# QCD Thermodynamics

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What is deconfinement in QCD ? What is the nature of the deconfined matter ?

Tools: screening of color charges, EoS, fluctuation of conserved quantum numbers

QGP: state of strongly interacting matter for
weakly interacting gas of quark and gluons ? $T \gg \Lambda_{QCD}, g \ll 1$

$$2\pi T \gg m_D \sim gT \gg g^2 T$$

EFT approach: EQCD

Magnetic screening scale:
non-perturbative

Perturbative series is an expansion is in $g$ and not $\alpha_s$
Loop expansion breaks down at some order (weak coupling may still work)

Problem : $g(\mu = 10^{16}\mathrm{GeV}) \simeq 1/2$

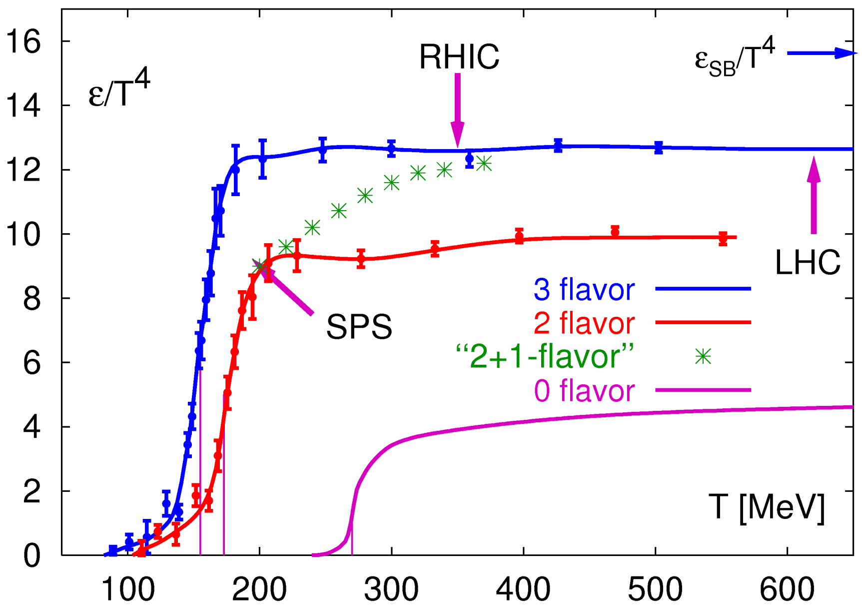
$$g(\mu = 10^2\mathrm{GeV}) = \sqrt{4\pi\alpha_s(\mu = 10^2\mathrm{GeV})} \simeq 1$$

Lattice QCD

Symposium on LGT and EFT, IAS, TUM, Garching, May 18-21, 2016

# Lattice QCD at T>0 now and then

Lattice QCD calculations at T>0 around 2002:




$T_c \simeq 173\text{MeV}$

for both chiral transition and deconfinement transition ( in terms of Polyakov loop)

Problems:

$N_\tau = 4 : a \equiv 1/(N_\tau a) = 1/(4T)$

$m_\pi = (500 - 800)\text{MeV}$

Continuum limit and physical masses are needed

$N_\tau \to \infty$

$m_\pi = 140\text{MeV}$

$costs \sim N_\tau^{11}$

$\sim 1/m_\pi^3$

This task can be accomplished using improved staggered fermions actions:
Highly Improved Staggered Quark (HISQ) or Stout action
Some calculations are performed using chiral (Domain Wall) fermions

Fluctuations of conserved charges: new look into deconfinement and QGP properties

# The temperature dependence of chiral condensate

Renormalized chiral condensate introduced by Budapest-Wuppertal collaboration

$$\langle\bar\psi\psi\rangle_q \Rightarrow \Delta_q^R(T) = m_s r_1^4 \left(\langle\bar\psi\psi\rangle_{q,T} - \langle\bar\psi\psi\rangle_{q,T=0}\right) + d, \quad q = l, s$$

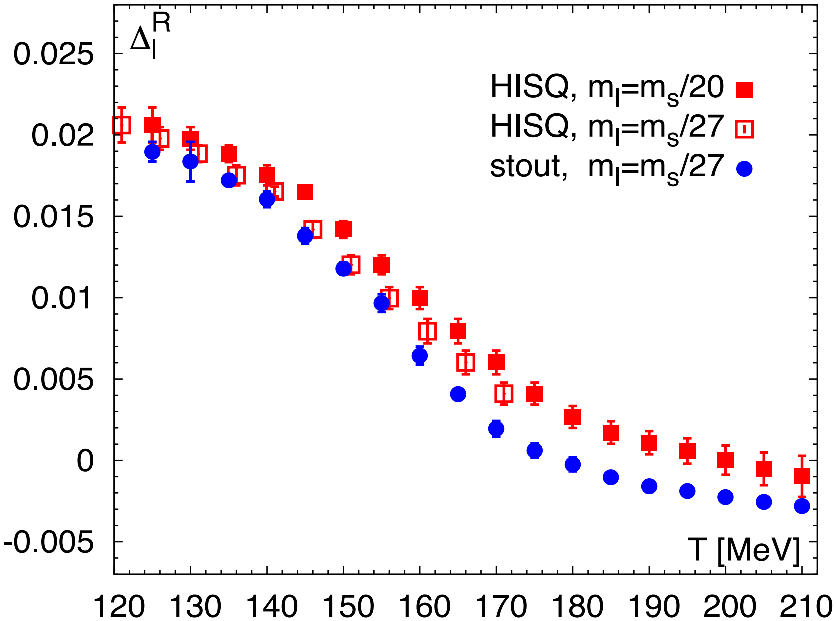
With choice : $d = \langle\bar\psi\psi\rangle_{m_q=0}^{T=0}$

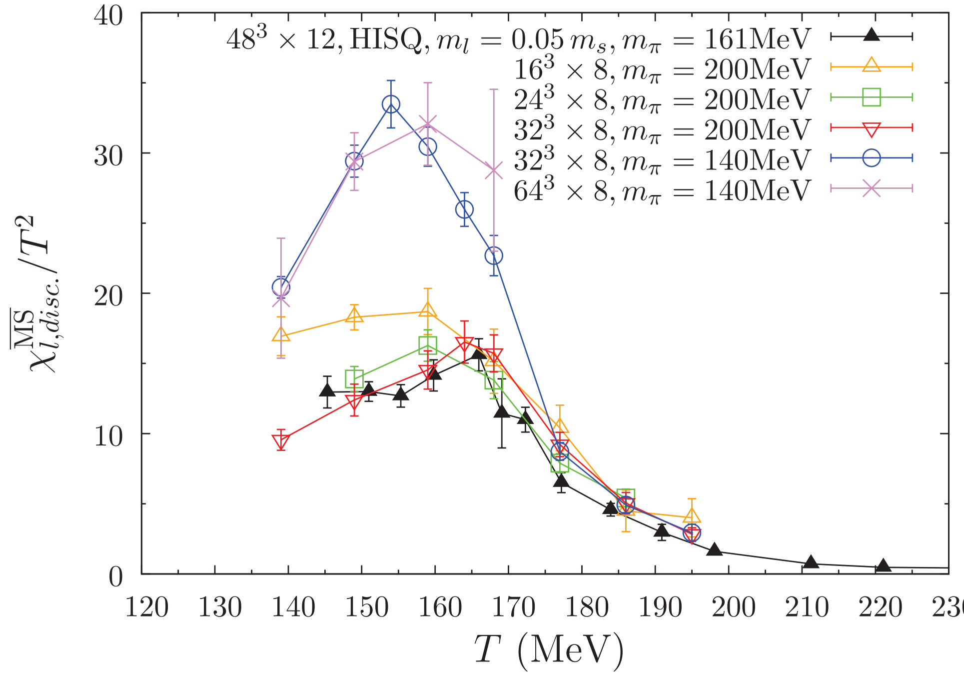
Bazavov et al (HotQCD), Phys. Rev. D85 (2012) 054503;
Bazavov et al, PRD 87(2013)094505,
Borsányi et al, JHEP 1009 (2010) 073

Calculations with chiral (Domain Wall) fermions:

$$\chi_{disc} = \frac{V}{T}\left(\langle(\bar\psi\psi)^2\rangle - \langle\bar\psi\psi\rangle^2\right)$$

Bhattacharya et al (HotQCD), PRL 113 (2014)082001




- O(4) scaling analysis and continuum limit:

$T_c = (154 \pm 8 \pm 1(scale))$MeV

$T_c = (155 \pm 8 \pm 1)$MeV

# $U_A(1)$ symmetry restoration ?

1[st] order chiral transition in QCD if $U_A(1)$ is restored at $T_c$

$$\chi_i = \int d^4x G_i(x)$$

$$\begin{array}{ccccc}
\chi_{5,con} & \pi:\ \bar{q}\gamma_5\frac{\tau}{2}q & \xleftrightarrow{SU(2)_L \times SU(2)_R} & \sigma:\ \bar{q}q & \chi_{con}+\chi_{disc} \\
& \Big\updownarrow U(1)_A & & \Big\updownarrow U(1)_A & \\
\chi_{con} & \delta:\ \bar{q}\frac{\tau}{2}q & \xleftrightarrow[SU(2)_L \times SU(2)_R]{} & \eta':\ \bar{q}\gamma_5 q & \chi_{5,con}-\chi_{5,disc}
\end{array}$$

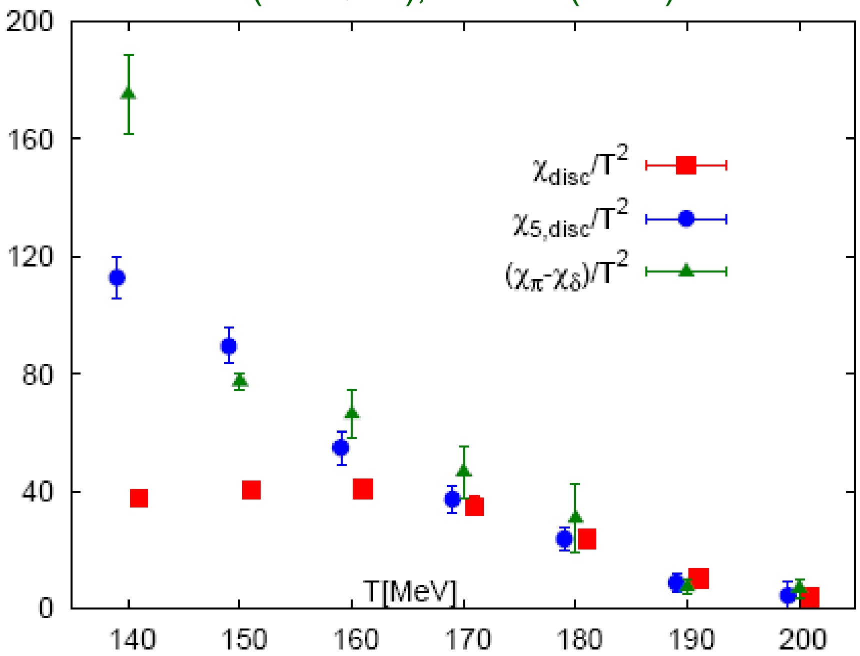
chiral: $\chi_{disc} = \chi_{5,disc}$

axial: $\chi_{disc} = -\chi_{5,disc}$

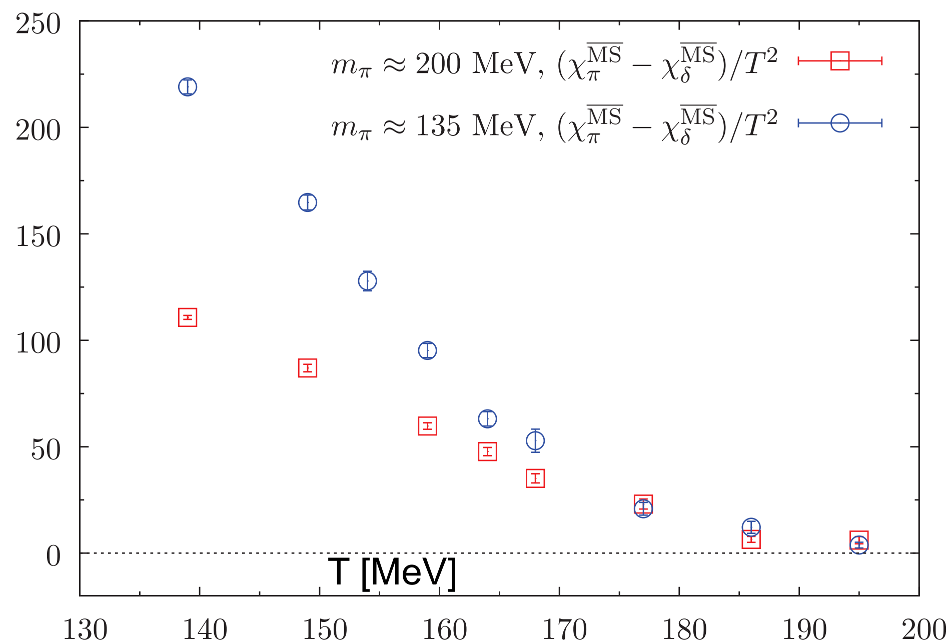
$\chi_{disc} = 0$

Relation to topology : $\chi_t = \left\langle \left(m_l \mathrm{Tr} M_l^{-1}\gamma^5\right)^2 \right\rangle / V = (m_l)^2 \chi_{\mathrm{disc},5}$

Bazavov et al (HotQCD), PRD86 (2012) 094503




Bhattacharya et al (HotQCD) PRL113 (2014) 082001




different measures of $U_A(1)$ breaking agree for $T>T_c$

axial symmetry broken $T=200$ MeV !

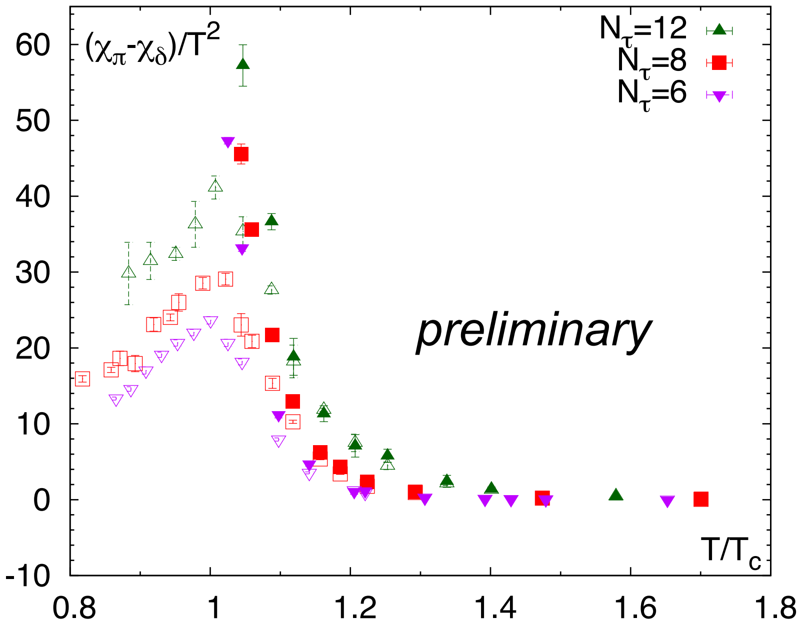
# Instanton gas at work ?

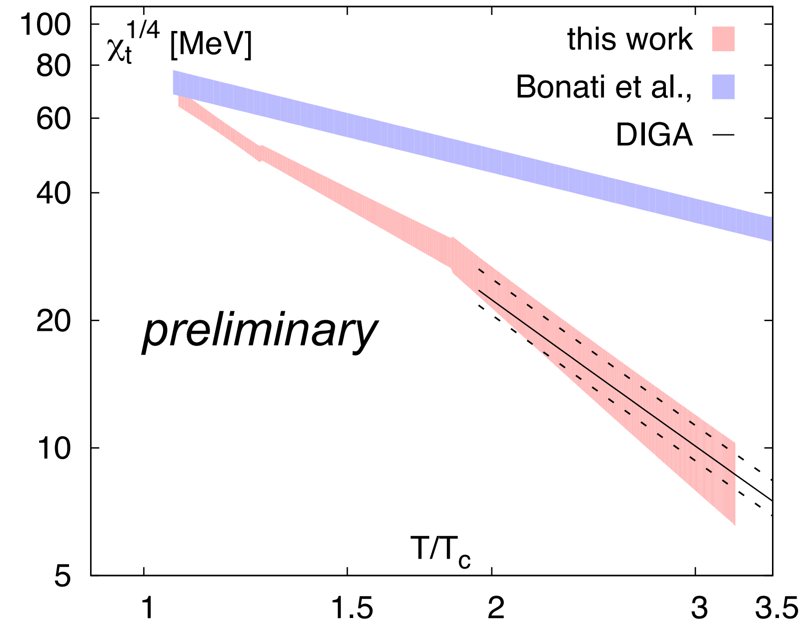
The amount of $U_A(1)$ breaking at high $T$ is reduce because of the reduced instanton density => dilute instanton gas approximation (DIGA), Gross et al, RMP 53 (1981) 43

Topological susceptibility with HISQ action using Symanzik flow

$$\chi_{top} = \frac{1}{V}\left(\langle Q^2\rangle - \langle Q\rangle^2\right) \qquad \chi_{top}/m_l^2 = \chi_{\mathrm{disc},5} \simeq \chi_{\mathrm{disc}}$$

Schadler, Sharma, PP




DIGA is not incompatible with the lattice results a K factor ~1.79 is included
Similar K factor was found for SU(3) gauge theory, Borsányi et al, PLB 752 (2016) 175

$U_A(1)$ is broken at any high temperatures due static chromo-magnetic fields
Laine, Vepsalainen, 2003; application to axion phenomenology ? See talk by Zoltan Fodor

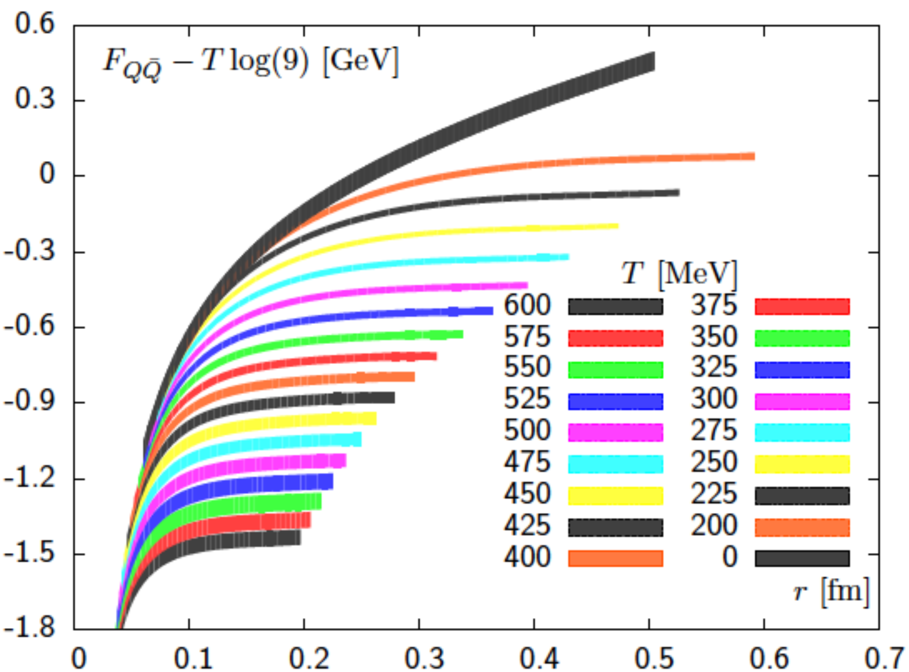
# Deconfinement and color screening

Onset of color screening is described by Polyakov loop (order parameter in SU(N) gauge theory)

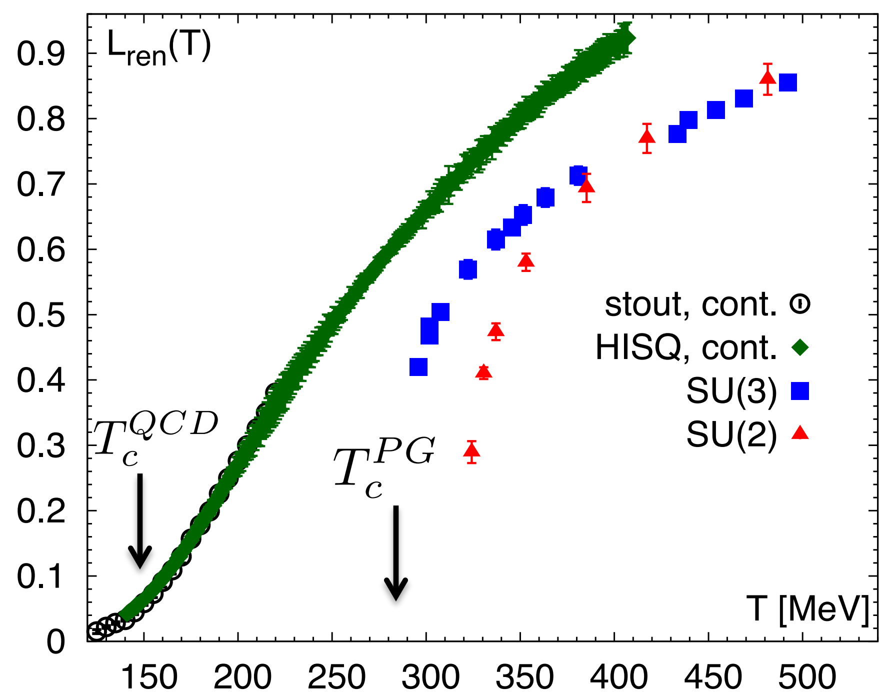
$$L = \mathrm{tr}\mathcal{P}e^{ig\int_0^{1/T} d\tau A_0(\tau,\vec{x})} \qquad \exp(-F_{Q\bar{Q}}(r,T)/T) = \frac{1}{9}\langle \mathrm{tr}L(r)\mathrm{tr}L^\dagger(0)\rangle$$

$$F_{Q\bar{Q}}(r\to\infty,T) = 2F_Q(T) \quad \Rightarrow \quad L_{ren} = \exp(-F_Q(T)/T)$$

2+1 flavor QCD, continuum extrapolated (see talk by Johannes Weber)




free energy of static quark anti-quark pair shows Debye screening at high temperatures




SU(N) gauge theory ≠ QCD !

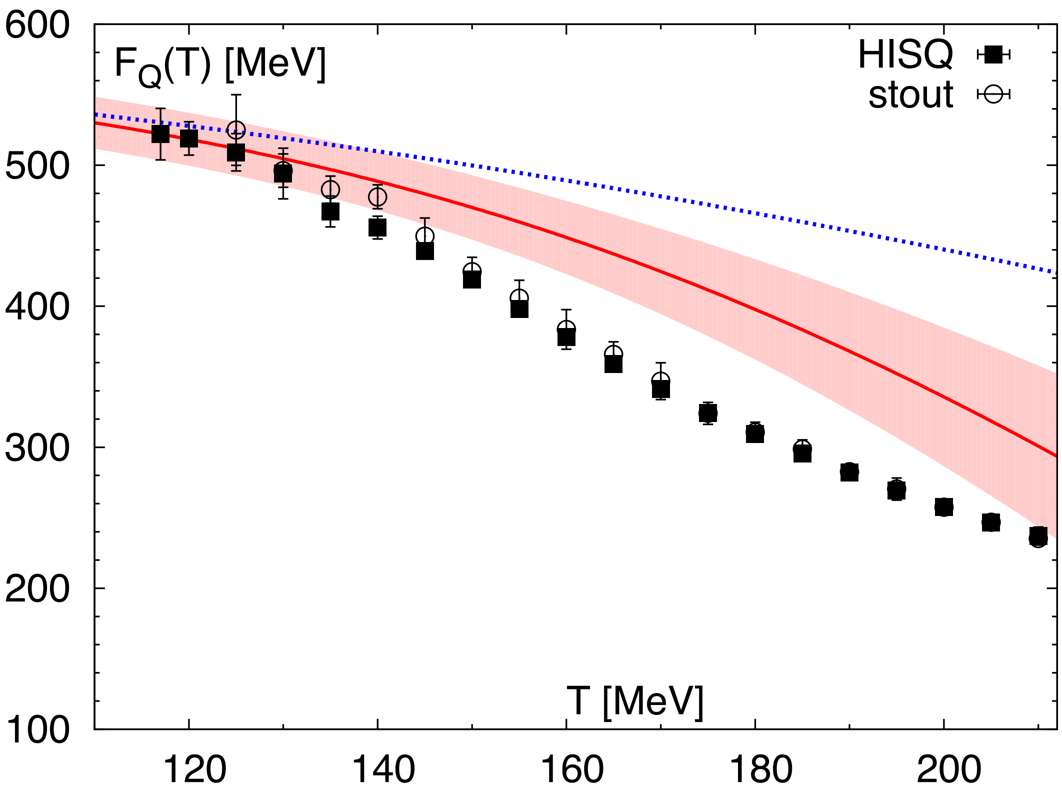
Similar results with stout action Borsanyi et al, JHEP04(2015) 138

# Polyakov and gas of static-light hadrons

$$Z_{Q\bar{Q}}(T)/Z(T) = \sum_n \exp(-E_n^{Q\bar{Q}}(r \to \infty)/T)$$

Energies of static-light mesons: $E_n^{Q\bar{Q}}(r \to \infty) = M_n - m_Q$

Free energy of an isolated static quark: $F_Q(T) = -\frac{1}{2}(T \ln Z_{Q\bar{Q}}(T) - T \ln Z(T))$




Megias, Arriola, Salcedo,
PRL 109 (12) 151601

Bazavov, PP, PRD 87 (2013) 094505

Ground state and first excited states are from lattice QCD
Michael, Shindler, Wagner,
arXiv1004.4235
Wagner, Wiese,
JHEP 1107 016,2011

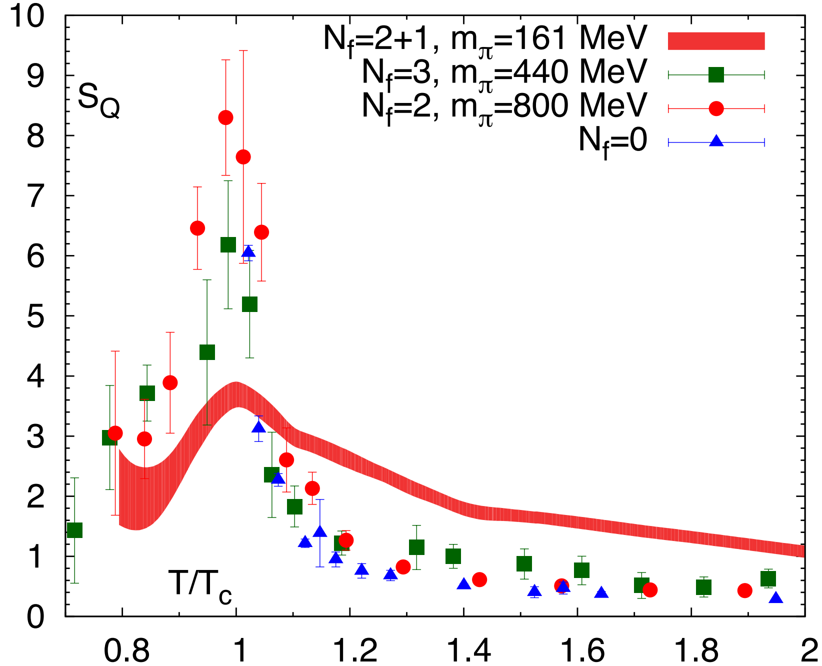
Higher excited state energies are estimated from potential model

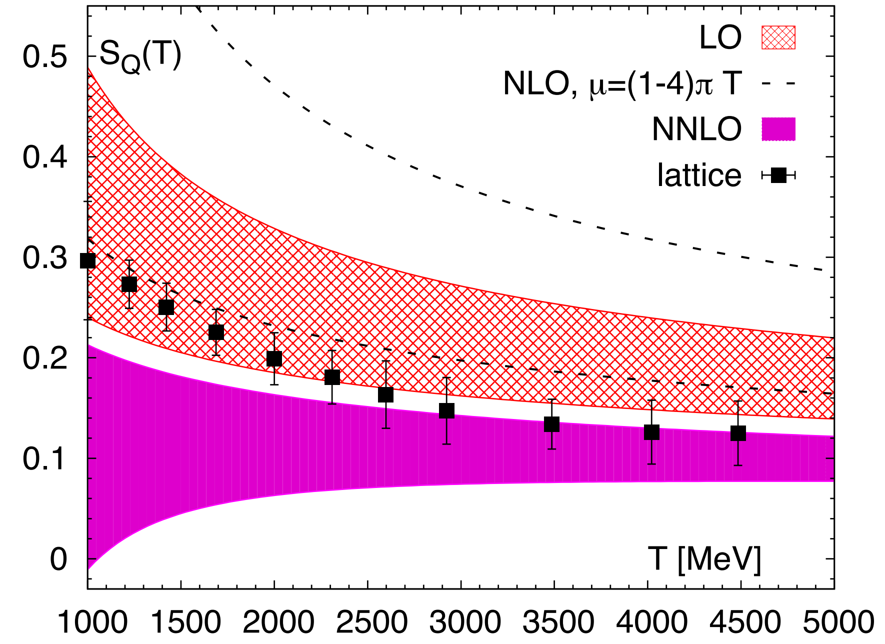
Gas of static-light mesons only works for *T < 145* MeV

# The entropy of static quark

TUMQCD, arXiv:1606.06637

$$S_Q = -\frac{\partial F_Q}{\partial T}$$




At low $T$ the entropy $S_Q$ increases reflecting the increase of states the heavy quark can be coupled to; at high temperature the static quark only "sees" the medium within a Debye radius, as $T$ increases the Debye radius decreases and $S_Q$ also decreases

The onset of screening corresponds to peak is $S_Q$ and its position coincides with $T_c$

Weak coupling (EQCD) calculations work only or $T > 1500$ MeV

# Casimir scaling of the Polyakov loop

Instead of fundamental representations consider Polyakov loop $P_n$ in arbitrary representation n

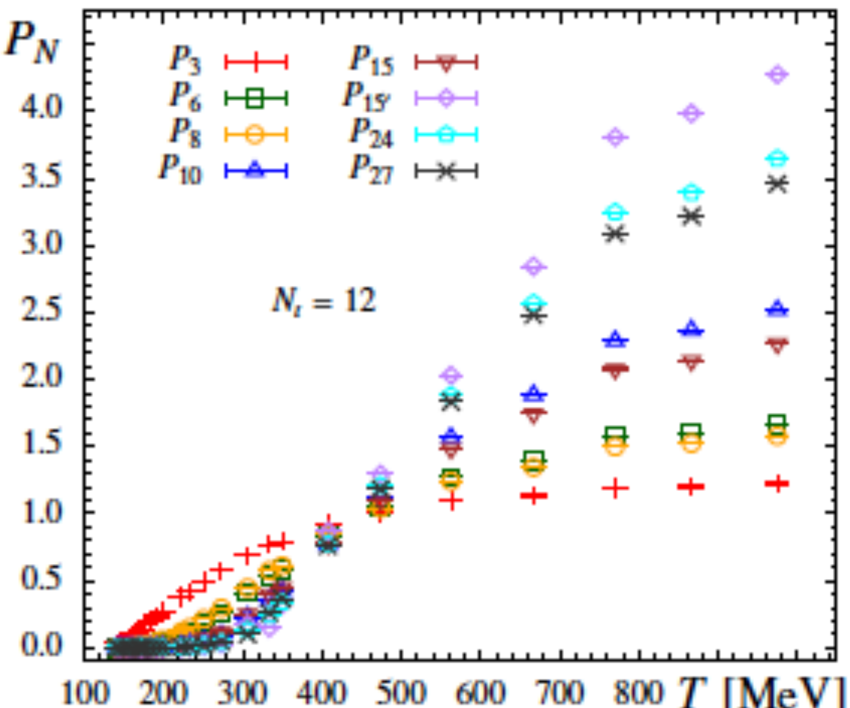
PP, Schadler, PRD92 (2015) 094517

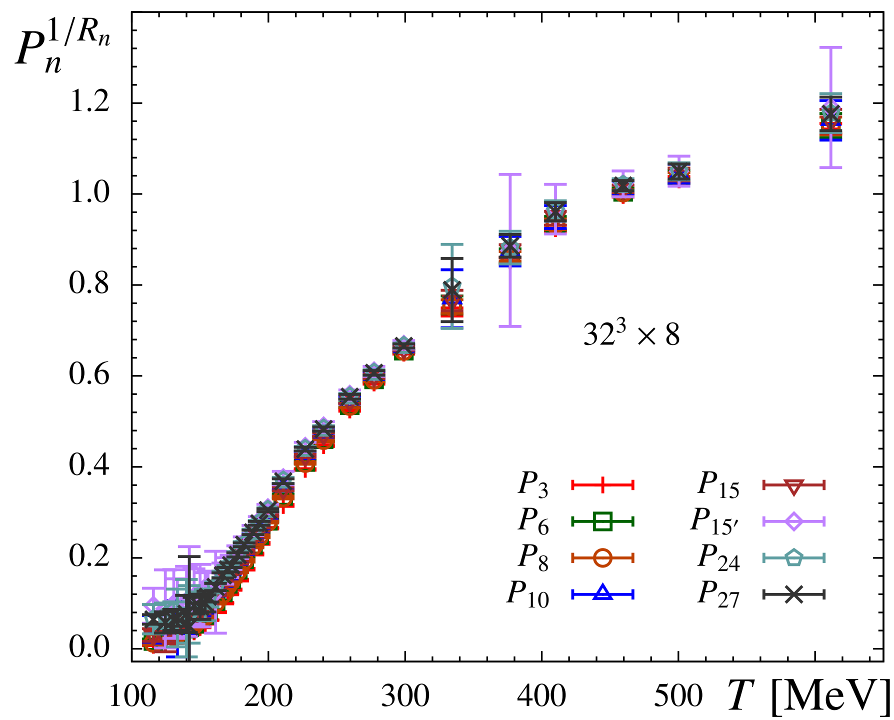
$$P_3 = L_{ren}$$

Use symanzik flow to renormalized the Polyakov loop and reduce the noise
Fodor et al, JHEP 1409 (2014) 018

Casimir scaling: free energy is proportional to qudratic
Casimir operator $C_n$ of rep *n*

$$R_n = C_n / C_3$$

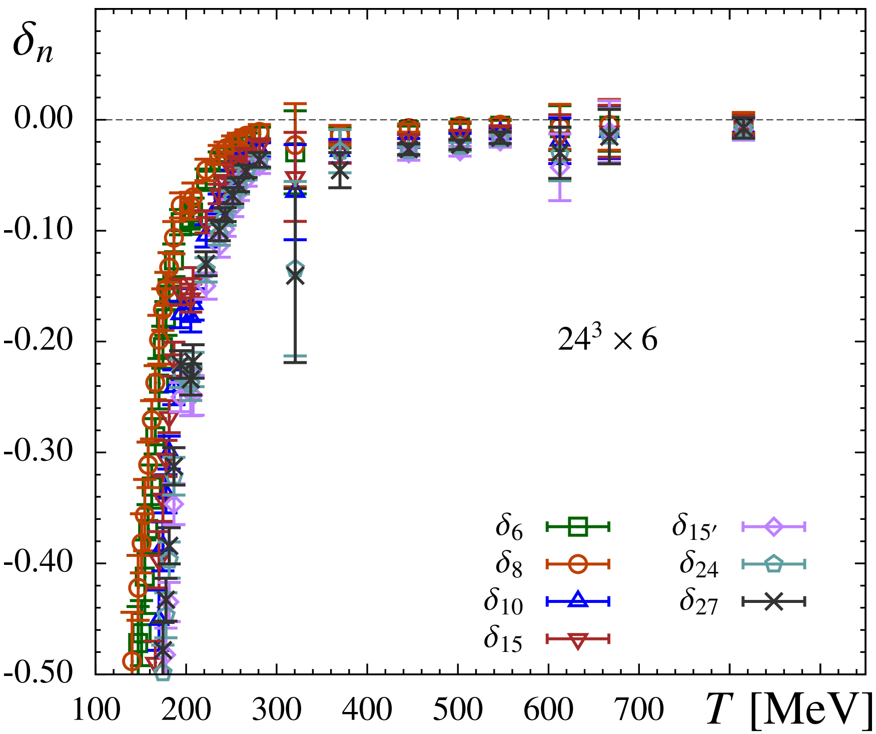


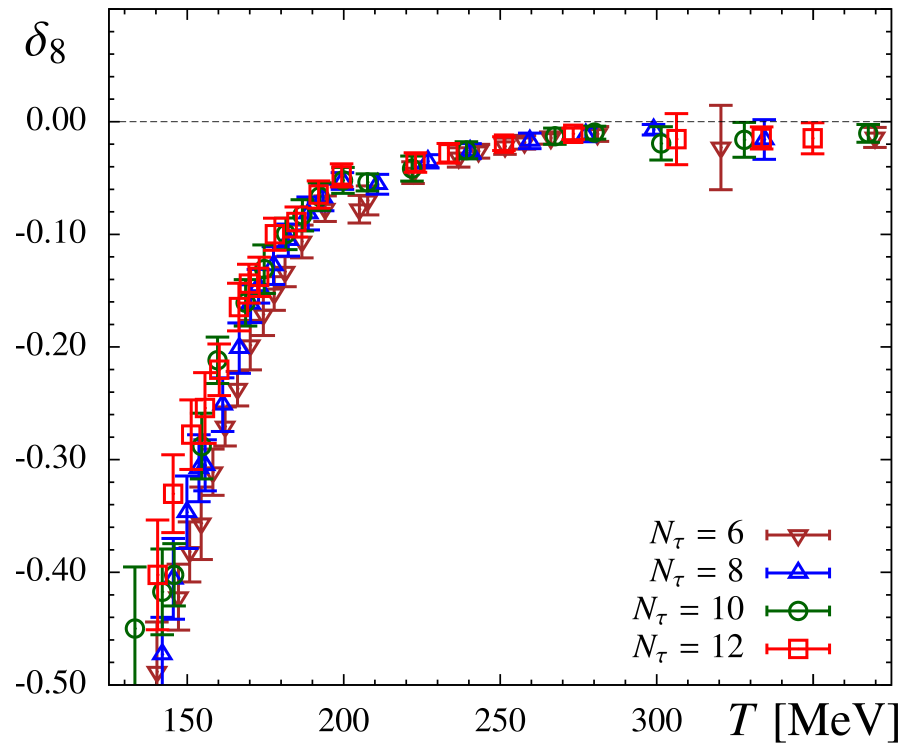




Expected in weak coupling expansion: e.g. at LO $F_Q^n = -C_n \alpha_s m_D$

# Casimir scaling of the Polyakov loop (con't)

$$\delta_n = 1 - P_n^{1/R_n}/P_3$$




Casimir scaling holds for *T>300* MeV color screening like in weakly coupled QGP ?

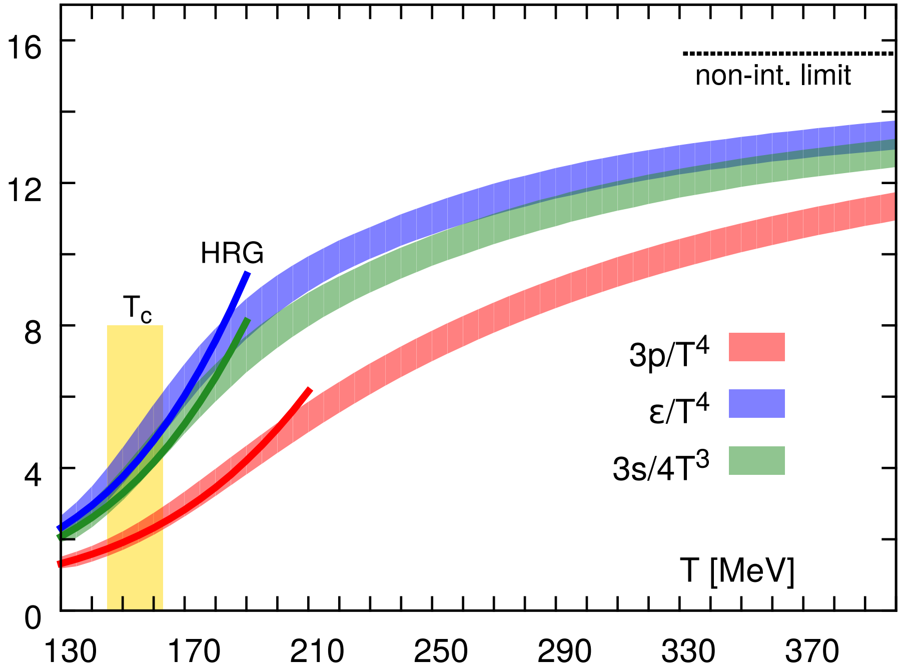
Breaking of Casimir scaling first appear at order $\alpha_s^4$ in the weak coupling expansion

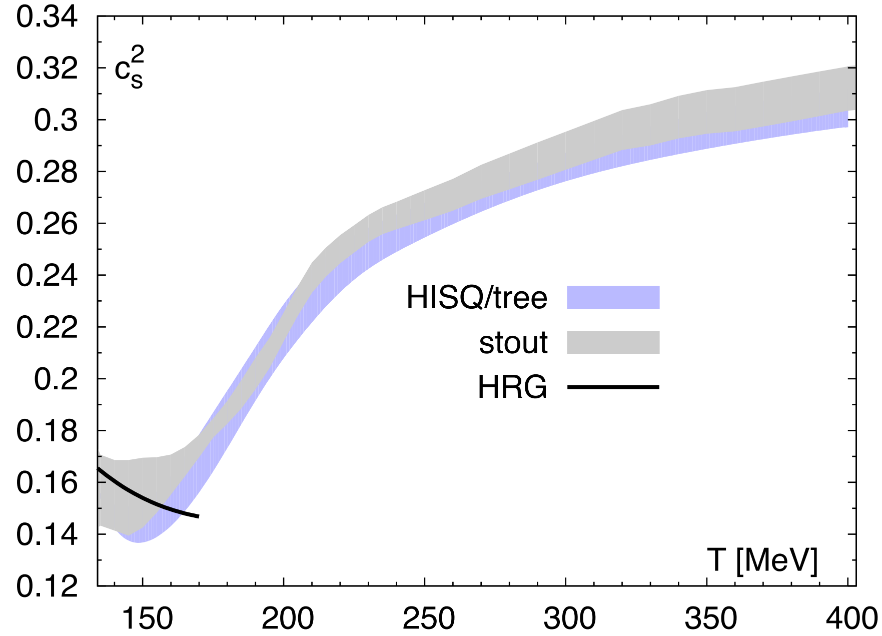
Berwein et al, PRD93 (2016) 034010

# Equation of state in the continuum limit

Equation of state has be calculated in the continuum limit up to *T=400* MeV using two different quark actions and the results agree well

Bazavov et al, PRD 90 (2014) 094503




Hadron resonance gas (HRG):
Interacting gas of hadrons = non-interacting gas of hadrons and hadron resonances
( virial expansion, Prakash & Venugopalan )

HRG agrees with the lattice for *T< 145* MeV

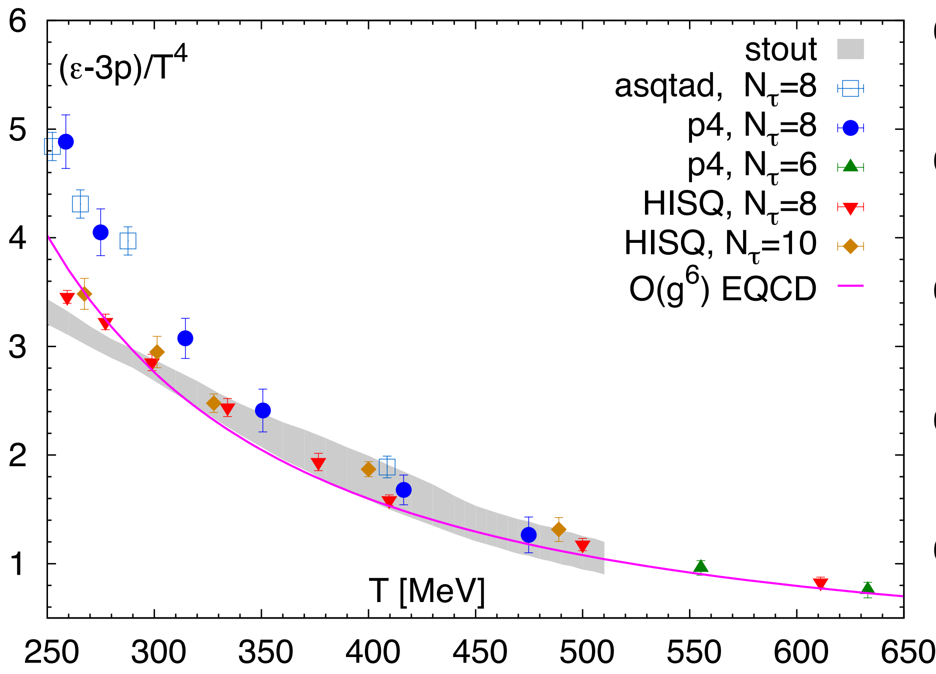
$$T_c = (154 \pm 9)\mathrm{MeV}$$

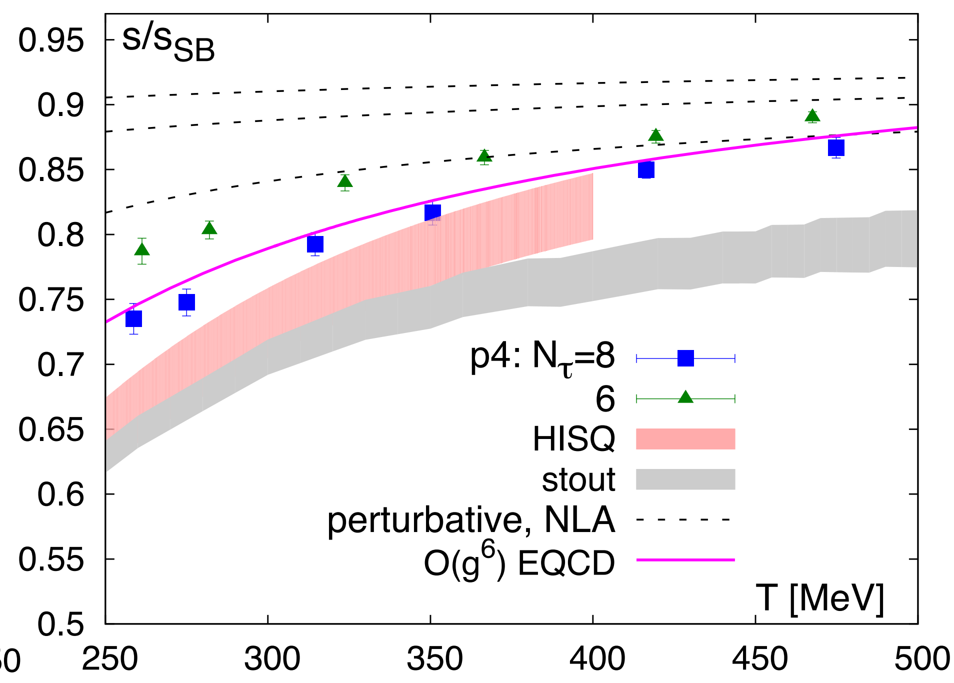
$$\epsilon_c \simeq 300\mathrm{MeV}/\mathrm{fm}^3$$

$$\epsilon_{low} \simeq 180\mathrm{MeV}/\mathrm{fm}^3 \leftrightarrow \epsilon_{nucl} \simeq 150\mathrm{MeV}/\mathrm{fm}^3$$

$$\epsilon_{high} \simeq 500\mathrm{MeV}/\mathrm{fm}^3 \leftrightarrow \epsilon_{proton} \simeq 450\mathrm{MeV}/\mathrm{fm}^3$$

# Equation of State on the lattice and in the weak coupling




The high temperature behavior of the trace anomaly is not inconsistent with weak coupling calculations (EQCD) for $T > 300$ MeV

For the entropy density the continuum lattice results are below the weak coupling calculations For $T < 500$ MeV

At what temperature can one see good agreement between the lattice and the weak coupling results ?

# QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \cdot \left(\frac{\mu_B}{T}\right)^i \cdot \left(\frac{\mu_Q}{T}\right)^j \cdot \left(\frac{\mu_S}{T}\right)^k$$ hadronic

$$\frac{p(T,\mu_u,\mu_d,\mu_s)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{uds} \cdot \left(\frac{\mu_u}{T}\right)^i \cdot \left(\frac{\mu_d}{T}\right)^j \cdot \left(\frac{\mu_s}{T}\right)^k$$ quark

$$\chi_{ijk}^{abc} = T^{i+j+k} \frac{\partial^i}{\partial \mu_a^i} \frac{\partial^j}{\partial \mu_b^j} \frac{\partial^k}{\partial \mu_c^k} \frac{1}{VT^3} \ln Z(T,V,\mu_a,\mu_b,\mu_c)|_{\mu_a=\mu_b=\mu_c=0}$$

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

$$\chi_2^X = \chi_X = \frac{1}{VT^3}(\langle X^2\rangle - \langle X\rangle^2) \qquad \chi_{11}^{XY} = \frac{1}{VT^3}(\langle XY\rangle - \langle X\rangle\langle Y\rangle)$$

↓

information about carriers of the conserved charges ( hadrons or quarks )

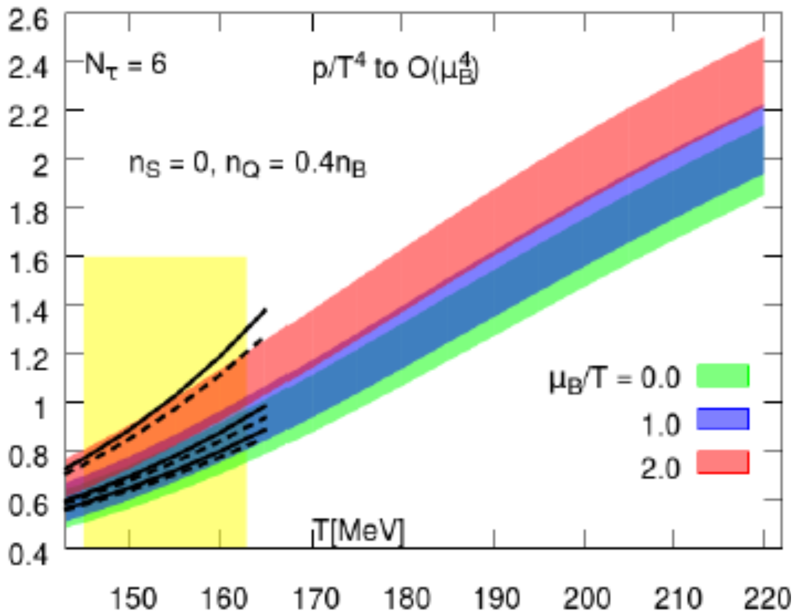
↓

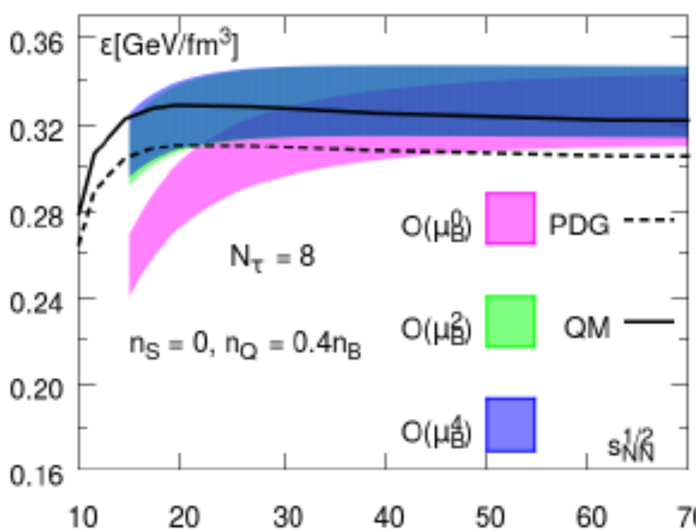
probes of deconfinement

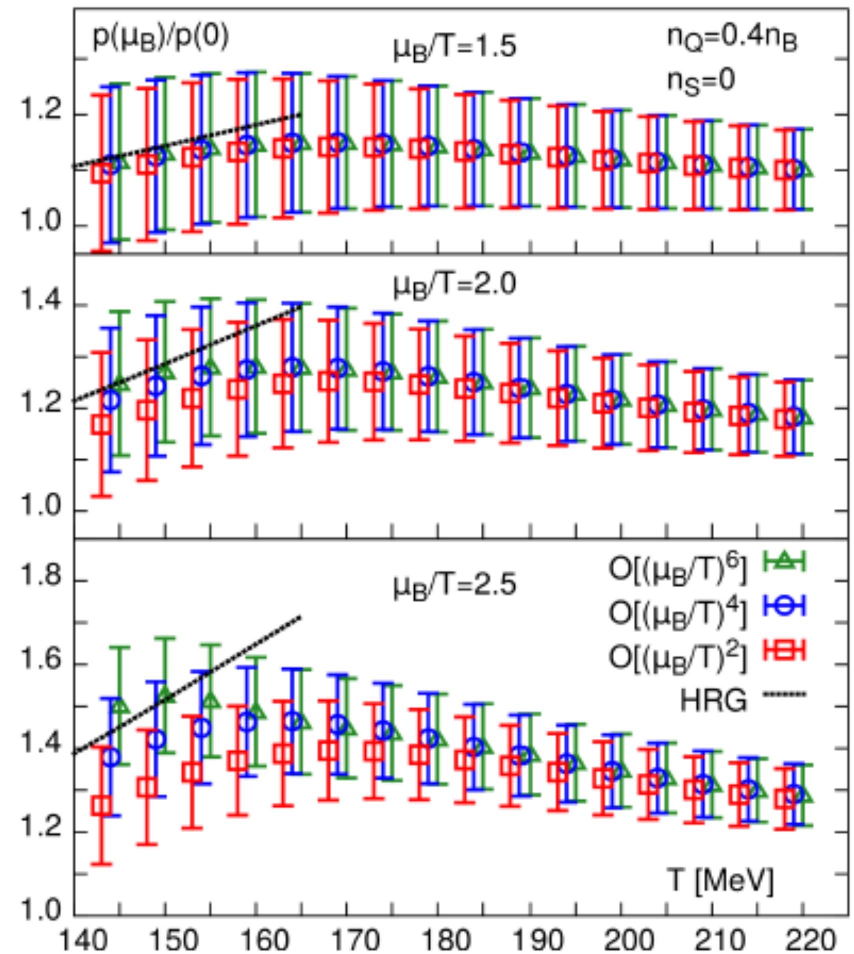
# Equation of state at non-zero baryon density

Taylor expansion up to 4th order for net zero strangeness $n_S = 0$ and $r = n_Q/n_B = Z/A = 0.4$

BNL-Bielefeld-CCNU




Moderate effects due to non-zero baryon density up to $\mu_B/T = 2 \leftrightarrow \sqrt{s} \sim 20\mathrm{GeV}$

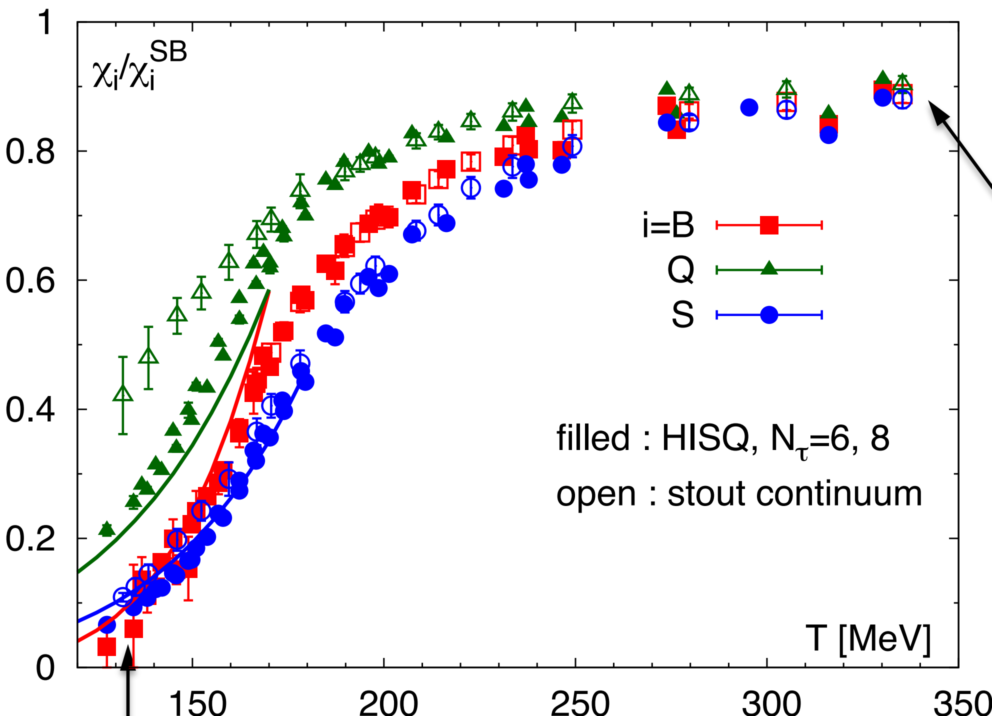
Energy density at freeze-out is independent of $\mu_B$

# Deconfinement : fluctuations of conserved charges

$$\chi_B = \frac{1}{VT^3}\left(\langle B^2\rangle - \langle B\rangle^2\right)$$ baryon number

$$\chi_Q = \frac{1}{VT^3}\left(\langle Q^2\rangle - \langle Q\rangle^2\right)$$ electric charge

$$\chi_S = \frac{1}{VT^3}\left(\langle S^2\rangle - \langle S\rangle^2\right)$$ strangeness




Ideal gas of massless quarks :

$$\chi_B^{SB} = \frac{1}{3} \qquad \chi_Q^{SB} = \frac{2}{3}$$

$$\chi_S^{SB} = 1$$

conserved charges carried by light quarks

conserved charges are carried by massive hadrons

HotQCD: PRD86 (2012) 034509

BW: JHEP 1201 (2012) 138,

# Deconfinement of strangeness

Partial pressure of strange hadrons in uncorrelated hadron gas:

$$P_S = \frac{p(T) - p_{S=0}(T)}{T^4} = M(T)\cosh\left(\frac{\mu_S}{T}\right) +$$

$$B_{S=1}(T)\cosh\left(\frac{\mu_B - \mu_S}{T}\right) + B_{S=2}(T)\cosh\left(\frac{\mu_B - 2\mu_S}{T}\right) + B_{S=3}(T)\cosh\left(\frac{\mu_B - 3\mu_S}{T}\right)$$

Strange hadrons are heavy treat them
As Boltzmann gas

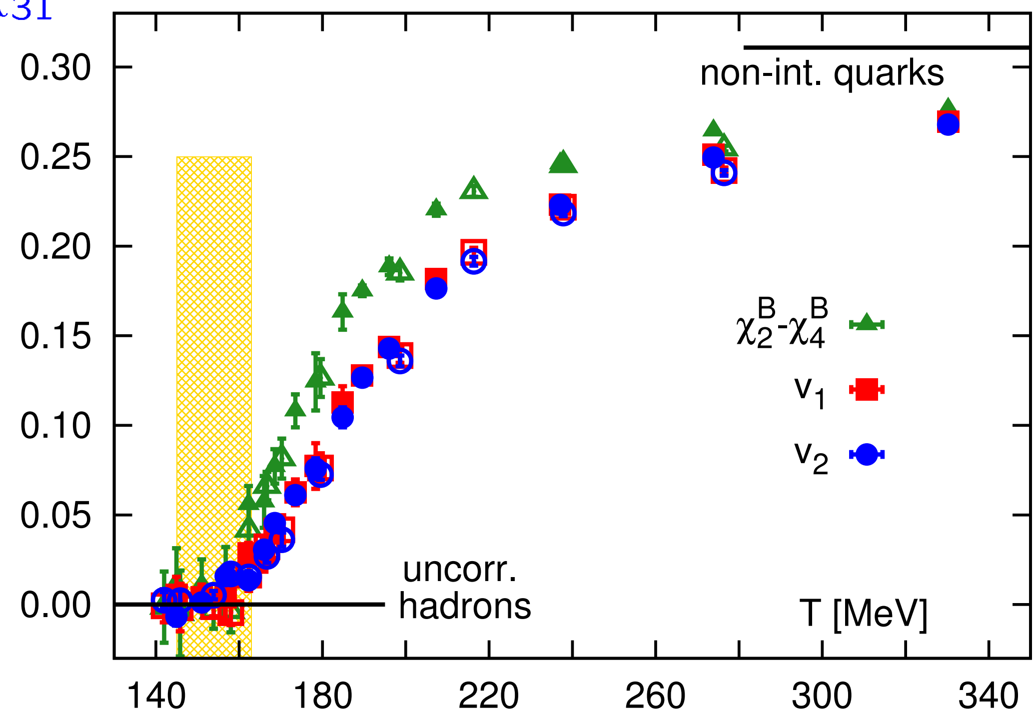
$$v_1 = \chi_{31}^{BS} - \chi_{11}^{BS}$$

$$v_2 = \frac{1}{3}\left(\chi_4^S - \chi_2^S\right) - 2\chi_{13}^{BS} - 4\chi_{22}^{BS} - 2\chi_{31}^{BS}$$

should vanish !

- $v_1$ and $v_2$ do vanish within errors at low $T$

- $v_1$ and $v_2$ rapidly increase above the transition region, eventually reaching non-interacting quark gas values

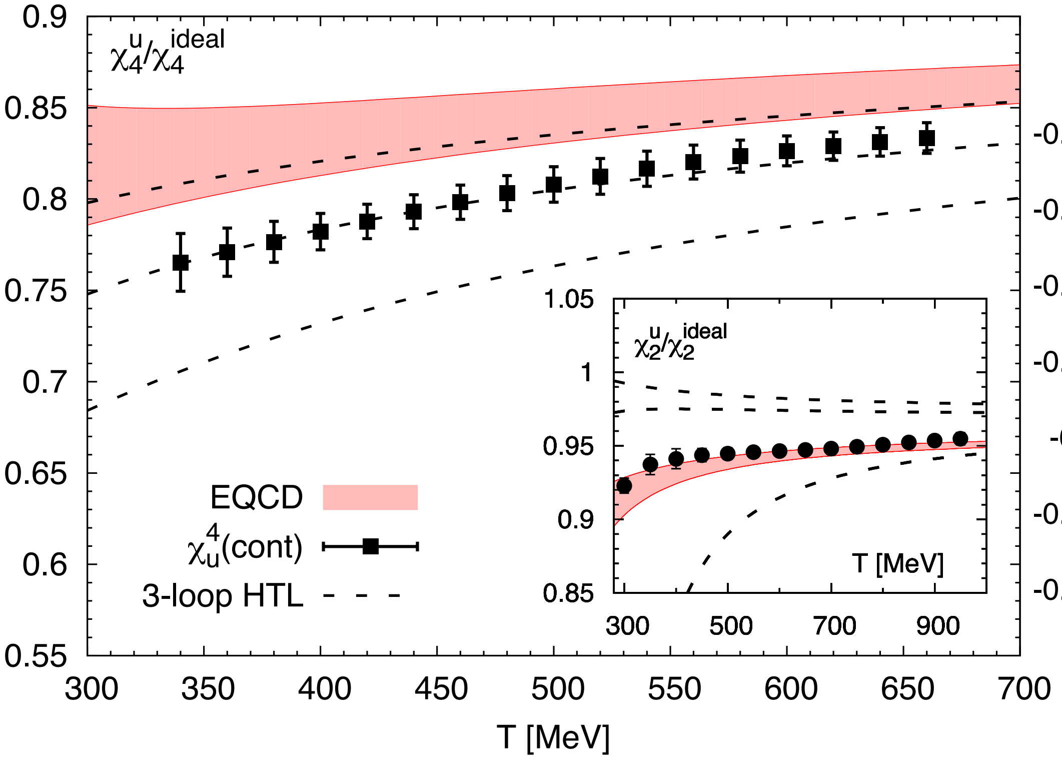



Bazavov et al, PRL 111 (2013) 082301

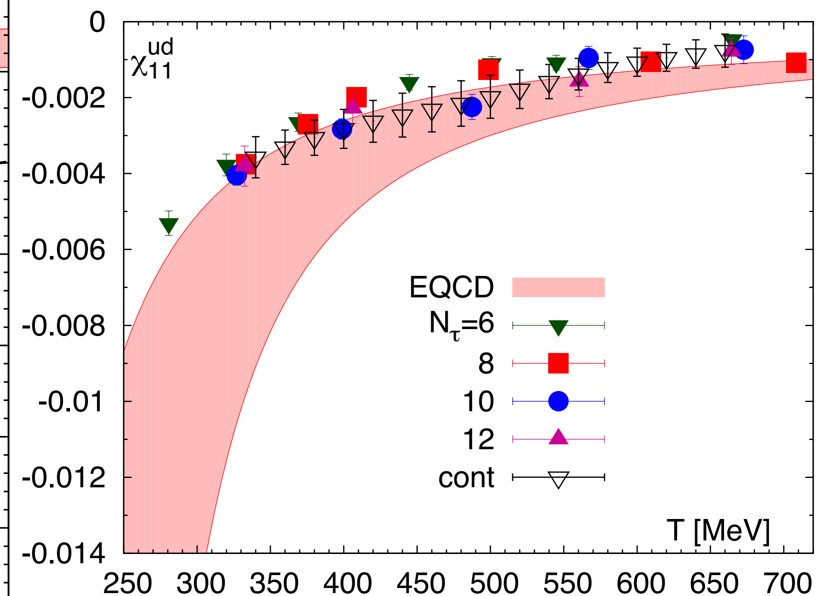
# Quark number fluctuations at high T

At high temperatures quark number fluctuations can be described by weak coupling approach due to asymptotic freedom of QCD

quark number fluctuations




quark number correlations




- Good agreement between continuum extrapolated lattice results and the weak coupling approach
- Quark number correlations vanish at any loop order but can be calculated in EQCD and the EQCD calculations agree with the continuum extrapolated lattice results

Bazavov et al, PRD88 (2013) 094021, Ding et at, PRD92 (2015) 074043

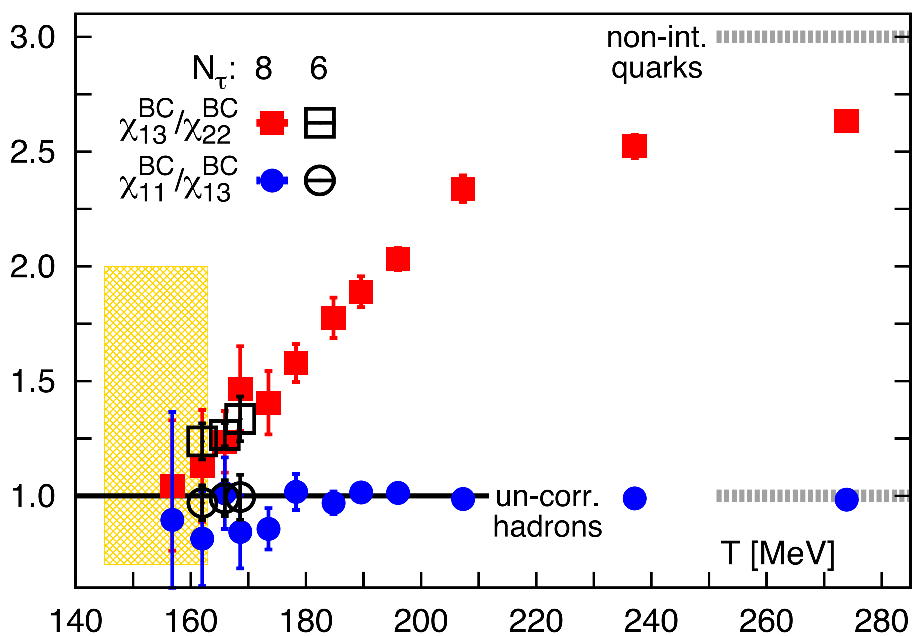
# What about charm hadrons ?

$$\chi^{XYC}_{nml} = T^{m+n+l} \frac{\partial^{n+m+l} p(T,\mu_X,\mu_Y,\mu_C)/T^4}{\partial\mu_X^n \partial\mu_Y^m \partial\mu_C^l}$$

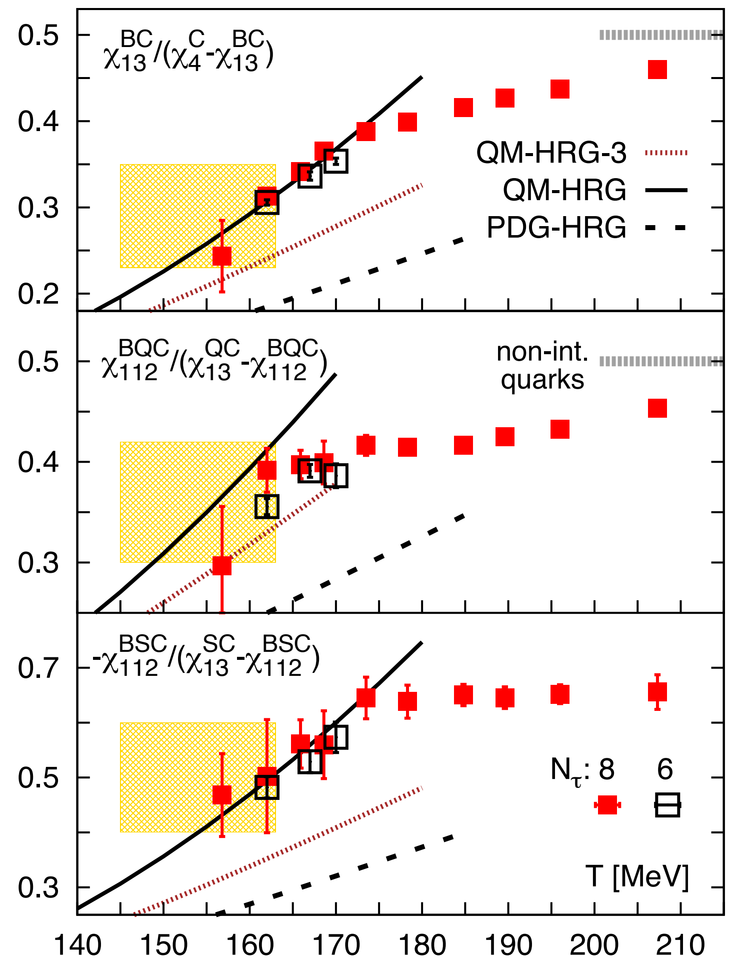
Bazavov et al, Phys.Lett. B737 (2014) 210

$m_c \gg T$ ⇨ only |*C*|=*1* sector contributes

In the hadronic phase all *BC*-correlations are the same !




*Charm baryon to meson pressure*




Hadronic description breaks down just above $T_c$
⇒ open charn deconfines above $T_c$

The charm baryon spectrum is not well known (only few states in PDG), HRG works only if the "missing" states are included

# Quasi-particle model for charm degrees of freedom

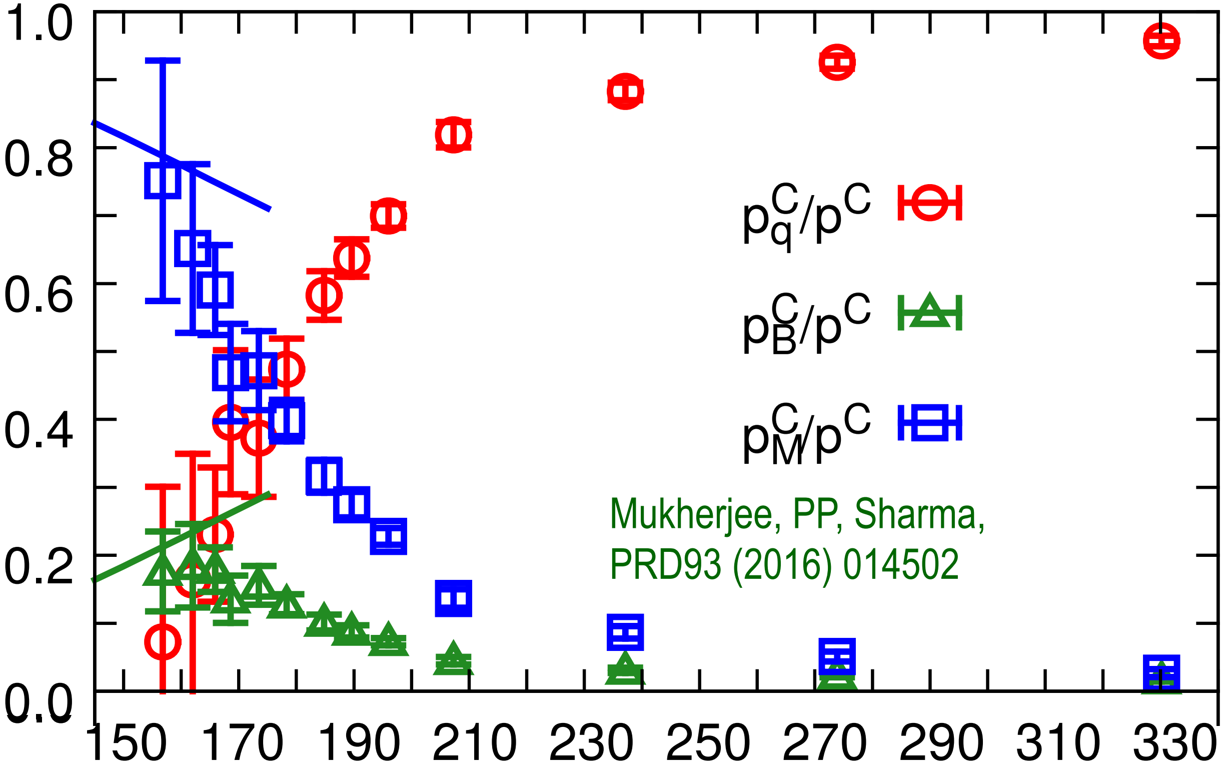
Charm dof are good quasi-particles at all $T$ because $M_c$>>$T$ and Boltzmann approximation holds

$$p^C(T,\mu_B,\mu_c) = p_q^C(T)\cosh(\hat{\mu}_C + \hat{\mu}_B/3) + p_B^C(T)\cosh(\hat{\mu}_C + \hat{\mu}_B) + p_M^C(T)\cosh(\hat{\mu}_C)$$

$$\hat{\mu}_X = \mu_X/T$$

$$\chi_2^C,\ \chi_{13}^{BC},\ \chi_{22}^{BC} \Rightarrow p_q^C(T), p_M^C(T), p_B^C(T)$$

Partial meson and baryon pressures described by HRG at $T_c$ and dominate the charm pressure then drop gradually, charm quark only dominant dof at $T>200$ MeV




Partial pressures drop because hadronic cxcitations become broad at high temperatures (bound state peaks merge with the continuum)

See
Jakovác, PRD88 (2013), 065012
Biró, Jakovác, PRD(2014)065012

Vice versa for quarks

# Summary

- The deconfinement transition temperature defined in terms of the free energy of static quark agrees with the chiral transition temperature for physical quark mass; $U_A(1)$ symmetry remains broken t high T

- Equation of state are known in the continuum limit up to $T=400$ MeV at zero baryon density and the effect of non vanishing baryon densities seem to be moderate.

- Hadron resonance gas (HRG) can describe various thermodynamic quantities at low temperatures

- Deconfinement transition can be studied in terms of fluctuations and correlations of conserved charges, it manifest itself as a abrupt breakdown of hadronic description that occurs around the chiral transition temperature

- Charm hadrons can exist above $T_c$ and are dominant dof for $T<180$ MeV

- For $T > (300\text{-}400)$ MeV weak coupling expansion works well for certain quantities but more work is needed to establish the validity of weak coupling methods

- Comparison of lattice and HRG results for certain charm correlations hints for existence of yet undiscovered excited baryons