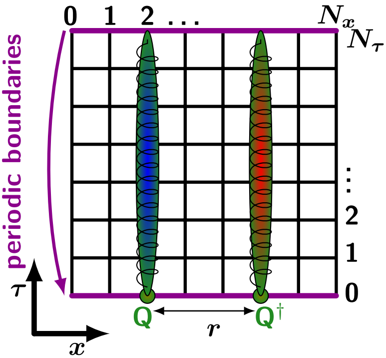
Encode **prior information** in information entropy S and minimize $\chi^2 + \alpha S$





[Burnier et al., arXiv:1411:3141, Asqtad, 2+1 flavors]

Wilson loops and Wilson line correlators



Color singlet Wilson line correlator

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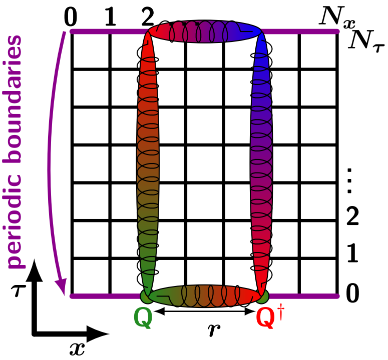
Coulomb gauge fixing is necessary.

Singlet Free energy:

$$F_S^b(\boldsymbol{r}, T) = -T \log C_S(N_\tau, \boldsymbol{r}, T)$$

Wilson loops and Wilson line correlators



Singlet Free energy:

$$F^b_{S\,W}(\boldsymbol{r}, T) = -T \log W_S(N_\tau, \boldsymbol{r}, T)$$

No ratio ⇒ divergences persist.

Cyclic Wilson loop

$$W_S(N_\tau, \boldsymbol{r}) = \square(N_\tau, \boldsymbol{r})$$

Gauge invariant . . .