Soft collinear effective theory for lepton-nucleon scattering

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- overview of proton radius puzzle
- Q^2 dependence of radius from electron-proton scattering
- soft-collinear effective theory for large logs in radiative corrections to lepton-nucleon scattering
- implications
- summary and outlook

based on 1605.02613, and related work with John Arrington, Gabriel Lee, Gil Paz

Some facts about the Rydberg constant puzzle (a.k.a. proton radius puzzle)

1) It has generated a lot of attention and controversy

The New York Times

2) The *most* mundane resolution necessitates:

- 5σ shift in fundamental Rydberg constant
- discarding or revising decades of results in e-p scattering and hydrogen spectroscopy

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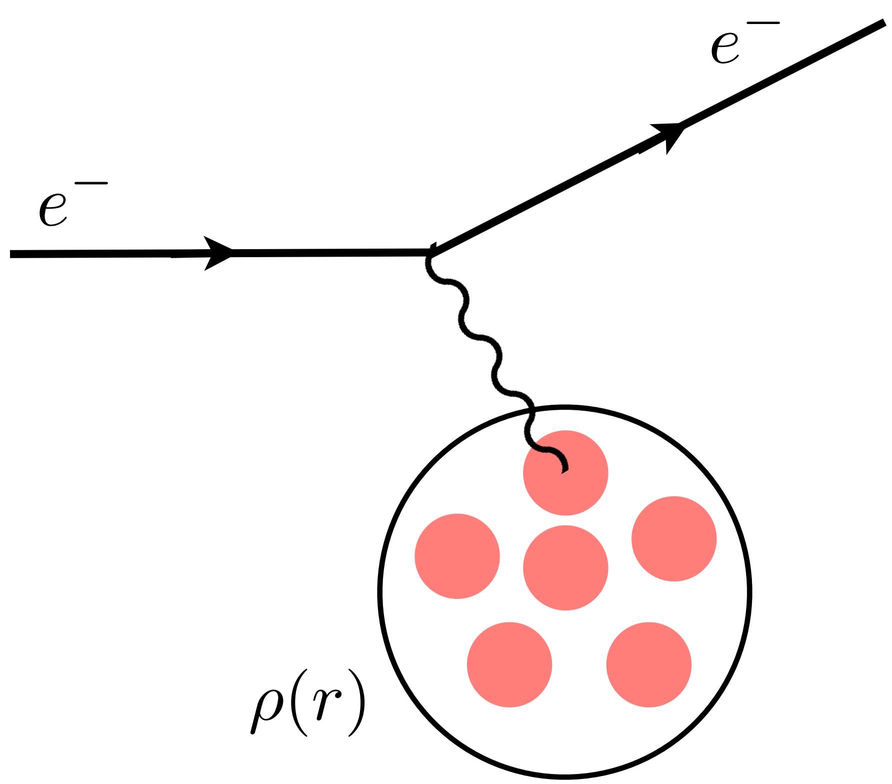
This problem has broad ownership, e.g.:

3) Systematic effects in electron-proton scattering impact neutrino-nucleus scattering, *at a level large compared to long baseline precision requirements*

What is the proton charge radius?

recall scattering from extended classical charge distribution:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{pointlike}} |F(q^2)|^2$$



$$F(q^2) = \int d^3r\, e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \rho(\boldsymbol{r})$$
$$= \int d^3r \left[1 + i\boldsymbol{q}\cdot\boldsymbol{r} - \frac{1}{2}(\boldsymbol{q}\cdot\boldsymbol{r})^2 + \ldots\right]\rho(r)$$
$$= 1 - \frac{1}{6}\langle r^2\rangle \boldsymbol{q}^2 + \ldots$$

for the relativistic, QM, case, *define* radius as slope of form factor

$$\langle J^\mu\rangle = \gamma^\mu F_1 + \frac{i}{2m_p}\sigma^{\mu\nu} q_\nu F_2$$

$$G_E = F_1 + \frac{q^2}{4m_p^2} F_2 \qquad G_M = F_1 + F_2$$

$$\boxed{r_E^2 \equiv 6 \frac{d}{dq^2} G_E(q^2)\Big|_{q^2=0}}$$

(up to radiative corrections)

Recall hydrogen spectrum:

$$E_n \sim \frac{R_\infty}{n^2} + \frac{r_E^2}{n^3}$$

$$hcR_\infty = \frac{m_e c^2 \alpha^2}{2} \approx 13.6\,\mathrm{eV}$$

proton charge radius

Disentangle 2 unknowns, R_∞ and r_E, using well-measured 1S-2S hydrogen transition *and*

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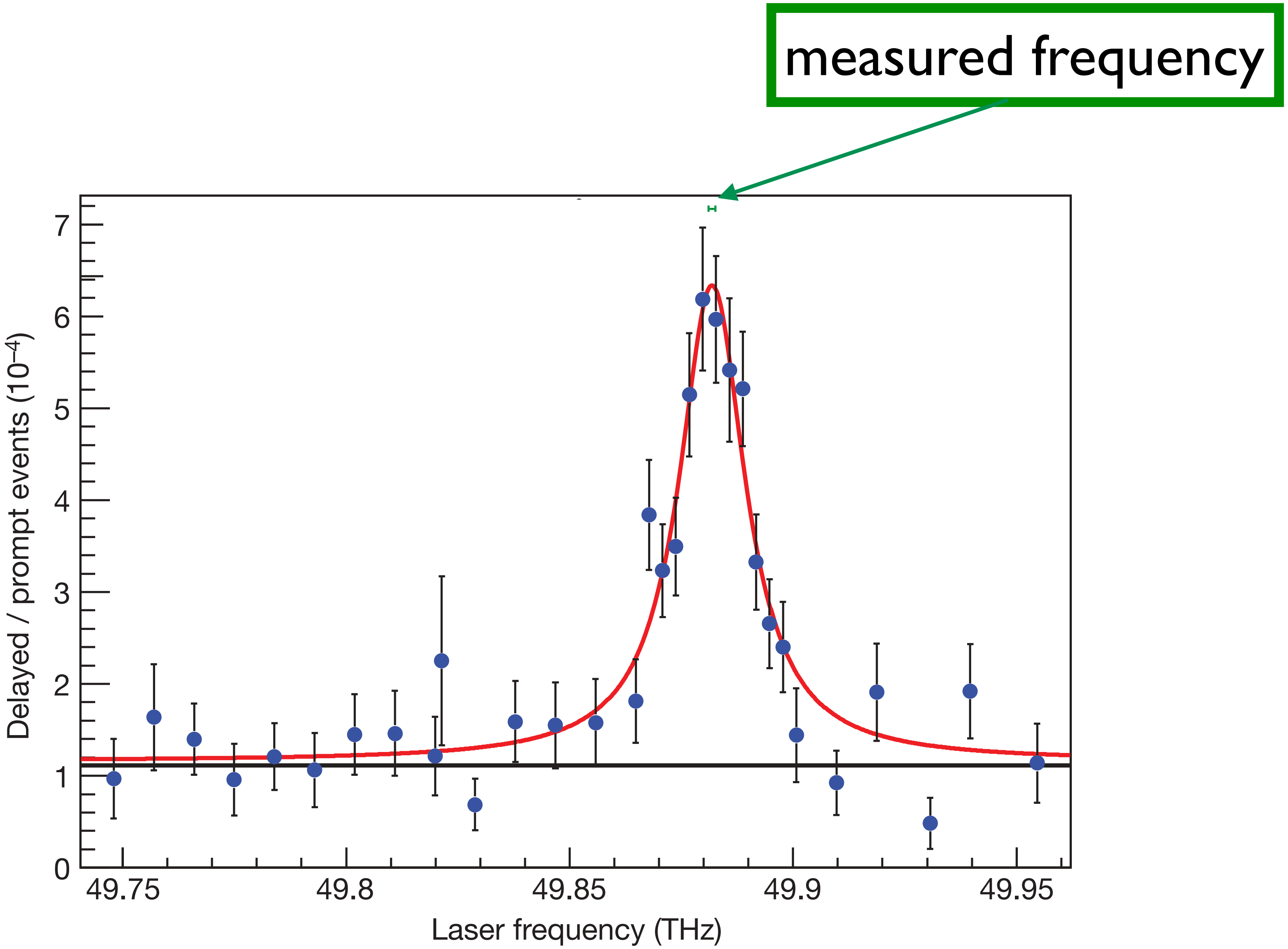
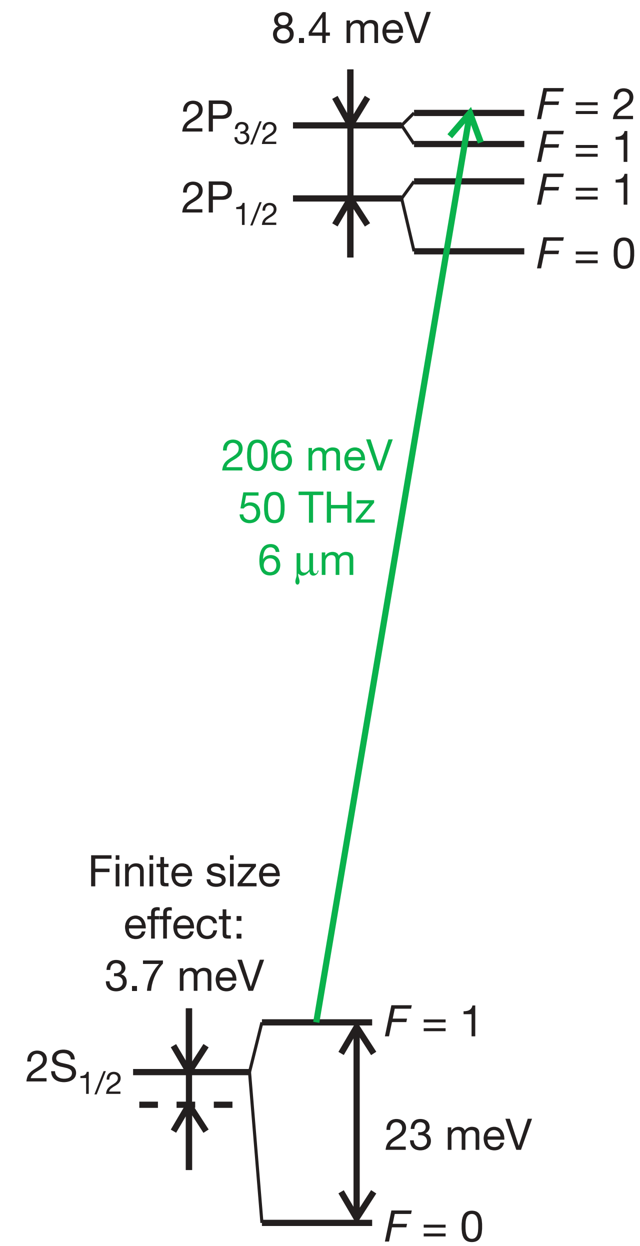
5σ discrepancy in Rydberg constant from (1+2) versus (3)

muonic hydrogen Lamb shift measurement

Pohl et al., Nature 466, 213 (2010)





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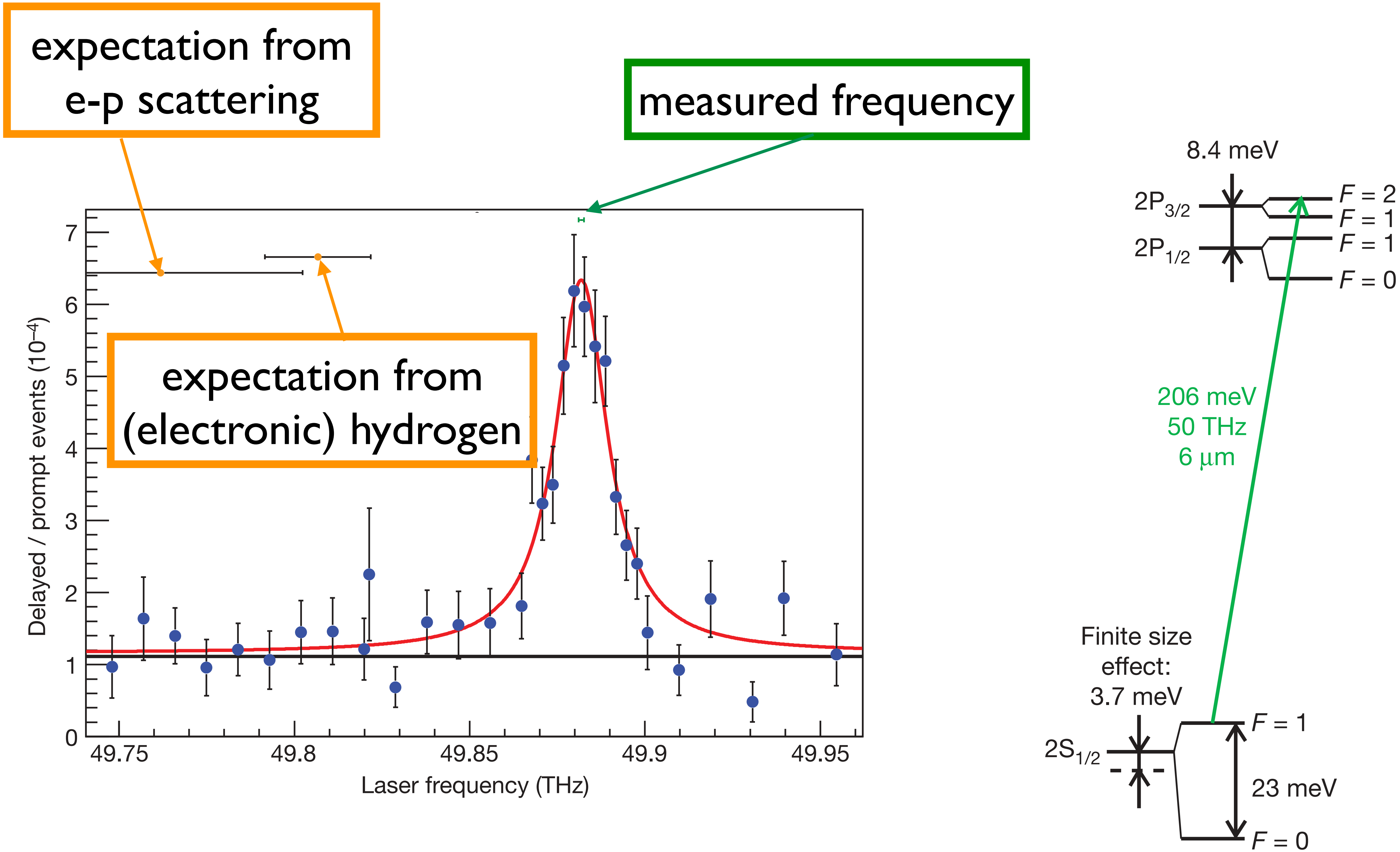
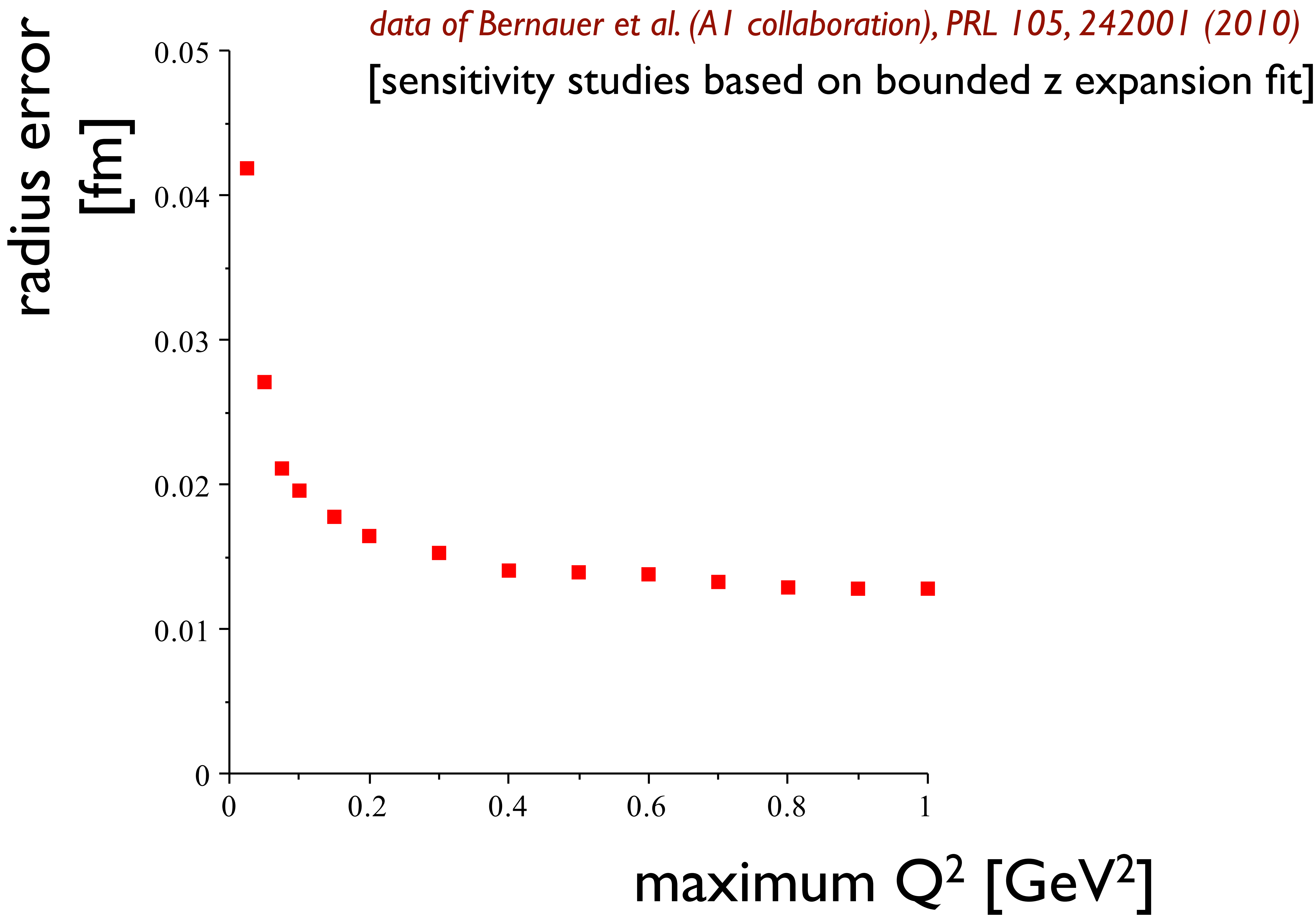


Radius defined as slope. Requires data over finite Q^2 range



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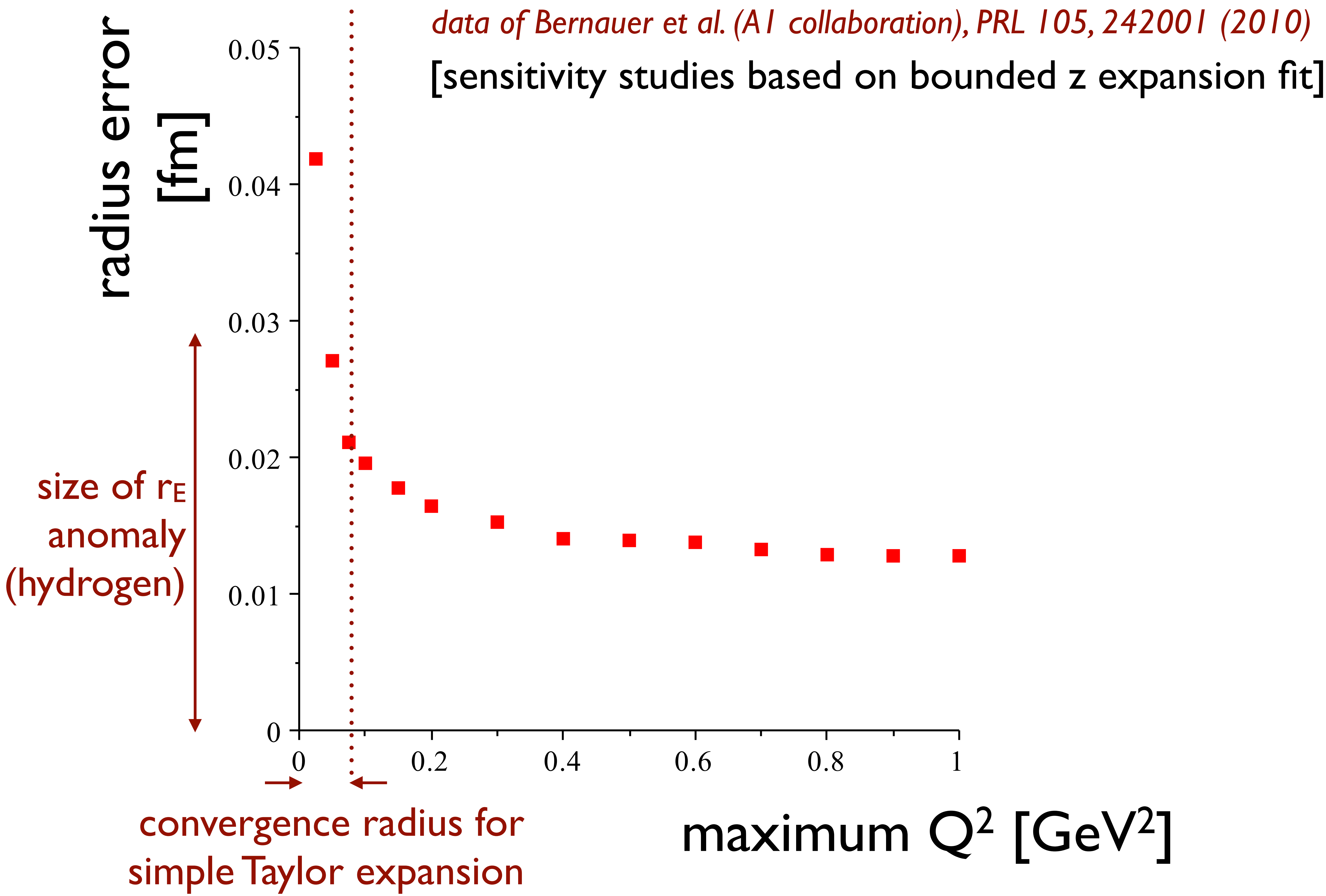

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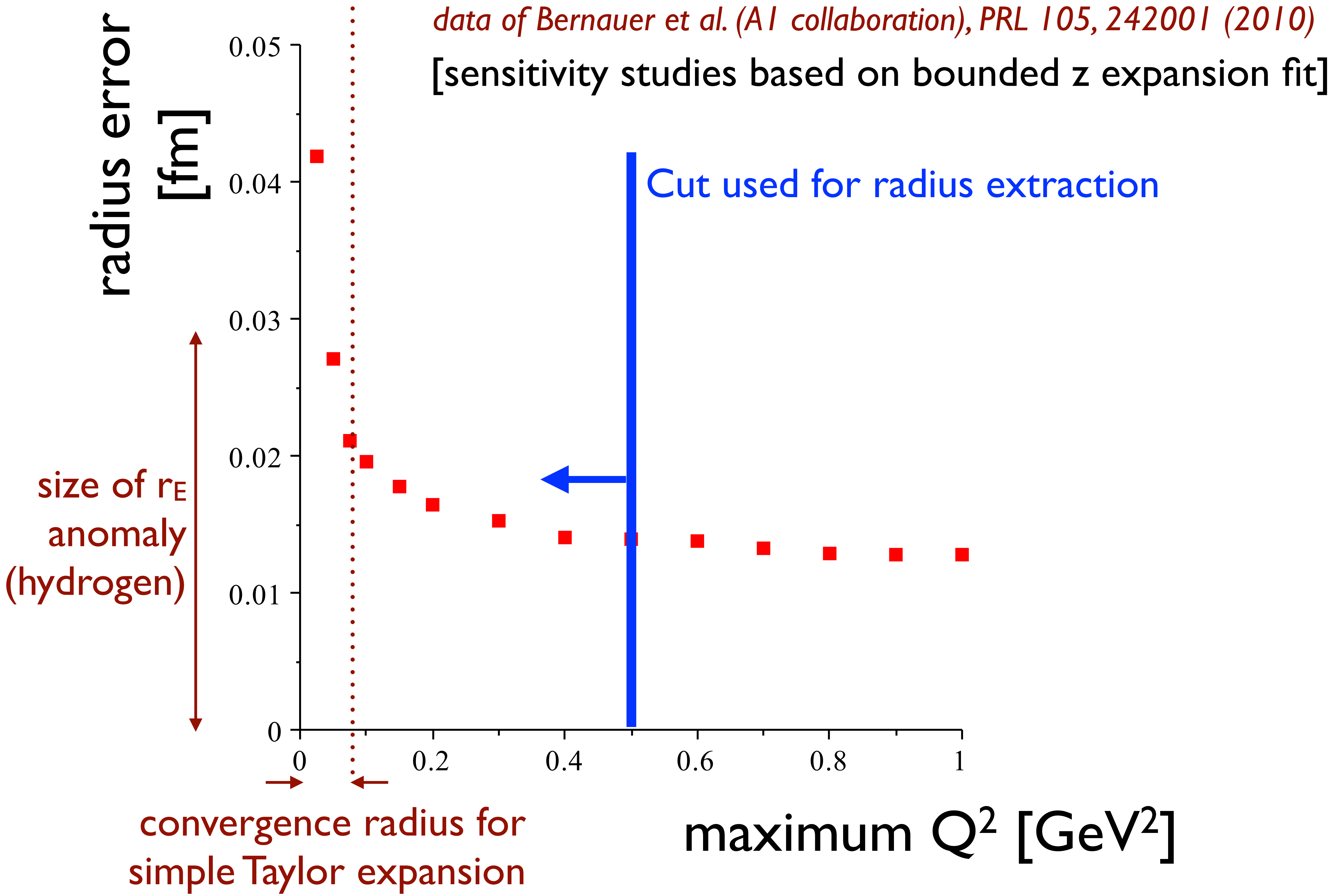
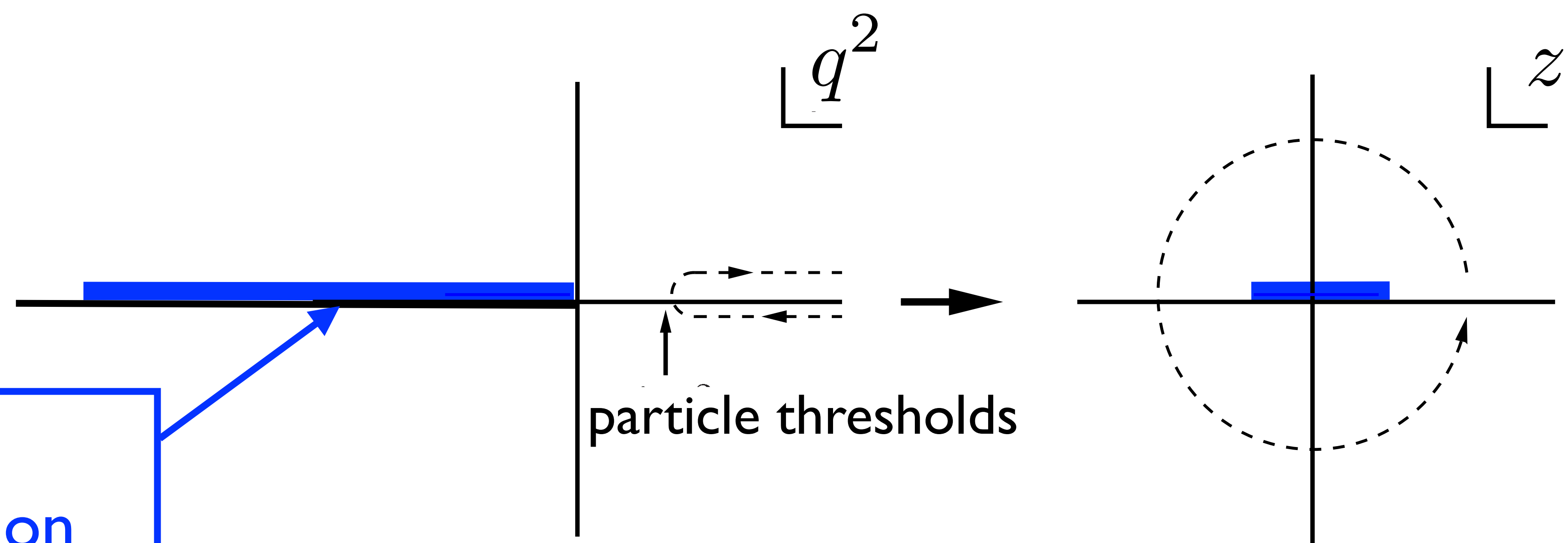


Underlying QCD tells us that Taylor expansion in appropriate variable is rapidly convergent



experimental kinematic region

$$F(q^2) = \sum_k a_k [z(q^2)]^k$$

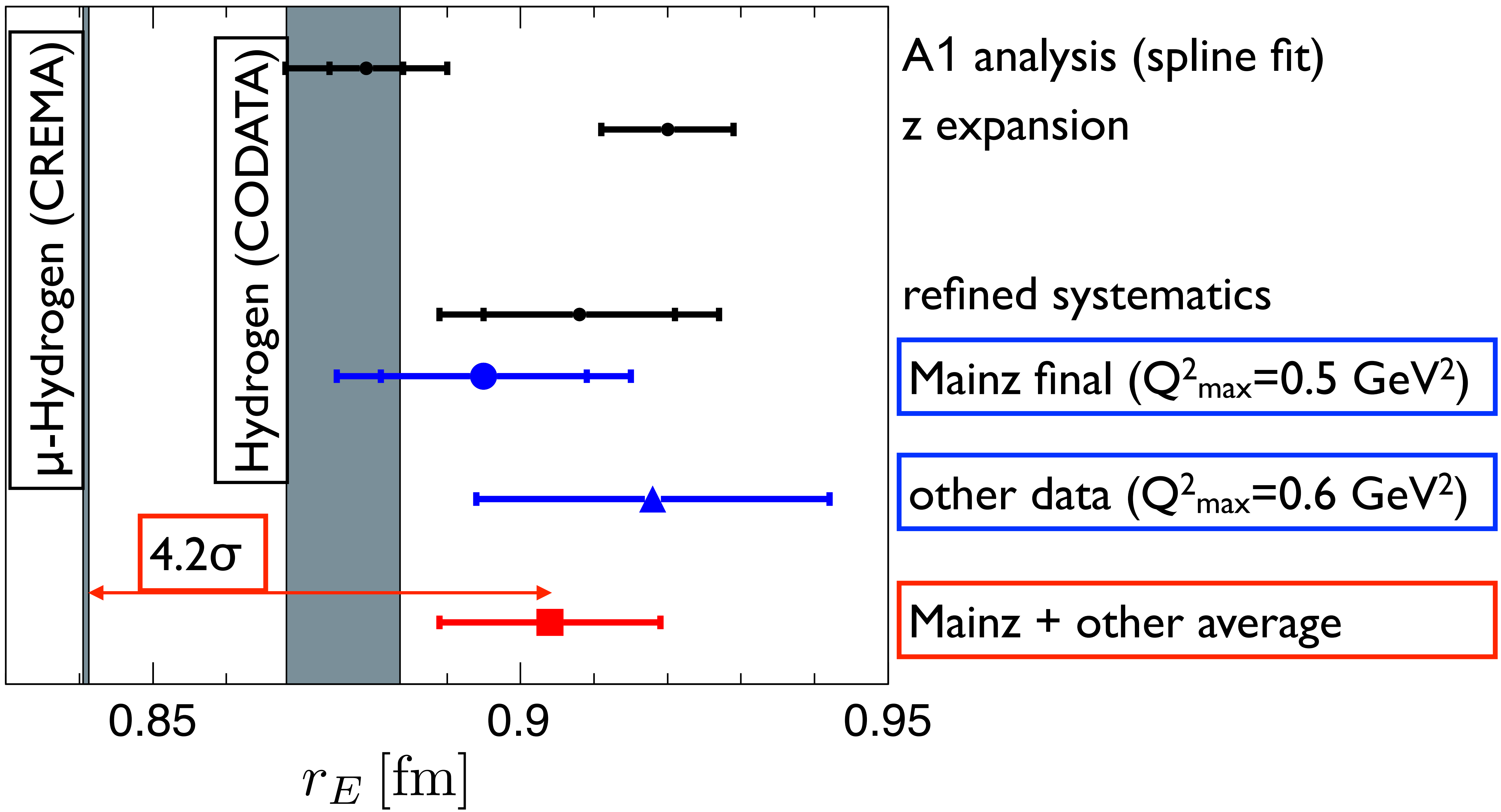
$$z(q^2, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - q^2} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - q^2} + \sqrt{t_{\rm cut} - t_0}}$$

coefficients in rapidly convergent expansion encode nonperturbative QCD

Systematically improvable, quantifiable uncertainties

experimental landscape: electron-proton scattering

from G. Lee, J. Arrington, RJH, 2015



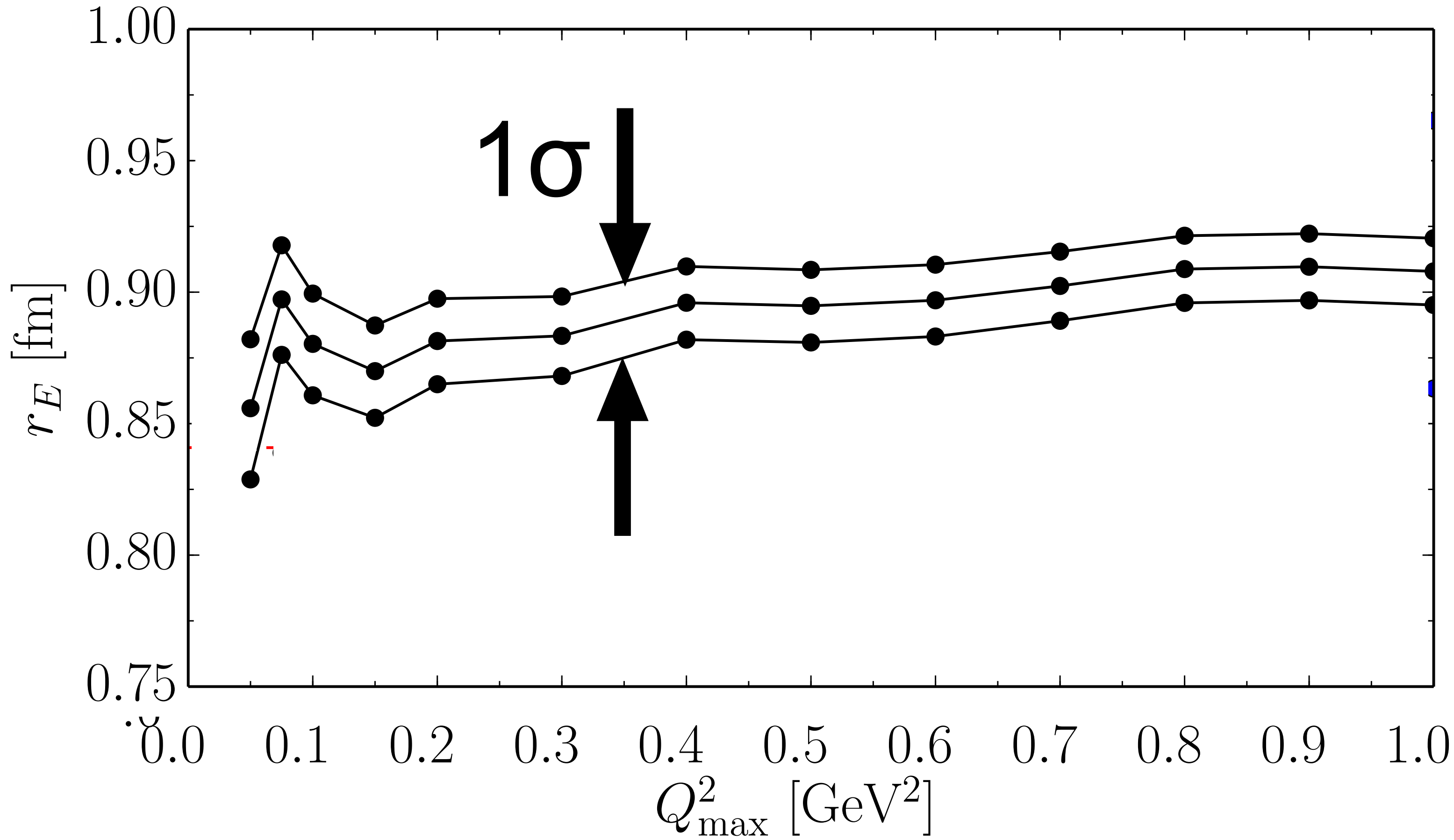
r_E^{Mainz} = 0.895(14)(14) fm

r_E^{other} = 0.918(24) fm

simple average: $r_E^{avg.}$ = 0.904(15) fm

muonic hydrogen: $r_E^{\mu H}$ = 0.841 fm

Tension between radius extracted from different Q^2 ranges



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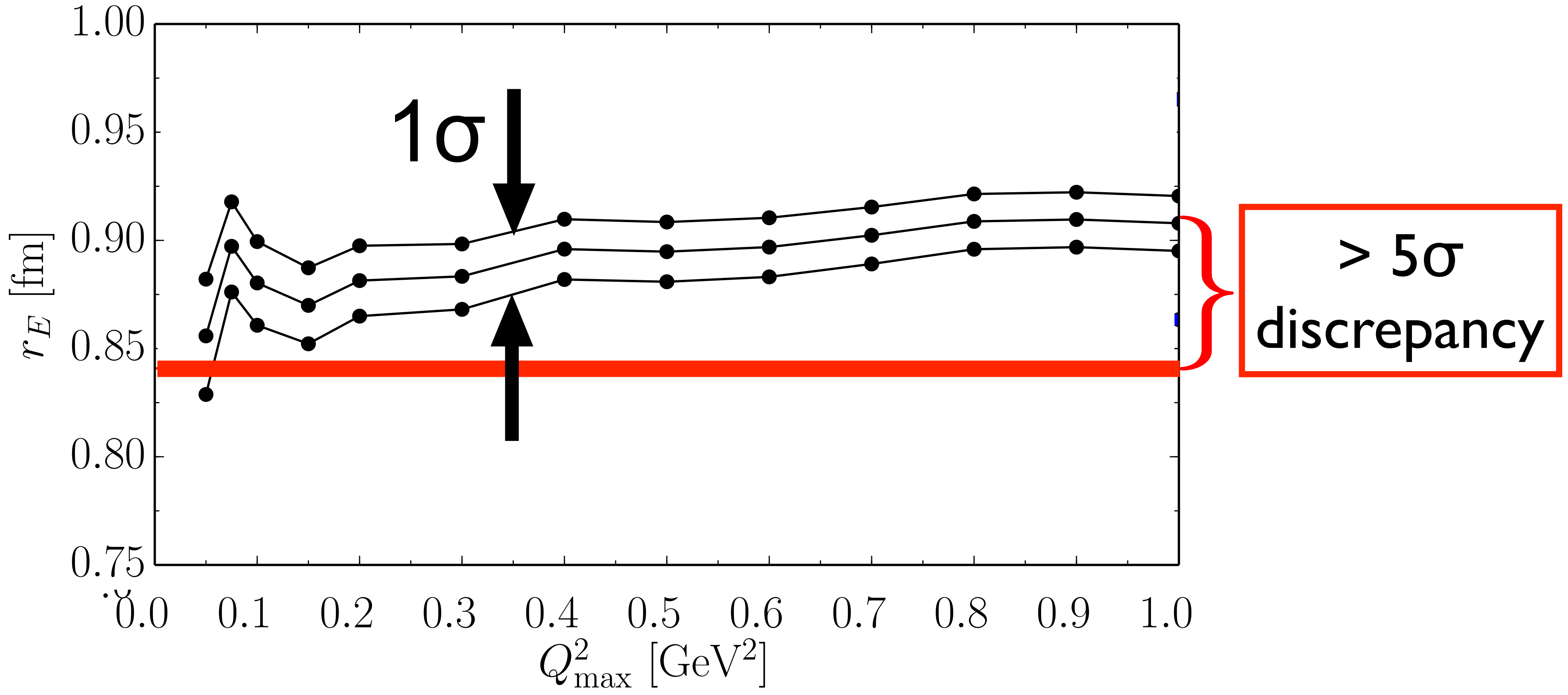
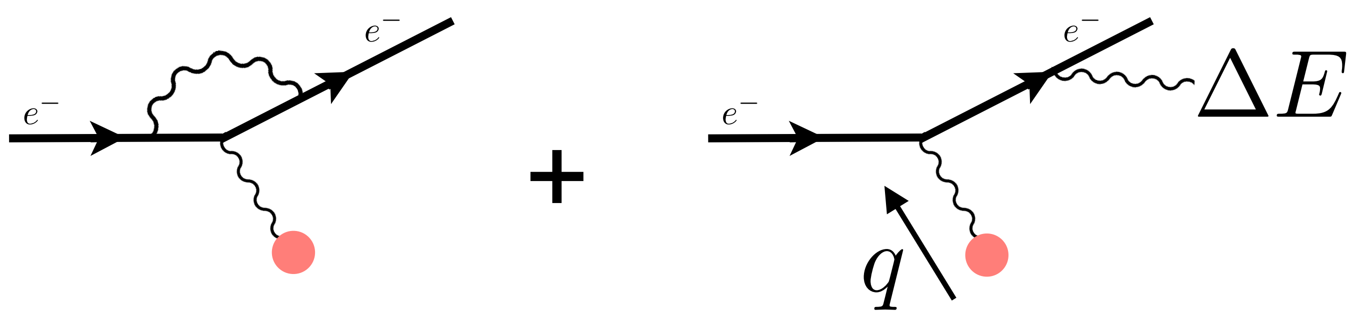


Are radiative corrections under control?

Large logarithms spoil QED perturbation theory when $-q^2=Q^2 \sim \mathrm{GeV}^2$

$$|F(q^2)|^2 \to |F(q^2)|^2 \left(1 - \underbrace{\frac{\alpha}{\pi} \log \frac{Q^2}{m_e^2} \log \frac{E^2}{(\Delta E)^2}}_{\approx 0.5} + \dots \right)$$



A standard ansatz sums leading logarithms by exponentiating 1st order:

$$|F(q^2)|^2 \left(1 - \frac{\alpha}{\pi} \log \frac{Q^2}{m_e^2} \log \frac{E^2}{(\Delta E)^2} + \dots \right) \to |F(q^2)|^2 \exp \left[- \frac{\alpha}{\pi} \log \frac{Q^2}{m_e^2} \log \frac{E^2}{(\Delta E)^2} \right]$$

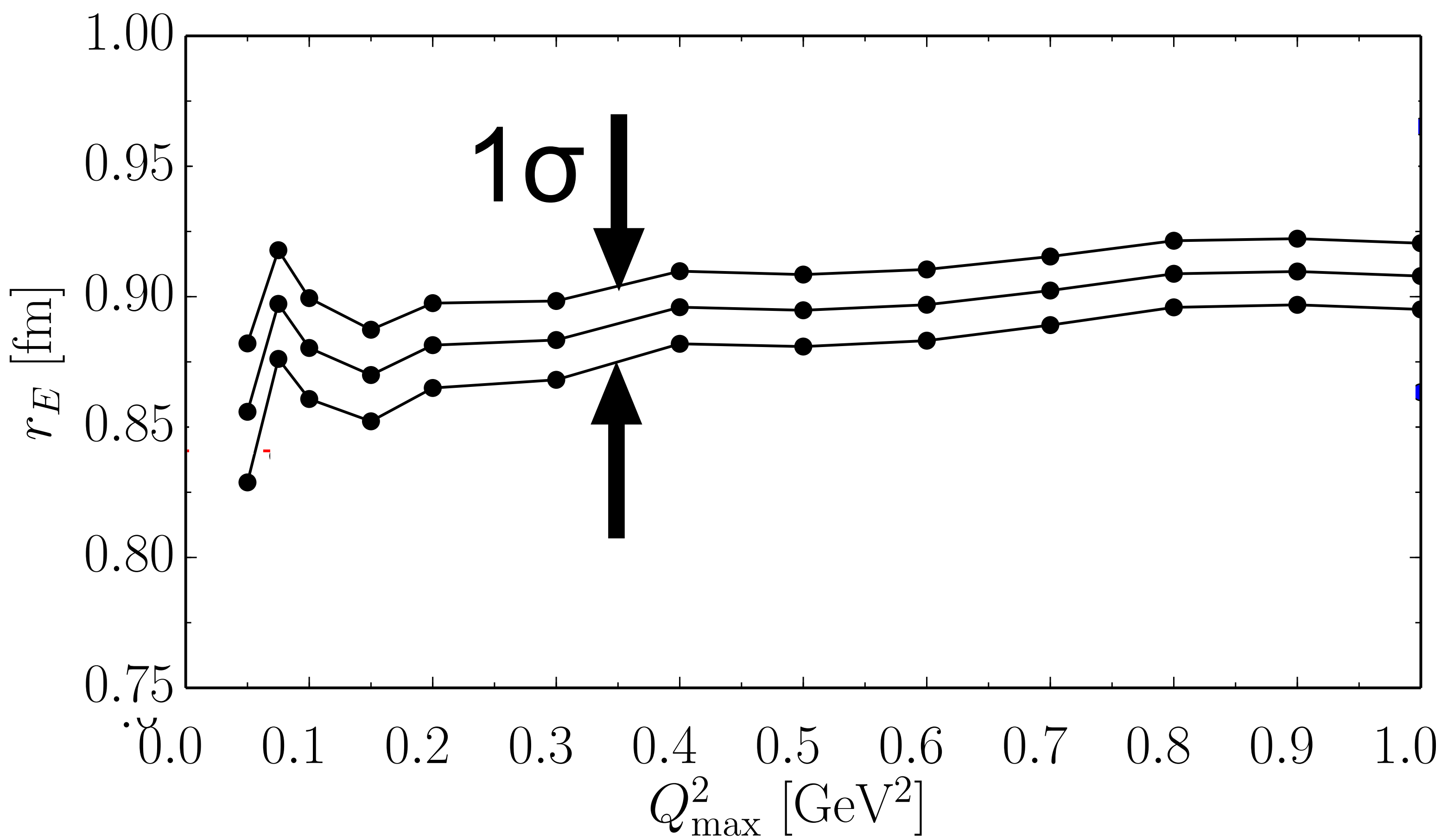
Yennie, Frautschi, Suura, 1961

Captures leading logarithms when

$$Q \sim E \,, \quad \Delta E \sim m_e$$

As consistency check, should find the same result for resumming:

$$\log^2 \frac{Q^2}{m_e^2} \qquad \text{vs.} \quad \log \frac{Q^2}{m_e^2} \log \frac{E^2}{(\Delta E)^2}$$

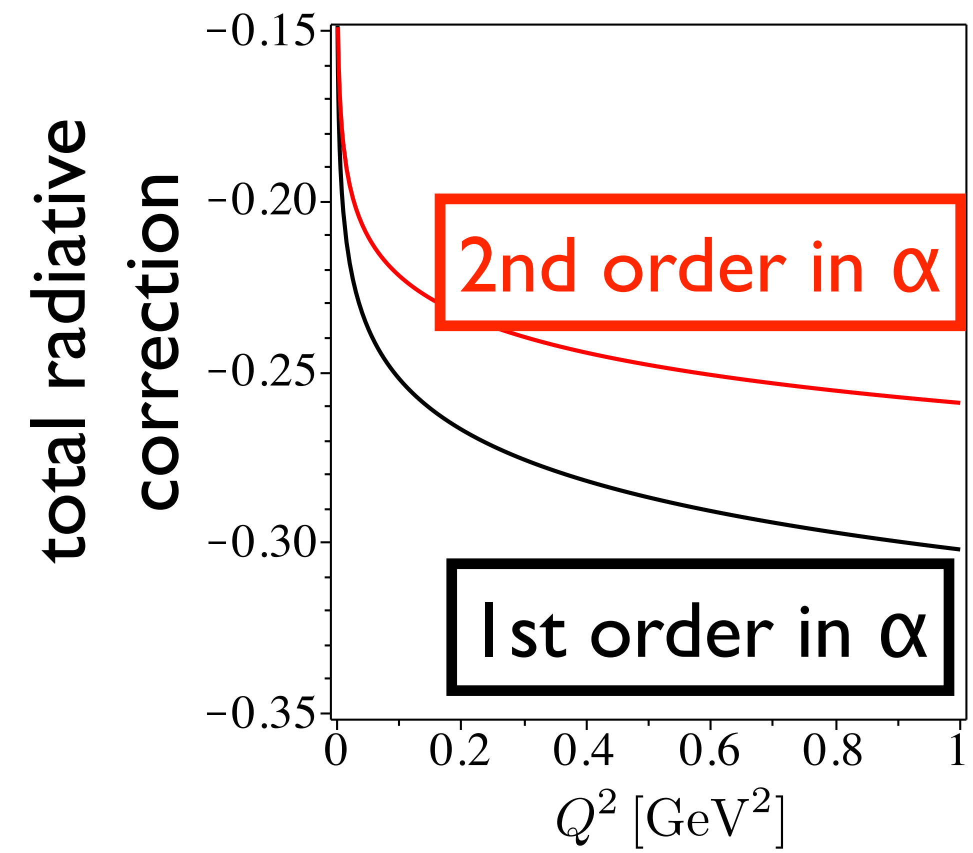


electron energy:

$E = 1\,\mathrm{GeV}$

electron energy loss cut:

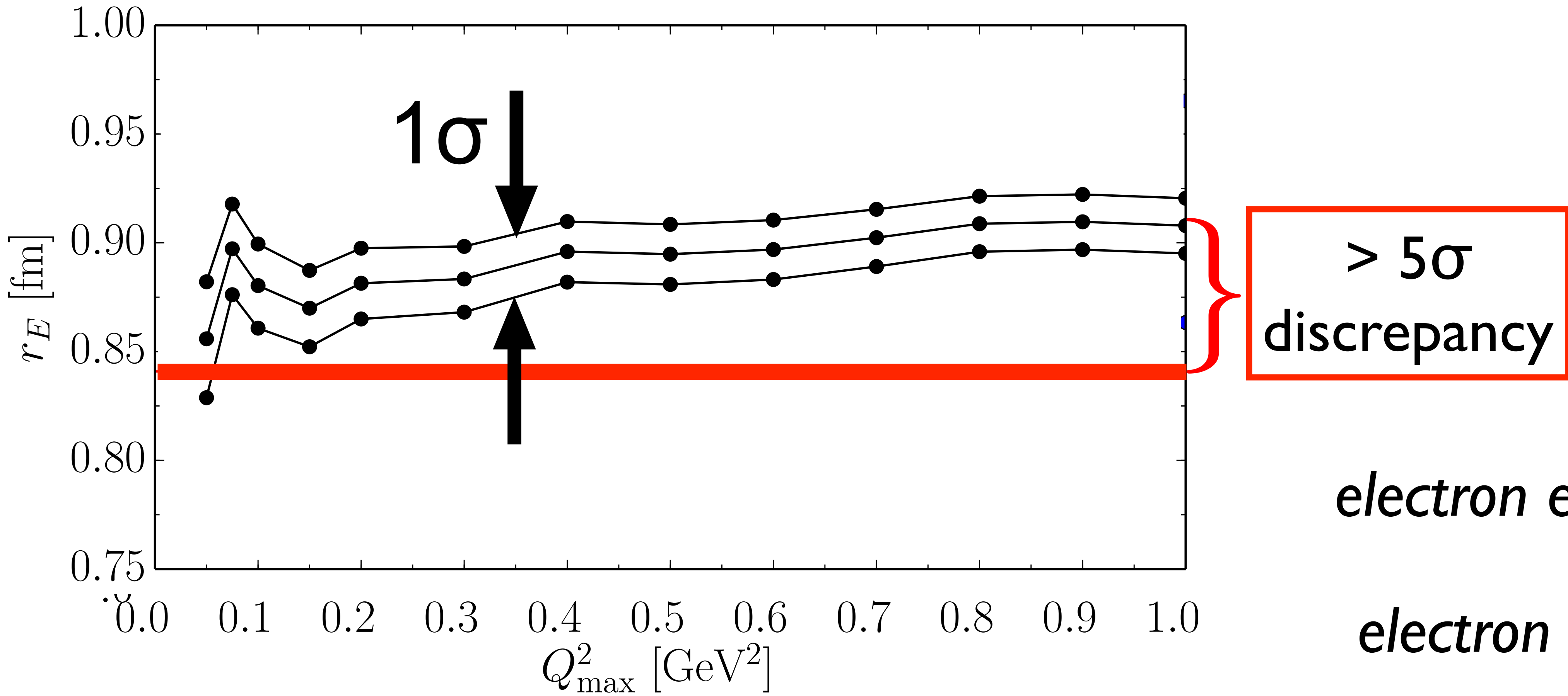
$\Delta E = 5\,\mathrm{MeV}$



- quoted systematics in A1 electron-proton scattering data are 0.2-0.5 %

- leading order radiative corrections ~30%

- need to systematically account for subleading logarithms, recoil, nuclear charge and structure



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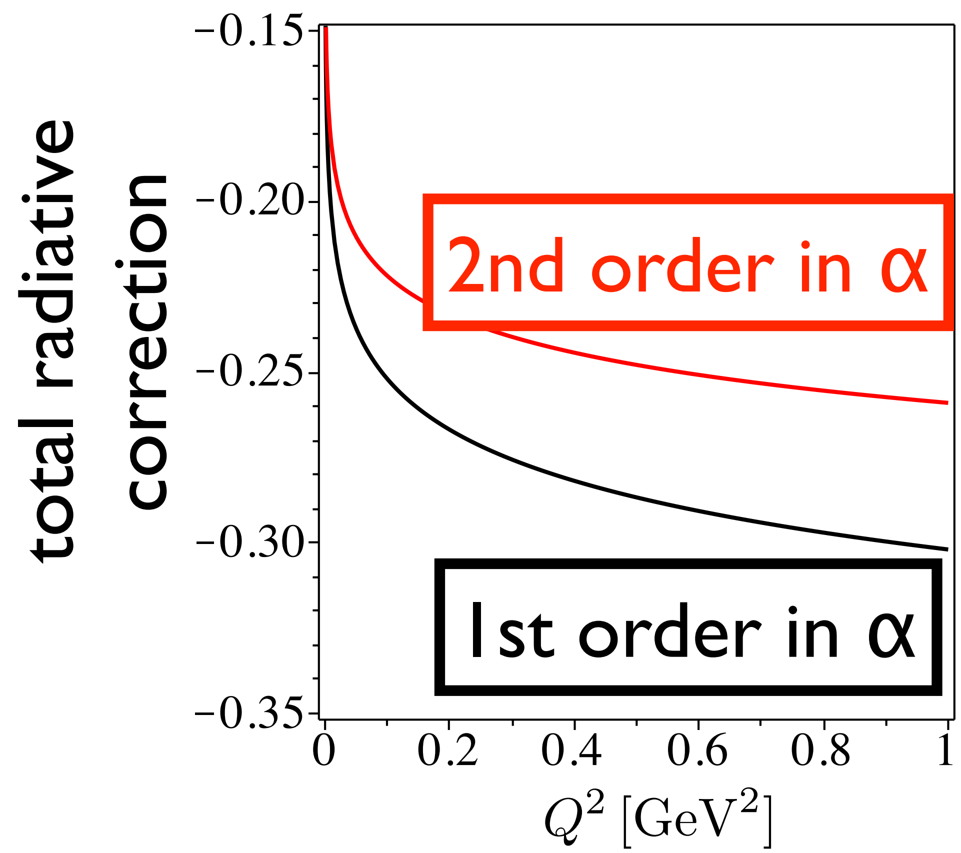

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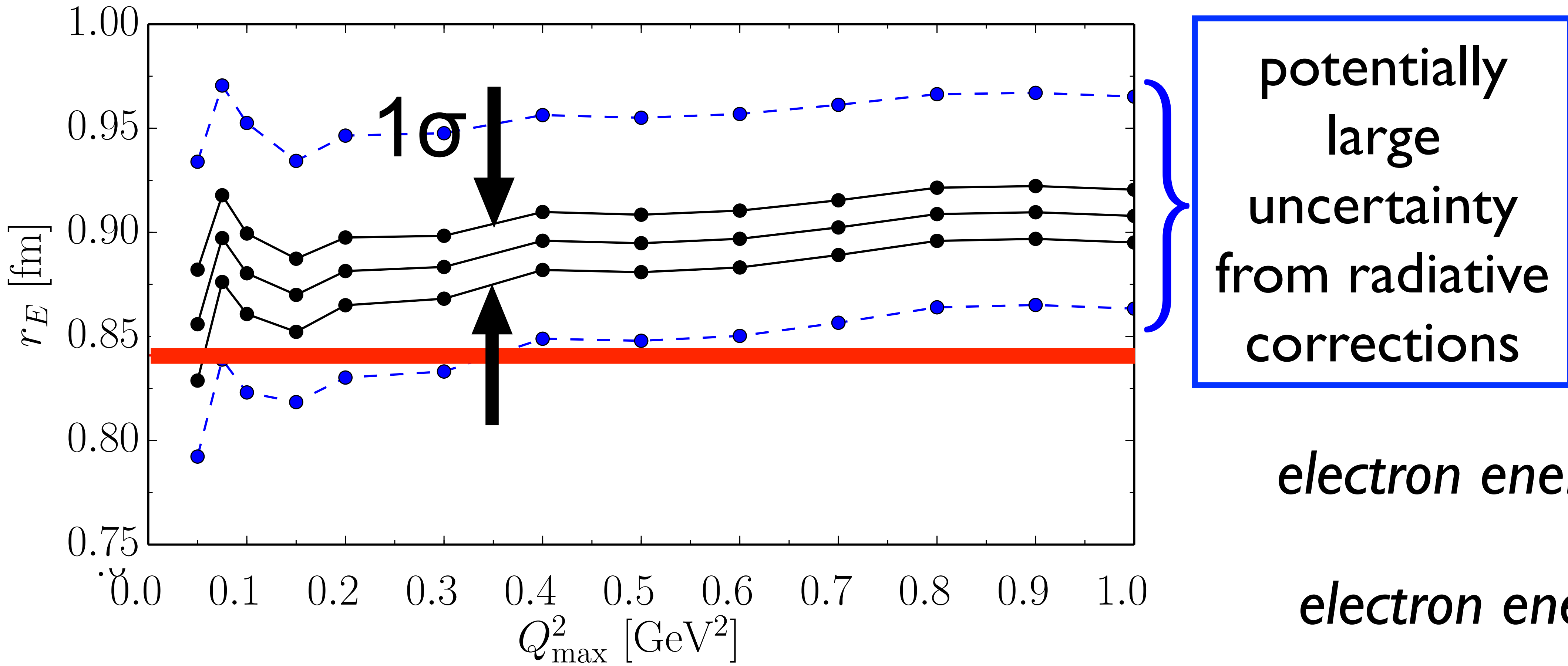

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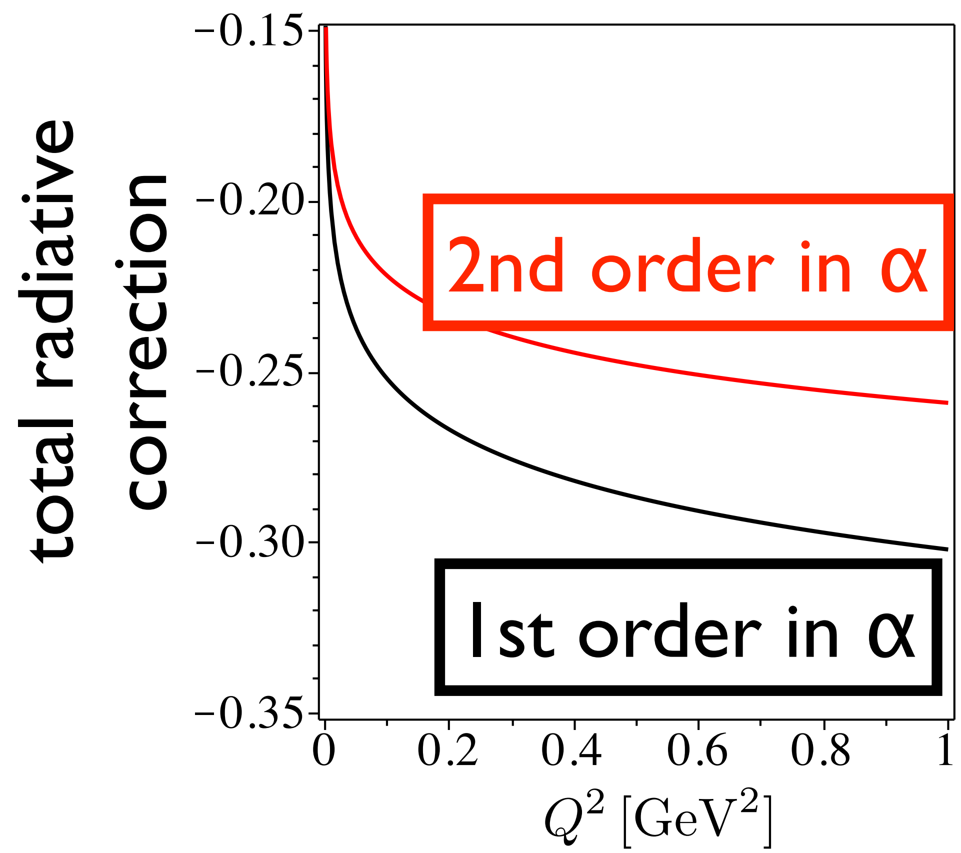

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Basics of soft-collinear effective theory

- degrees of freedom:

  - hard momenta → Wilson coefficients of effective operators

  - soft and collinear fields constrained by multiple gauge symmetries, organized by power counting in m_e/Q

Bauer et al. hep-ph/0005275,0011336; Chay and Kim hep-ph/0201197; Beneke et al. hep-ph/0206152; Hill and Neubert hep-ph/0211018, …

- factorization

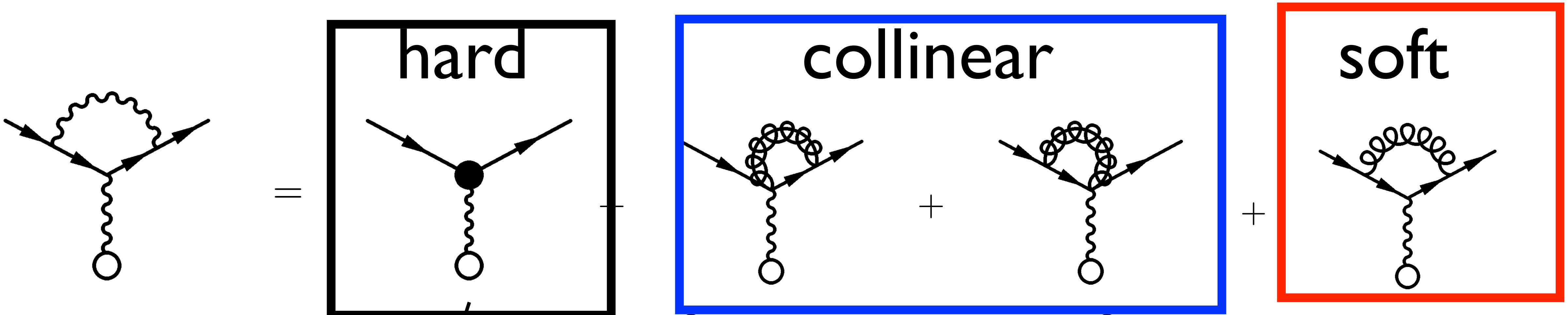
$$d\sigma \sim (\mathrm{hard}) \times (\mathrm{collinear}) \times (\mathrm{soft})$$

- resummation

  - governed by universal anomalous dimensions

Becher, RJH, Lange, Neubert hep-ph/0309227; Becher and Neubert 0903.1126, 0904.1021; Beneke, Falgari and Schwenn 0907.1443, …

- factorization



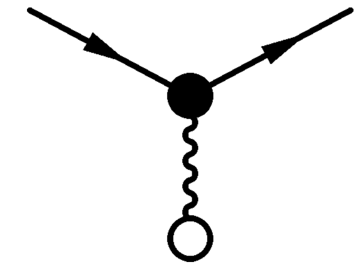
$$d\sigma \propto \boxed{H\left(\frac{Q^2}{\mu^2}\right)}\,\boxed{J\left(\frac{m^2}{\mu^2}\right) R\left(\frac{m^2}{\mu^2},\frac{p\cdot p'}{m^2}\right)}\,\boxed{S\left(\frac{\Delta E}{\mu},\frac{p\cdot p'}{m^2},\frac{E}{m},\frac{E'}{m}\right)}$$

hadron structure
(Born form factors, ...)

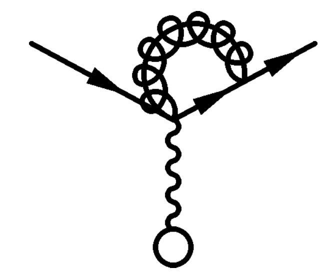
[remainder function starting
at 2-loop (collinear anomaly)]

Sudakov form factor at one loop:

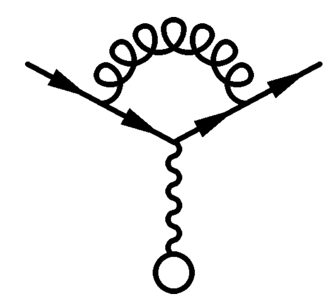
Hard $$F_H(\mu) = 1 + \frac{\alpha}{4\pi}\left[-\log^2\frac{Q^2}{\mu^2} + 3\log\frac{Q^2}{\mu^2} - 8 + \frac{\pi^2}{6}\right]$$

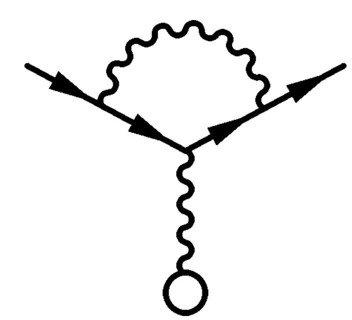


Collinear $$F_J(\mu) = 1 + \frac{\alpha}{4\pi}\left[\log^2\frac{m^2}{\mu^2} - \log\frac{m^2}{\mu^2} + 4 + \frac{\pi^2}{6}\right]$$



Soft $$F_S(\mu) = 1 + \frac{\alpha}{4\pi}\left[2\log\frac{\lambda^2}{\mu^2}\left(\log\frac{Q^2}{m^2} - 1\right)\right]$$





Large logarithms regardless of choice for μ

F_S: exponentiates (evaluate at any scale)

F_J: evaluate at μ~m

F_H: evaluate at μ~M~Q

$$F = F_H F_J F_S$$

Two photon exchange

• Nuclear charge corrections introduce new spin structures (helicity counting: 3 amplitudes at leading power in me/Q)

$$F_H(\mu)\gamma^\mu \otimes \gamma_\mu \to \sum_{i=1}^{3} c_i(\mu)\,\Gamma_i^{(e)} \otimes \Gamma_i^{(p)}$$

• In principle, can use e+ and e- data to separately determine 1-photon exchange and 2-photon exchange contributions to c_i

• However, with available data, measure combination of 1- and 2-photon contributions.

• Regardless of treatment of hard coefficients, remaining radiative corrections are universal

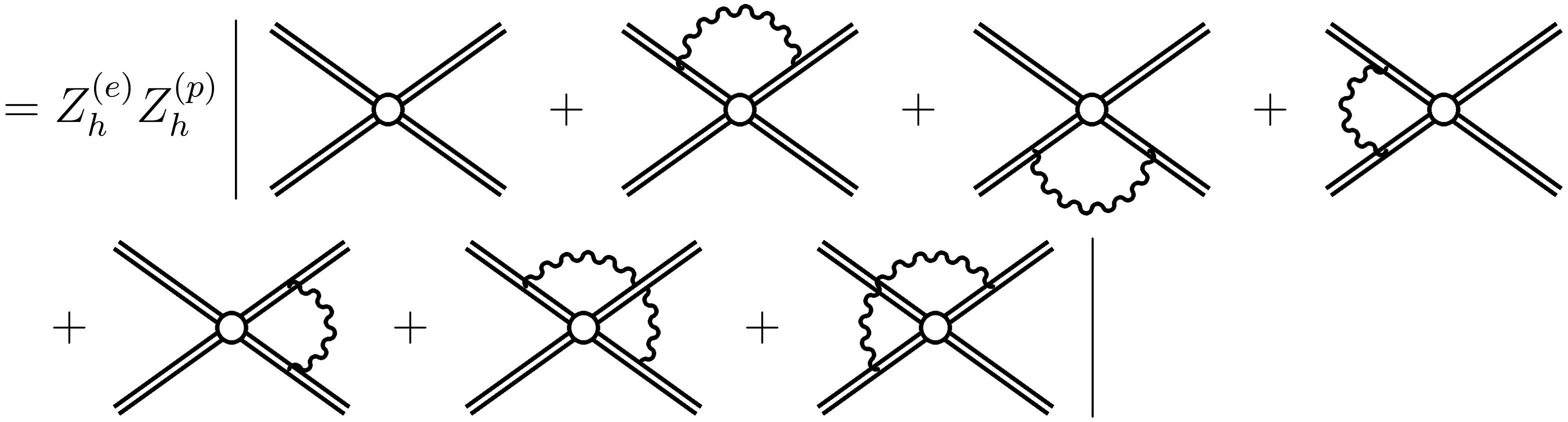
$$d\sigma = H(M) \times \underbrace{\frac{H(\mu)}{H(M)} \times J(\mu) \times S(\mu)}_{\text{correct data by this factor}}$$

want to extract this

- J: refers to collinear region, same as before

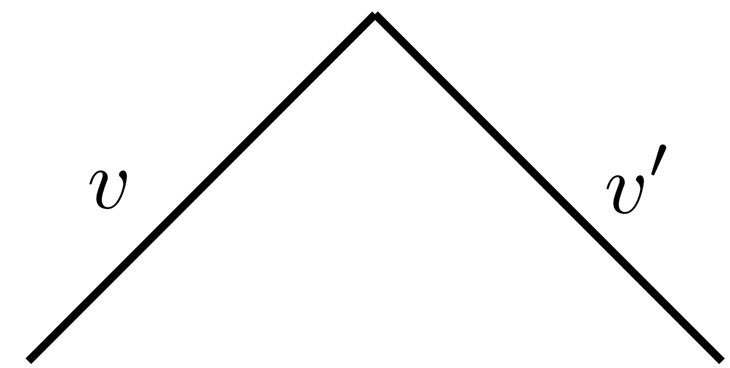
- S: include nuclear charge for general soft function (computed through 2-loop order)

$$\sqrt{S(\mu, \Delta E = 0)} = Z_h^{(e)} Z_h^{(p)}$$



- H(μ)/H(M): must now account for large logs in this factor

- resumation



governed by Wilson loops with cusps:

$$\bar{h} iv \cdot D h \to \bar{h}^{(0)} S_v^\dagger iv \cdot D S_v h^{(0)} = \bar{h}^{(0)} iv \cdot \partial h^{(0)} \,, \quad S_v(x) = P \exp\left[i \int_{-\infty}^0 ds v \cdot A_s(x+sv) \right]$$

renormalization of hard function of interest:

$$\frac{d \log H}{d \log \mu} = 2 \left[\gamma_{\rm cusp}(\bar\alpha) \log \frac{Q^2}{\mu^2} + \gamma_{\rm cusp}(v\cdot v', \bar\alpha) + 2\gamma_{\rm cusp}(\bar\alpha) \log \frac{v\cdot p'}{-v\cdot p - i0} + \gamma(\bar\alpha) \right] .$$

universal functions

$$\text{electron} : \; p^\mu$$
$$\text{proton} : \; M v^\mu$$

solution, summing large logarithms:

$$\log \frac{H(\mu_L)}{H(\mu_H)} = -\frac{\alpha}{2\pi} \log^2 \frac{\mu_H^2}{\mu_L^2} + \dots$$

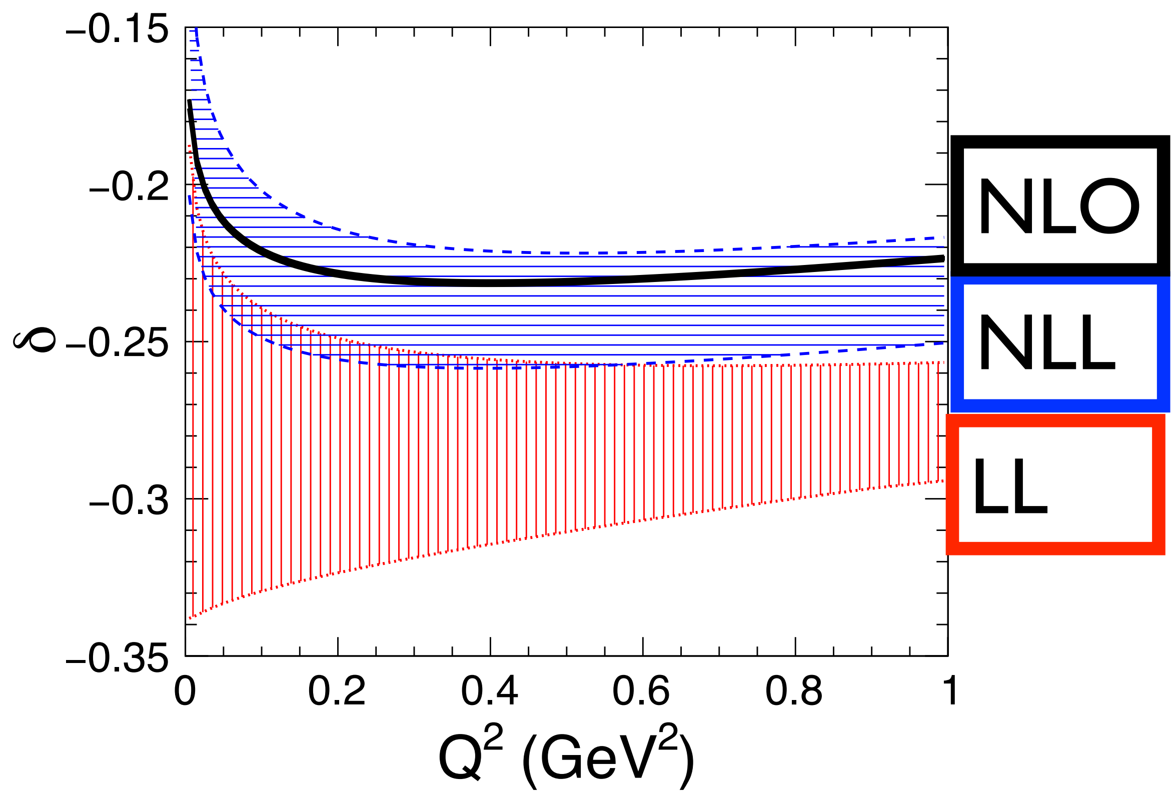
$$d\sigma = H(M) \times \underbrace{\frac{H(\mu)}{H(M)} \times J(\mu) \times S(\mu)}_{\text{total radiative correction}}$$

numerically: $\alpha L^2 = \alpha \log^2 \frac{Q^2}{m^2} \sim 1 \quad \Rightarrow \quad \alpha L \sim \alpha^{\frac{1}{2}}$, etc.

electron energy: $E = 1\,\mathrm{GeV}$

electron energy loss cut: $\Delta E = 5\,\mathrm{MeV}$

total radiative correction



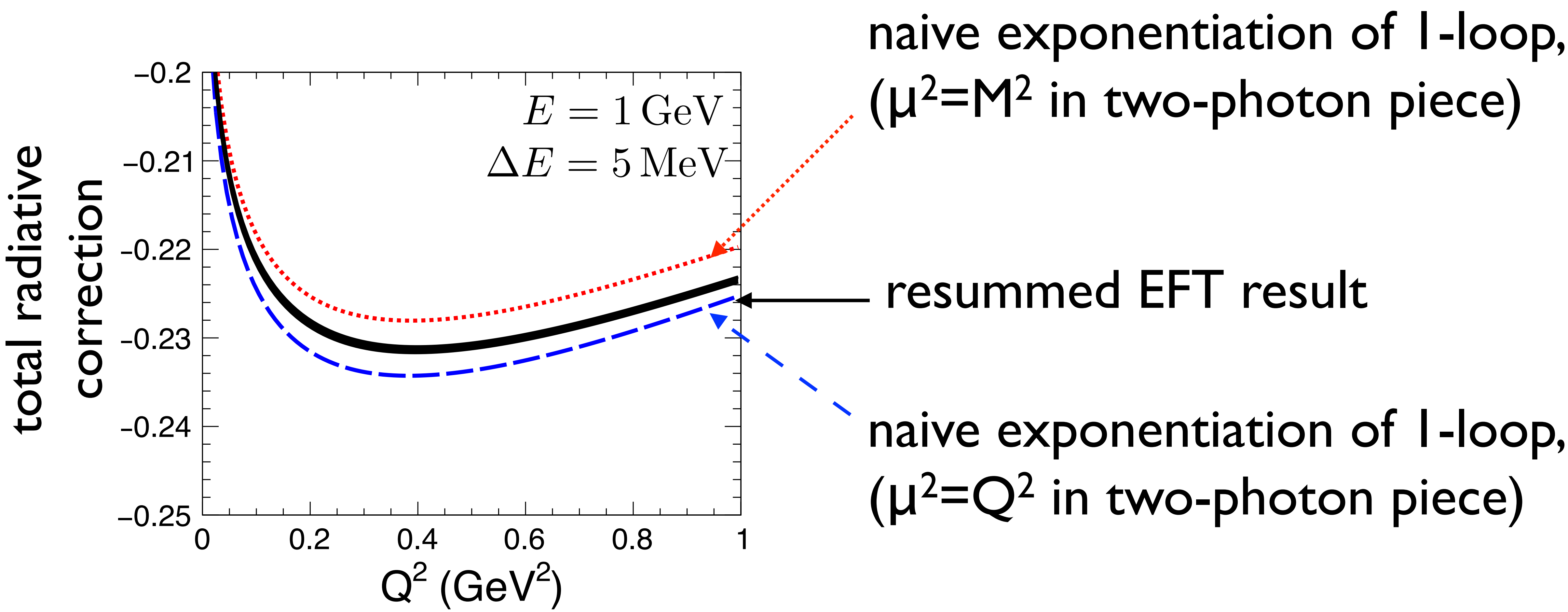
correct through:

NLO: $\mathcal{O}(\alpha)$

NLL: $\mathcal{O}(\alpha^{\frac{1}{2}})$

LL: $\mathcal{O}(1)$

Comparison to previous implementations of radiative corrections, e.g. in A1 analysis of electron-proton scattering data



- discrepancies at 0.5-1% compared to currently applied radiative correction models (cf. 0.2-0.5% systematic error budget of A1)

- conflicting implicit scheme choices for 1PE and 2PE

- complete analysis: account for floating normalizations, correlated shape variations when fitting together with backgrounds

EFT analysis clarifies several issues involving conflicting and/or implicit conventions and scheme choices

1) Scheme choice and definition of radius and "Born" form factors

2) Scheme dependence of two-photon exchange

3) Limitations of naive exponentiation

1) Scheme choice and definition of radius and "Born" form factors

$$\langle J^\mu \rangle = \bar{u}_{v'} \left[\tilde{F}_1 \gamma^\mu + \tilde{F}_2 \frac{i}{2} \sigma^{\mu\nu} (v'_\nu - v_\nu) \right] u_v$$

Massive particle form factor (e.g. for proton):

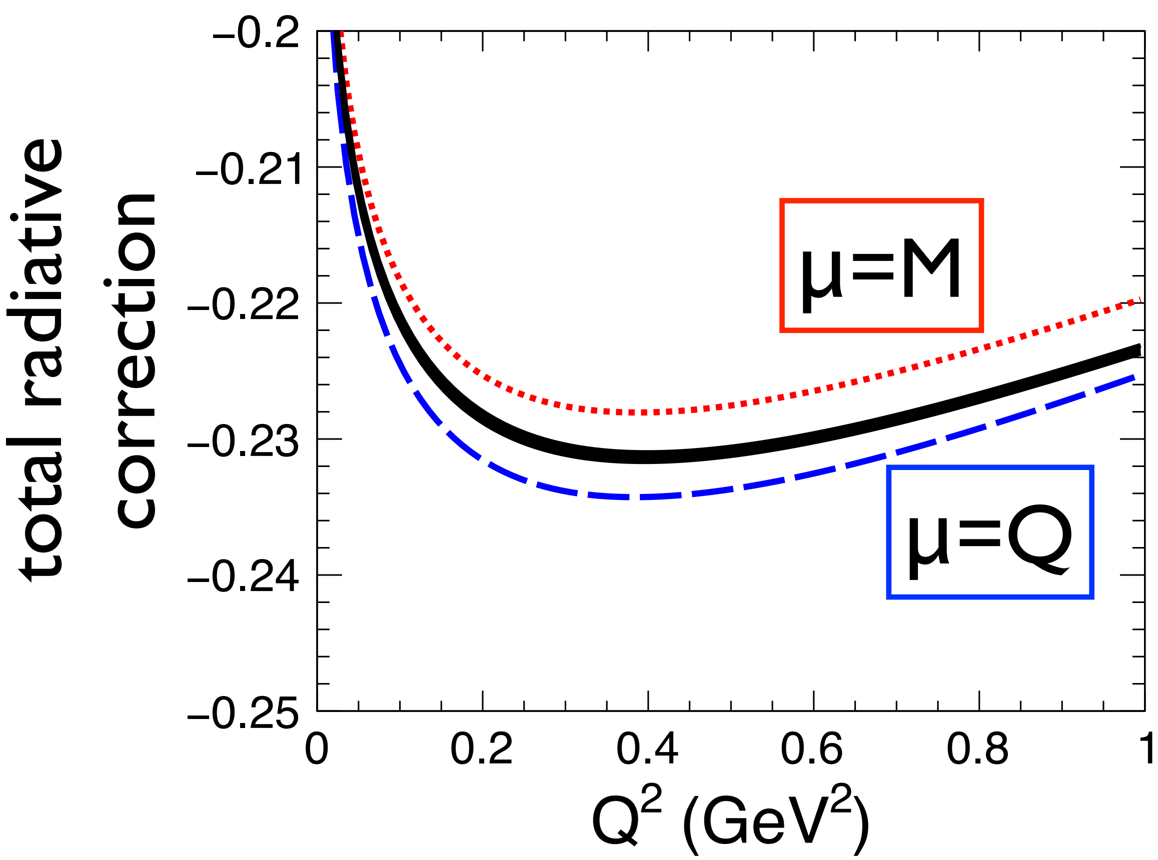
$$\tilde{F}_i = F_H F_S$$

$$F_H(q^2, \mu = M) \equiv \underbrace{F_i(q^2)^{\text{Born}}}_{\text{hard coefficient}} \equiv \tilde{F}_i(q^2) \underbrace{F_S^{-1}(w, \mu = M)}_{\text{soft function}}$$

Multiple conventions in the literature. Different "Born" form factors, different radii (differences typically below current precision)

2) Scheme dependence of two-photon exchange

As for form factors, define hadronic functions in the general 2→2 scattering process as the hard component in the factorization formula at factorization scale μ=M



Prevailing conventions have used conflicting μ=M for 1 photon exchange, μ=Q for 2 photon exchange

A crude estimate of uncertainty in the 2 photon exchange subtraction

3) Limitations of naive exponentiation

- Renormalization analysis for subleading logs :

$$\log \frac{H(\mu_L)}{H(\mu_H)} = -\frac{\alpha}{2\pi} \log^2 \frac{\mu_H^2}{\mu_L^2} + \ldots$$

$\Rightarrow$ New terms at order $\alpha^2 L^3$, $\alpha^2 L^2$, $\alpha^3 L^4$, …

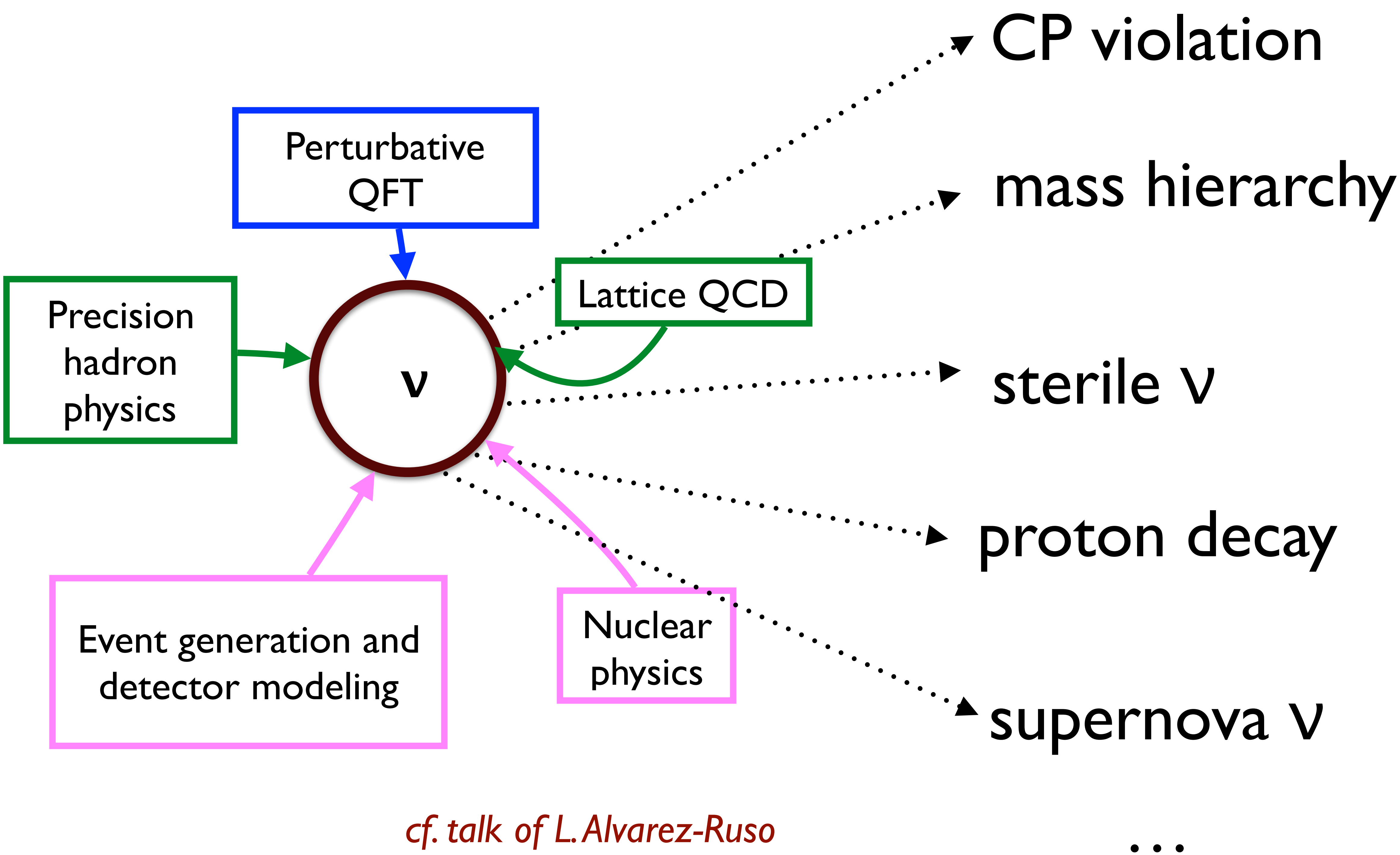
- *Total* versus *individual* real photon energy below ΔE :

$$S^{(2)} = \frac{1}{2!}[S^{(1)}]^2 - \frac{16\pi^2}{3}(L-1)^2 \qquad S = \sum_n \left(\frac{\alpha}{4\pi}\right)^n S^{(n)}$$

$\Rightarrow$ New terms at order $\alpha^2 L^2$

complete analysis: account for floating normalizations, correlated shape variations when fitting together with backgrounds. stay tuned

Broader context: QCD in many regimes critical to extracting fundamental physics in the neutrino sector



cf. talk of L. Alvarez-Ruso

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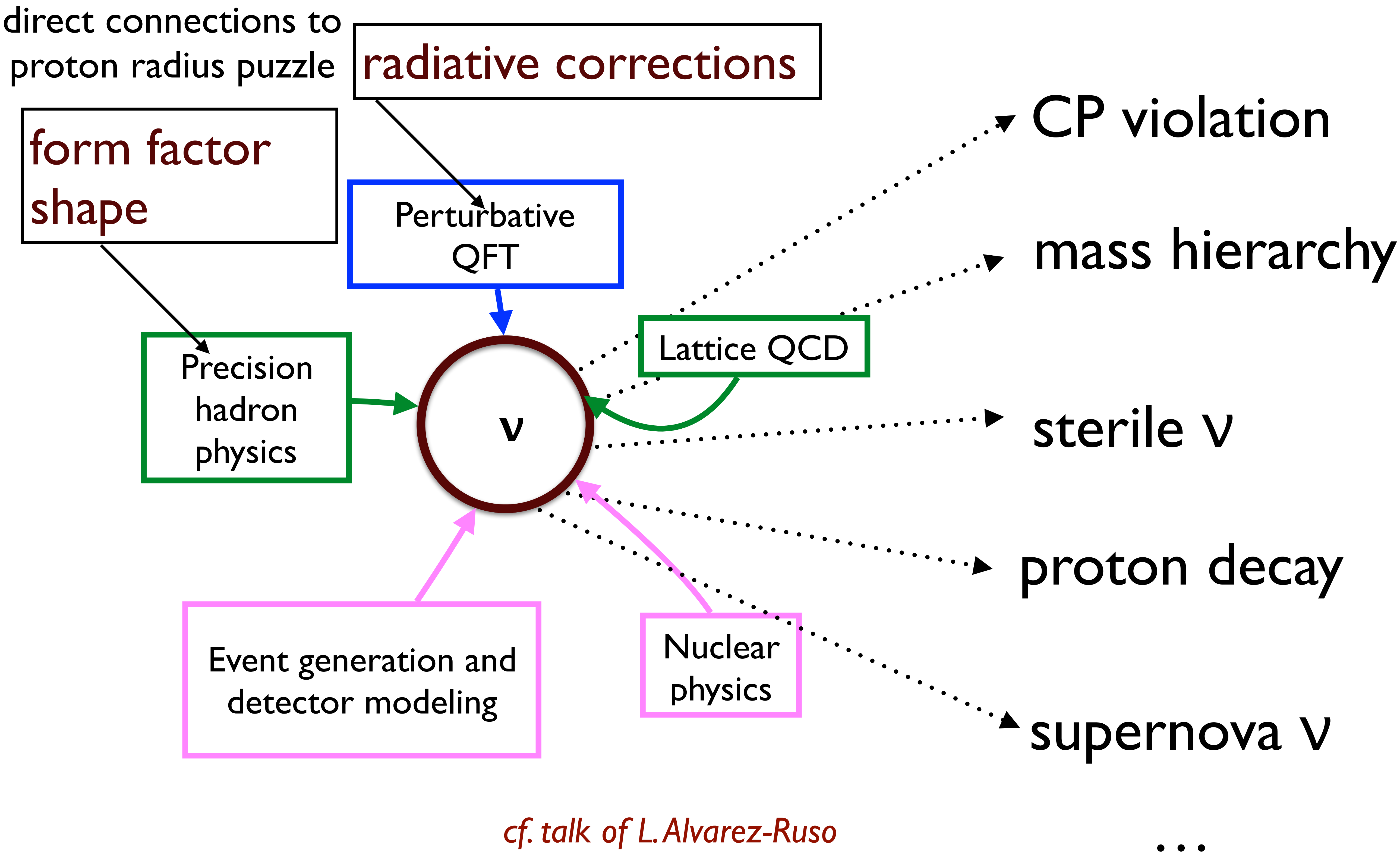


cf. talk of L. Alvarez-Ruso

Broader context: Sudakov logs ubiquitous, appear whenever kinematic invariants large compared to particle masses. Poor convergence, or even breakdown of fixed order perturbation theory

- massive boson production at proton collider	$\alpha_s \log^2 \frac{m_Z^2}{q_T^2}$	$q_T \sim \mathrm{GeV}$
- dark matter annihilation	$\alpha_2 \log^2 \frac{M_{\mathrm{DM}}^2}{m_W^2}$	$M_{\mathrm{DM}} \sim \mathrm{TeV}$
- Lepton-nucleon scattering	$\alpha \log^2 \frac{Q^2}{m_e^2}$	$Q \sim \mathrm{GeV}$

Effective theories differ in detail. For lepton-nucleon scattering: explicit lepton mass, bremsstrahlung energy cut, nuclear recoil and charge corrections

Summary: soft-collinear effective theory for lepton-nucleon scattering

- internal data tensions in electron-proton scattering indicate potential underestimated systematic

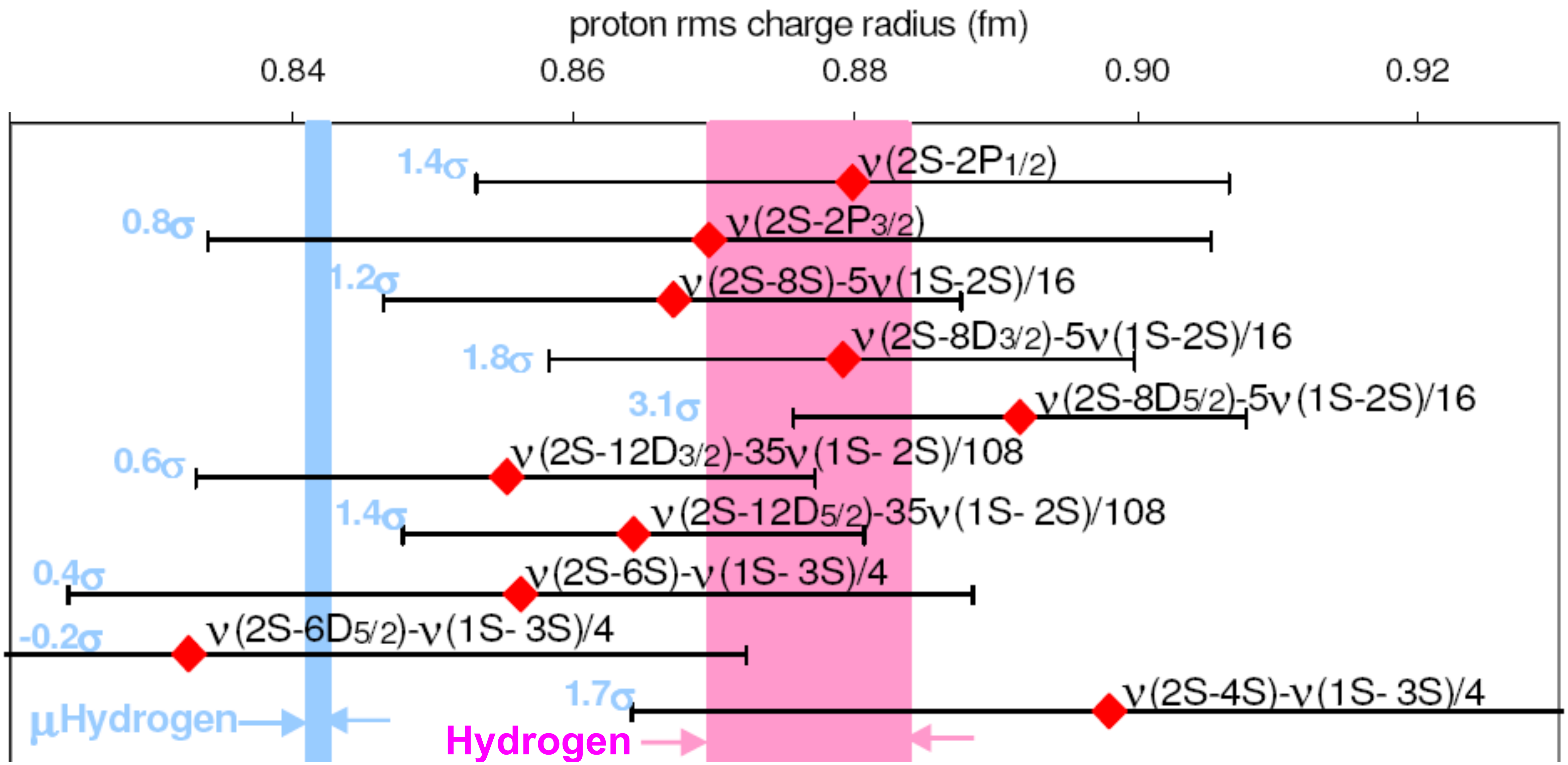
- developed general effective theory for radiative corrections to lepton hadron scattering

- control over large logarithms involving multiple scales

- further work underway to implement with precise experimental conditions, backgrounds and analysis strategy

- related applications: neutrino charged current scattering; $e^+e^- \to$ hadrons for $(g-2)_\mu$; parity-violating electron-proton scattering, connecting lattice amplitudes to experiment, …

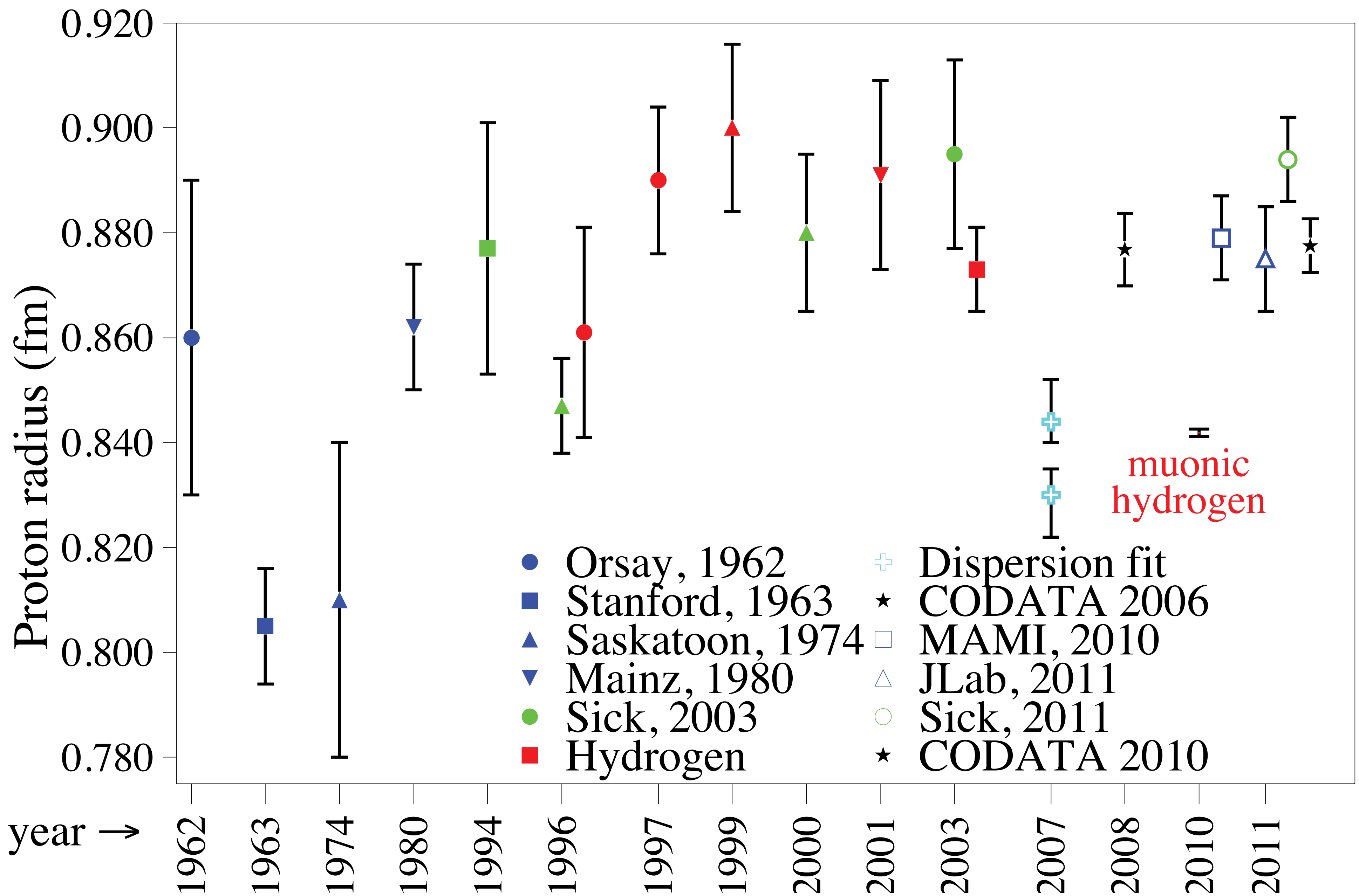
Experimental landscape: hydrogen



From E. Hessels, proton radius workshop 2014

- no straightforward systematic explanation identified, but ~5σ deviation results from summing many ~2σ effects

Experimental landscape: historical e-p extractions



From Pohl et al., Ann.Rev.Nucl.Part.Sci. 63, 175