# Nucleon matrix elements

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**Symposium on Effective Field Theories and Lattice Gauge Theory**

# Outline

1. **Introduction**
   - Current status of simulations
   - Computational cost
2. **Nucleon observables**
   - Nucleon charges: $g_A$, $g_s$, $g_T$
   - Nucleon $\sigma$-terms
   - Electromagnetic form factors
   - Parton Distributions
   - Electric Dipole Moment
3. **Conclusions**

# Quantum ChromoDynamics (QCD)

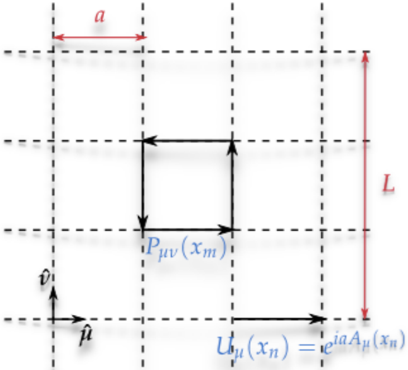
QCD-Gauge theory of the strong interaction

Lagrangian: formulated in terms of quarks and gluons

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f \left( i\gamma^\mu D_\mu - m_f \right) \psi_f$$

$$D_\mu = \partial_\mu - ig\frac{\lambda^a}{2} A^a_\mu$$

Choice of fermion discretisation scheme e.g. Clover, Twisted Mass, Staggered, Overlap, Domain Wall
Each has its advantages and disadvantages




Eventually,

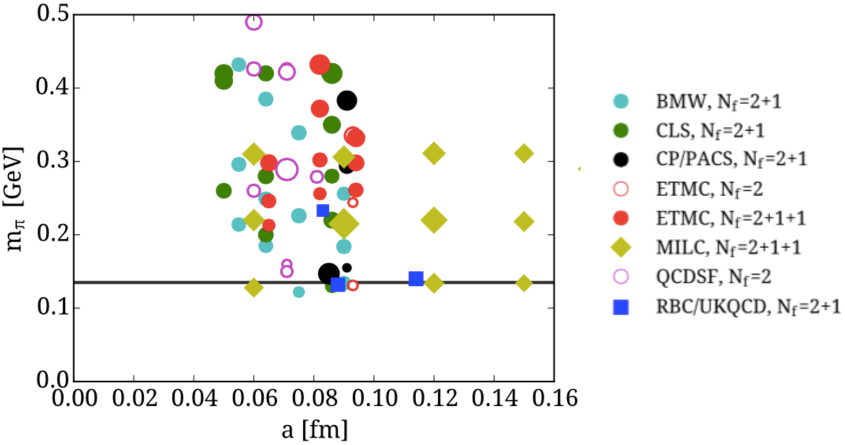
- all discretization schemes must agree in the continuum limit $a \to 0$
- observables extrapolated to the infinite volume limit $L \to \infty$

# Why nucleon structure?

With simulations at the physical value of the pion mass there is a number of interesting questions we want to address:

- Can we reproduce known quantities?
- Can we reproduce the excited spectrum of the nucleon and its associated resonances?
- Can we resolve the long-standing issue of the spin content of the nucleon?
- Can we determine accurately enough the charge radius of the proton?
- Can we provide input for experimental searches for new physics?

## Status of simulations



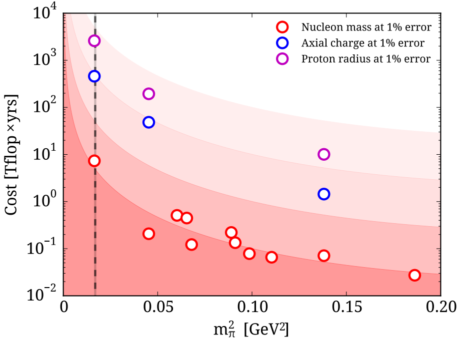

Size of the symbols according to the value of $m_\pi L$: smallest value $m_\pi L \sim 3$ and largest $m_\pi L \sim 6.7$.

In this talk: Show results from an analysis of $N_f = 2$ simulations with twisted mass Wilson fermions including a clover term at physical values of the light quark masses, (ETMC) A. Abdel-Rehim *et al.*, arXiv:1311.4522, arXiv:1507.05068

$\rightarrow$ first results at physical point, (ETMC) A. Abdel-Rehim *et al.*, Phys. Rev. D92 (2015), 114513 , arXiv:1507.04936

# Observables at physical quark mass

For the analysis of the physical ensemble with a volume of $48^3 \times 96$, methods to reduce the statistical error are essential




$\sim e^{(m_p - \frac{3}{2} m_\pi) t_s}$

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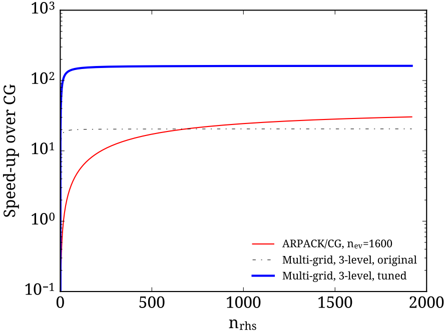
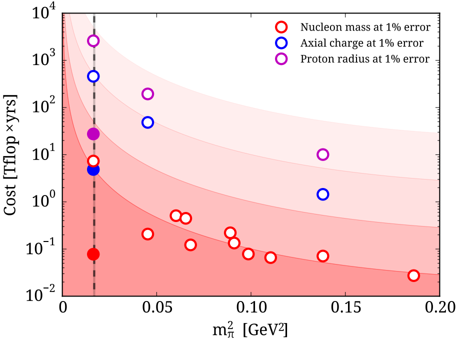



Speed-up of Inversion (for a lattice of $48^3 x64$)

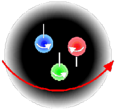
# Observables at physical quark mass

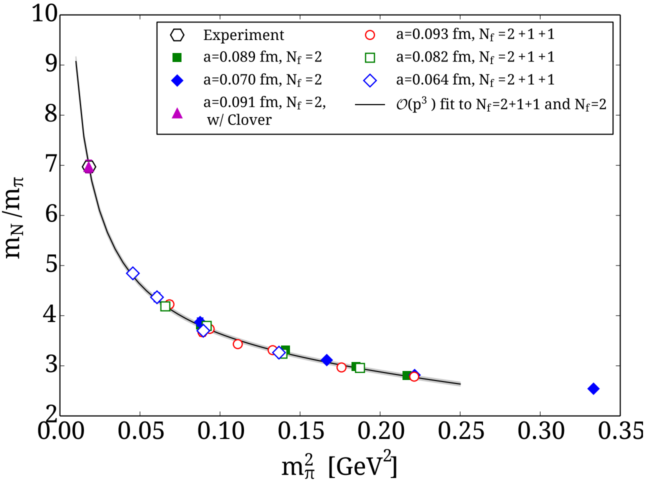
For the analysis of the physical ensemble with a volume of $48^3 \times 96$, methods to reduce the statistical error are essential




Speed-up of Inversion with multi-grid

# The nucleon



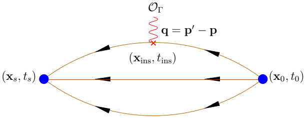



- Cut-off effects small for these lattice spacings
- LO fit with $m_\pi < 375$ MeV does not include the physical point
- Determine lattice spacing using the $\mathcal{O}(p^3)$ result

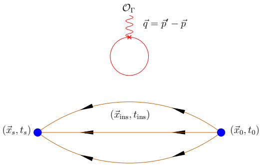
# Evaluation of matrix elements

Three-point functions:

$$G^{\mu\nu}(\Gamma,\vec{q},t_s,t_{\rm ins}) = \sum_{\vec{x}_s,\vec{x}_{\rm ins}} e^{i\vec{x}_{\rm ins}\cdot\vec{q}}\,\Gamma_{\beta\alpha}\,\langle J_\alpha(\vec{x}_s,t_s)\mathcal{O}^{\mu\nu}_\Gamma(\vec{x}_{\rm ins},t_{\rm ins})\bar{J}_\beta(\vec{x}_0,t_0)\rangle$$

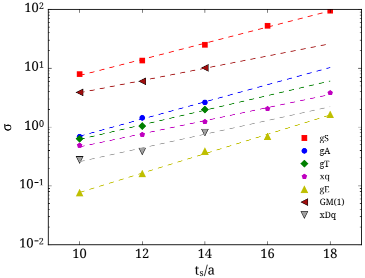



connected contribution




disconnected contribution

$$R(t_s,t_{\rm ins},t_0)\ \xrightarrow[(t_s-t_{\rm ins})\Delta\gg1]{(t_{\rm ins}-t_0)\Delta\gg1}\ \mathcal{M}[1+\ldots e^{-\Delta(\mathbf{p})(t_{\rm ins}-t_0)}+\ldots e^{-\Delta(\mathbf{p}')(t_s-t_{\rm ins})}]$$




- $\mathcal{M}$ the desired matrix element
- $t_s$, $t_{\rm ins}$, $t_0$ the sink, insertion and source time-slices
- $\Delta(\mathbf{p})$ the energy gap with the first excited state

To ensure ground state dominance need multiple sink-source time separations ranging from 0.9 fm to 1.5 fm

# Extracting nucleon matrix elements

- Plateau method:

$$R(t_s, t_{\rm ins}, t_0) \xrightarrow[(t_s - t_{\rm ins})\Delta \gg 1]{(t_{\rm ins} - t_0)\Delta \gg 1} \mathcal{M}[1 + \ldots e^{-\Delta(\mathbf{p})(t_{\rm ins} - t_0)} + \ldots e^{-\Delta(\mathbf{p}')(t_s - t_{\rm ins})}]$$

  - $\mathcal{M}$ the desired matrix element
  - $t_s, t_{\rm ins}, t_0$ the sink, insertion and source time-slices
  - $\Delta(\mathbf{p})$ the energy gap with the first excited state

- Excited states contributions are different for different operators and pion mass $\to$ need to carefully check
- Need to include disconnected contributions unless shown to be negligible

- Summation method: Summing over $t_{\rm ins}$:

$$\sum_{t_{\rm ins}=t_0}^{t_s} R(t_s, t_{\rm ins}, t_0) = \text{Const.} + \mathcal{M}[(t_s - t_0) + \mathcal{O}(e^{-\Delta(\mathbf{p})(t_s - t_0)}) + \mathcal{O}(e^{-\Delta(\mathbf{p}')(t_s - t_0)})].$$

  Excited state contributions are suppressed by exponentials decaying with $t_s - t_0$, rather than $t_s - t_{\rm ins}$ and/or $t_{\rm ins} - t_0$
  However, one needs to fit the slope rather than to a constant or take differences and then fit to a constant
  L. Maiani, G. Martinelli, M. L. Paciello, and B. Taglienti, Nucl. Phys. B293, 420 (1987); S. Capitani *et al.*, arXiv:1205.0180

- Fit keeping the first excited state, T. Bhattacharya *et al.*, arXiv:1306.5435

All should yield the same answer in the end of the day!

# Nucleon charges: $g_A$, $g_S$, $g_T$

- axial-vector operator: $\mathcal{O}_A^a = \bar{\psi}(x)\gamma^\mu\gamma_5\frac{\tau^a}{2}\psi(x)$
- scalar operator: $\mathcal{O}_S^a = \bar{\psi}(x)\frac{\tau^a}{2}\psi(x)$
- tensor operator: $\mathcal{O}_T^a = \bar{\psi}(x)\sigma^{\mu\nu}\frac{\tau^a}{2}\psi(x)$

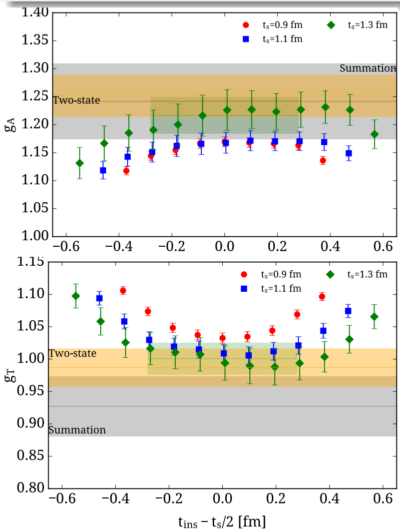
$\Longrightarrow$ extract from ratio: $\langle N(\vec{p}')\mathcal{O}_X N(\vec{p})\rangle|_{q^2=0}$

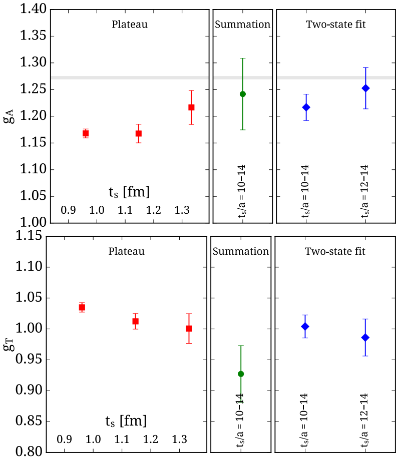
• Axial charge $g_A$ • Scalar charge $g_S$ • Tensor charge $g_T$

(i) isovector combination has no disconnect contributions; (ii) $g_A$ well known experimentally, $g_T$ to be measured at JLab

# Nucleon charges: $g_A$, $g_S$, $g_T$

- $N_f = 2$ TMF with clover term $a = 0.093(2)$ fm with $m_\pi = 133$ MeV; Connected part with $\sim 6800$ statistics
First results with 1500 statistics, A. Abdel-Rehim et al. Phys.Rev. D92 (2015) no.11, 114513, Erratum: Phys.Rev. D93 (2016) no.3, 039904

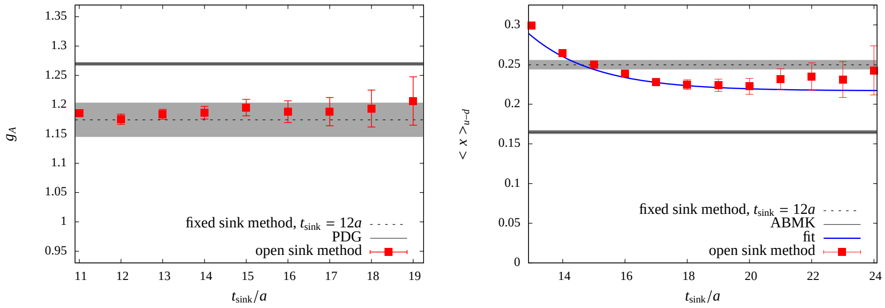


# Nucleon axial charge $g_A$

A previous study using $N_F = 2+1+1$ TM with $m_\pi = 373$ MeV and $a = 0.08$ fm

Vary source- sink separation:




$g_A$ unaffected, $\langle x \rangle_{u-d}$ 10% lower

$\Longrightarrow$ Excited contributions are operator dependent

S. Dinter, C.A., M. Constantinou, V. Drach, K. Jansen and D. Renner, arXiv: 1108.1076

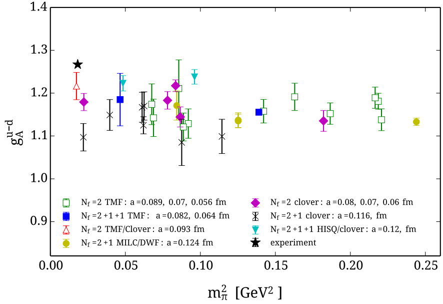
# Nucleon Axial-vector charge $g_A$

Comparison of lattice results

Axial-vector FFs: $A^3_\mu = \bar{\psi}\gamma_\mu\gamma_5\frac{\tau^3}{2}\psi(x) \Longrightarrow \frac{1}{2}\bar{u}_N(\vec{p}')\left[\gamma_\mu\gamma_5 G_A(q^2) + \frac{q^\mu\gamma_5}{2m}G_p(q^2)\right]u_N(\vec{p})|_{q^2=0}$

$\rightarrow$ yields $G_A(0) \equiv g_A$: i) well known experimentally, & ii) no quark loop contributions

Isovector

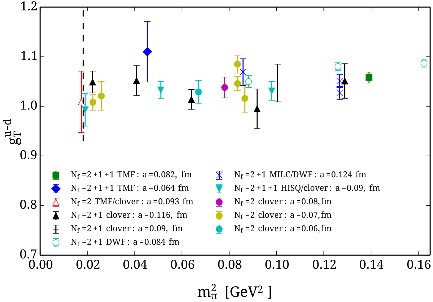



- $g_A$ at the physical point using $\sim$ 6800 at sink-source time separation 1.3 fm slightly below the physical value $\rightarrow$ important to increase sink-source time separation at constant statistical error but also volume effects should also be checked - compute $g_A$ on a $64^3 \times 128$ volume at the same light quark masses
- many results from other collaborations, e.g.
  - $N_f = 2 + 1$ Clover, J. R. Green *et al.*, arXiv:1209.1687
  - $N_f = 2$ Clover, R.Hosley *et al.*, arXiv:1302.2233
  - $N_f = 2$ Clover, S. Capitani *et al.* arXiv:1205.0180
  - $N_f = 2 + 1$ Clover, B. J. Owen *et al.*, arXiv:1212.4668
  - $N_f = 2 + 1 + 1$ Mixed action (HISQ/Clover), T. Bhattacharya *et al.*, arXiv:1306.5435

# Nucleon tensor $g_T$, $g_S$

- scalar operator: $\mathcal{O}_S^a = \bar{\psi}(x)\frac{\tau^a}{2}\psi(x)$
- tensor operator: $\mathcal{O}_T^a = \bar{\psi}(x)\sigma^{\mu\nu}\frac{\tau^a}{2}\psi(x)$

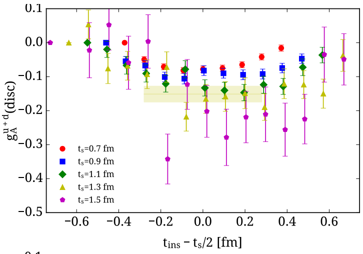
Isovector

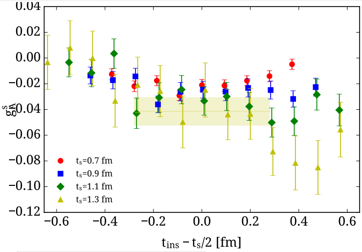


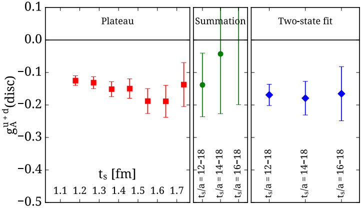

- Experimental value of $g_T^{u-d} \sim 0.64^{+0.30}_{-0.13}$ from global analysis of HERMES, COMPASS and Belle $e^+e^-$ data, M. Anselmino *et al.* (2013).
  New analysis of COMPASS and Belle data :$g_T^{u-d} = 0.81(44)$, M. R. A. Courtoy, A. Bacchettad, M. Guagnellia, arXiv: 1503.03495
- $g_s^{u-d}$ is very noisy. Currently $g_S^{u-d} \sim 1 \pm 0.5$

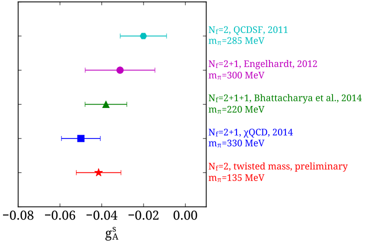
# Nucleon charges: $g_A^q$, $g_T^q$, $g_S^q$

Include disconnected contributions, $\sim 202,000$ statistics



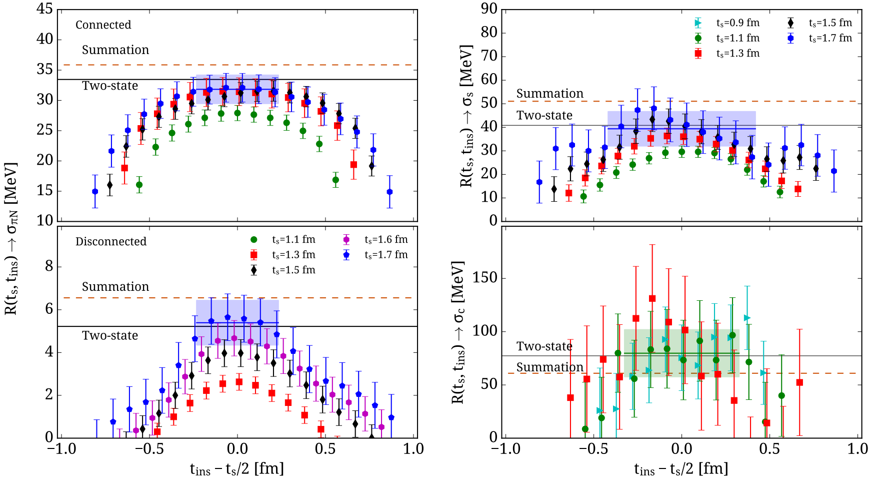




- Disconnected contribution to $g_A^{u+d}$ cannot be neglected, $\sim$ 10% of connected
- Disconnected contribution to $g_T^{u+d}$ negligible
- Disconnected contribution to $g_S^{u+d}$ are $\sim$ 25% of the connected.

# The quark content of the nucleon or nucleon $\sigma$-terms

- $\sigma_f \equiv m_f \langle N|\bar{q}_f q_f|N\rangle$: measures the explicit breaking of chiral symmetry
  Largest uncertainty in interpreting experiments for dark matter searches - Higgs-nucleon coupling depends on $\sigma$, J. Ellis, K. Olive, C. Savage, arXiv:0801.3656
- In lattice QCD:
  i) Feynman-Hellmann theorem: $\sigma_l = m_l \frac{\partial m_N}{\partial m_l}$
  Similarly $\sigma_s = m_s \frac{\partial m_N}{\partial m_s}$
  ii) Direct computation of the scalar matrix element, A. Abdel-Rehim *et al.* arXiv:1601.3656



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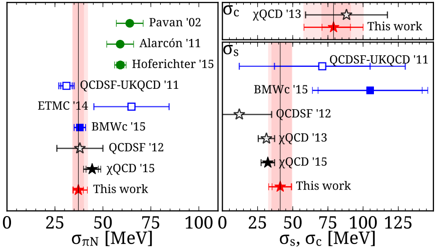
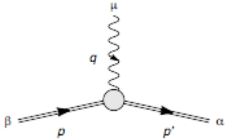
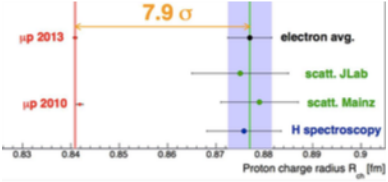


# Electromagnetic form factors

$$\langle N(p',s')|j^\mu(0)|N(p,s)\rangle = \bar{u}_N(p',s')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u_N(p,s)$$





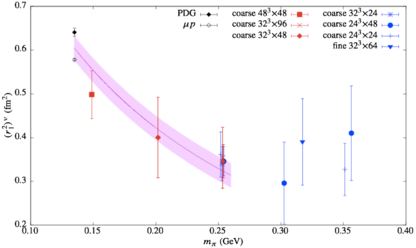


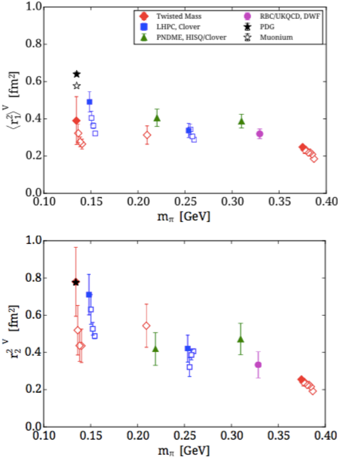

- Proton radius extracted from muonic hydrogen is 7.7 $\sigma$ different from the one extracted from electron scattering, R. Pohl *et al.*, Nature 466 (2010) 213
- Muonic measurement is ten times more accurate

# Dirac and Pauli radii

Dipole fits: $\frac{G_0}{(1+Q^2/M^2)^2} \Rightarrow \langle r_i^2 \rangle = -\frac{6}{F_i}\frac{dF_i}{dQ^2}|_{Q^2=0} = \frac{12}{M_i^2}$

Need better accuracy at the physical point




Using results from summation method, J. M. Green *et al.*, 1404.4029

# Position methods

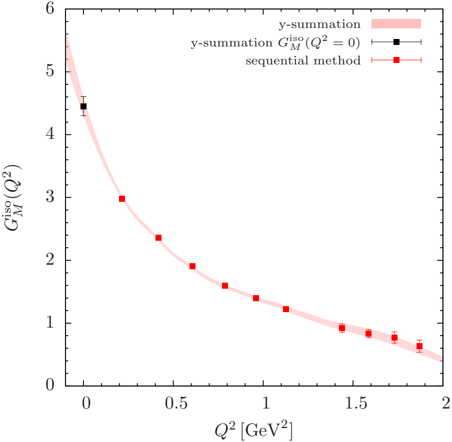
- Avoid model dependence-fits
- Application to Sachs form factors $\rightarrow$ nucleon isovector magnetic moment $G_M^{\rm iso}(0)$
- Isovector rms charge radius of the nucleon
- Neutron electric dipole moment

As a first step we calculated $G_M(0)$ (equivalently $F_2(0)$) at $m_\pi = 373$ MeV.

C.A., G. Koutsou, K. Ottnad, M. Petschlies, PoS(Lattice2014), 144

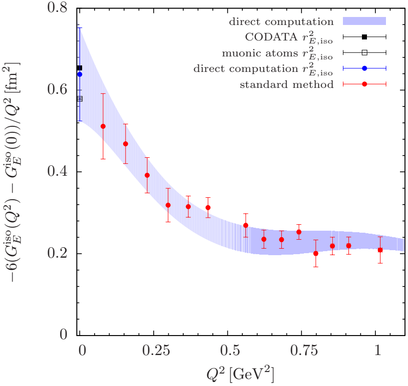
# Magnetic moment $G_M^{\text{iso}}(0)$

- In principle, values at larger $Q^2$ have very little influence
- Value for $G_M^{\text{iso}} = 4.45(15)_{\text{stat}}$ larger than result from dipole fit $3.99(9)_{\text{stat}}$
- Closer to exp. value (4.71)




$G_M^{\text{iso}}(0)$ from $\mathcal{O}(4700)$ gauge confs of B55; $t_s/a = 14$

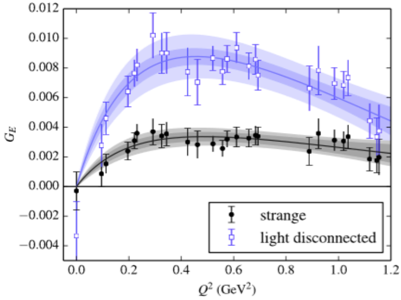
# Results for $r_E^{\mathrm{iso}}$

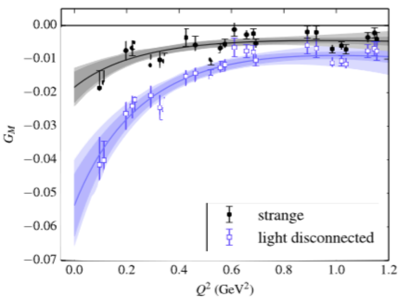



- We use an ETMC $48^3 \times 96$, $N_f = 2$ ensemble with **physical pion mass**
- Data shown in plot are for $O(1400)$ confs
- $t_s/a = 14$ **compatible with experiment!**
- Unfortunately errors are still not small enough to distinguish the two experimental values

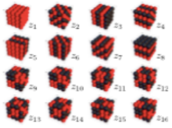
# Strange Electromagnetic form factors

New methods for disconnected fermion loops: hierarchical probing, A. Stathopoulos, J. Laeuchli, K. Orginos, arXiv:1302.4018




$N_f = 2 + 1$ clover fermions, $m_\pi \sim 320$ MeV, J. Green et al., Phys.Rev. D92 (2015) 3, 031501, arXiv: 1505.01803



Sampling of the fermion propagator using site colouring schemes

# Moments of Generalized Parton Distributions

Factorization leads to matrix elements of local operators:

- vector operator

$$\mathcal{O}^{\mu_1\cdots\mu_n}_{V^a} = \bar{\psi}(x)\gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2} \dots i \overleftrightarrow{D}^{\mu_n\}} \frac{\tau^a}{2}\psi(x)$$

- axial-vector operator

$$\mathcal{O}^{\mu_1\cdots\mu_n}_{A^a} = \bar{\psi}(x)\gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2} \dots i \overleftrightarrow{D}^{\mu_n\}} \gamma_5 \frac{\tau^a}{2}\psi(x)$$

- tensor operator

$$\mathcal{O}^{\mu_1\cdots\mu_n}_{T^a} = \bar{\psi}(x)\sigma^{\{\mu_1,\mu_2} i \overleftrightarrow{D}^{\mu_3} \dots i \overleftrightarrow{D}^{\mu_n\}} \frac{\tau^a}{2}\psi(x)$$

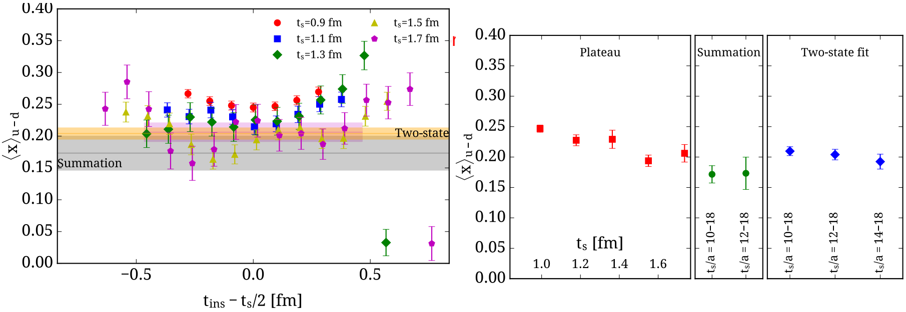
Special cases:

- no-derivative $\rightarrow$ nucleon form factors
- For $Q^2 = 0 \rightarrow$ parton distribution functions
  one-derivative $\rightarrow$ first moments e.g. average momentum fraction $\langle x \rangle$
  Generalized form factor decomposition:

$$\langle N(p',s')|\mathcal{O}^{\mu\nu}_{V3}|N(p,s)\rangle = \bar{u}_N(p',s')\left[A_{20}(q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m}\right]\frac{1}{2}u_N(p,s)$$

Nucleon spin $J^q = \frac{1}{2}\left[A_{20}(0) + B_{20}(0)\right]$ and $\langle x \rangle_q = A_{20}(0)$

# Momentum fraction and the nucleon spin




- $\langle x \rangle_{u-d}$ approach physical value for bigger source-sink separations $\rightarrow$ need an equivalent high statistics study e.g. $t_s \sim 1.5$ fm we used $\sim 48000$ statistics
- Can provide a prediction for $\langle x \rangle_{\delta u - \delta d}$

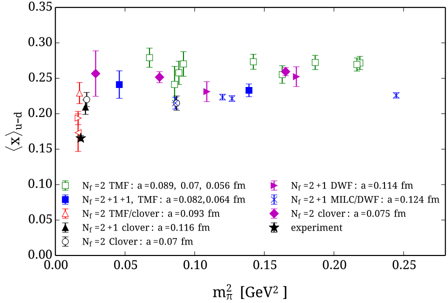
Experimental value:

- $\langle x \rangle_{u-d}$ from S. Alekhin *et al.* arXiv:1202.2281

# Momentum fraction and the nucleon spin

What is the distribution of the nucleon momentum among the nucleon constituents?

$\langle x \rangle$ obtained in the $\overline{MS}$ scheme at $\mu = 2$ GeV.




Near the physical point we show results from:

- $N_f = 2$ twisted mass plus clover-impoved from ETMC fermions, A. Abdel-Rehim *et al.* 1507.05068
- $N_f = 2+1$ clover fermions with 2-HEX smearing from LHPC, J. Green *et al.*, 1209.1687
- $N_f = 2$ clover fermions, G. Bali *et al.*, 1408.6850
- $N_f = 2$ clover fermions from QCDSF/UKQCD, D. Pleiter *et al.*, 1101.2326

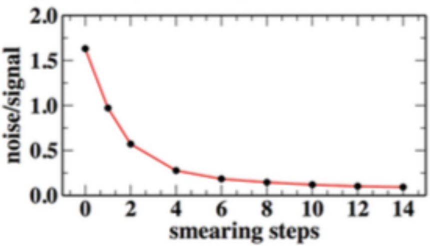
- $\langle x \rangle_{u-d}$ approach physical value for bigger source-sink separations $\rightarrow$ need an equivalent high statistics study e.g. $t_s \sim 1.5$ fm we used $\sim 48000$ statistics
- Can provide a prediction for $\langle x \rangle_{\delta u - \delta d}$

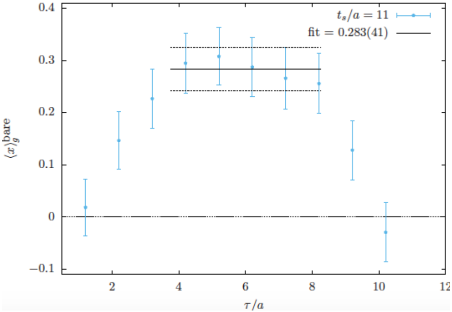
Experimental value:

- $\langle x \rangle_{u-d}$ from S. Alekhin *et al.* arXiv:1202.2281

# Nucleon gluon moment

- $N_f = 2+1+1$ twisted mass, $a$ = 0.082 fm, $m_\pi$ = 373 MeV, $\sim$ 34,470 statistics
- $N_f = 2$ twisted mass plus clover, $a = 0.093$ fm, $m_\pi = 132$ MeV, $\sim$155,800 statistics



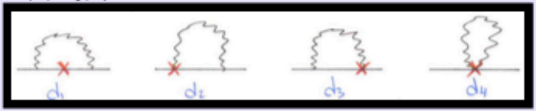




- Matrix element of the gluon operator $O_{\mu\nu} = -\mathrm{Tr}[G_{\mu\rho}G_{\nu\rho}]$
- We consider $\langle N|O_{44} - \frac{1}{3}O_{jj}|N\rangle$ at zero momentum, which yields directly $\langle x\rangle_g$
- HYP-smearing to reduce noise
- Perturbative renormalization

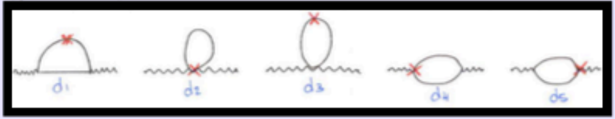
# Nucleon gluon moment-Renormalization

Mixing with $\langle x \rangle_{u+d} \Longrightarrow$ Perturbation theory - M. Constantinou and H. Panagopoulos

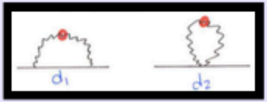
$\times Z_{qq}$ : $\Lambda_{qq} = \langle q | \mathcal{O}_q | q \rangle$



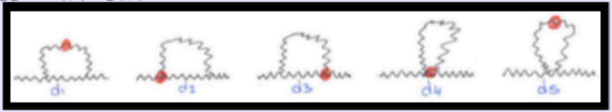

$\times Z_{qg}$ : $\Lambda_{qg} = \langle g | \mathcal{O}_q | g \rangle$




$\bullet Z_{gq}$ : $\Lambda_{gq} = \langle q | \mathcal{O}_g | q \rangle$




$\bullet Z_{gg}$ : $\Lambda_{gg} = \langle g | \mathcal{O}_g | g \rangle$



## Nucleon gluon moment-Renormalization

Mixing with $\langle x\rangle_{u+d} \Longrightarrow$ Perturbation theory - M. Constantinou and H. Panagopoulos

× $Z_{qq}$ : $\Lambda_{qq} = \langle q|\mathcal{O}_q|q\rangle$

$$\boxed{Z_{gg}=1+\frac{g^2}{16\pi^2}\left(1.0574\, N_f + \frac{-13.5627}{N_c} - \frac{2\,N_f}{3}\log(a^2\bar{\mu}^2)\right)}$$

× $Z_{qg}$ : $\Lambda_{qg} = \langle g|\mathcal{O}_q|g\rangle$

$$\boxed{Z_{gq}=0+\frac{g^2\,C_f}{16\pi^2}\left(0.8114 + 0.4434\, c_{SW} - 0.2074\, c_{SW}^2 + \frac{4}{3}\log(a^2\bar{\mu}^2)\right)}$$

• $Z_{gq}$ : $\Lambda_{gq} = \langle q|\mathcal{O}_g|q\rangle$

$$\boxed{Z_{qq}=1+\frac{g^2}{16\pi^2}\left(-1.8557 + 2.9582\, c_{SW} + 0.3984\, c_{SW}^2 - \frac{8}{3}\log(a^2\bar{\mu}^2)\right)}$$

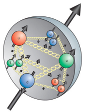
• $Z_{gg}$ : $\Lambda_{gg} = \langle g|\mathcal{O}_g|g\rangle$

$$\boxed{Z_{qg}=0+\frac{g^2\,N_f}{16\pi^2}\left(0.2164 + 0.4511\, c_{SW} + 1.4917\, c_{SW}^2 - \frac{4}{3}\log(a^2\bar{\mu}^2)\right)}$$

- Preliminary value: $\langle x\rangle_g = 0.282(39)$ for the physical ensemble in $\overline{\rm MS}$ at $\mu = 2$ GeV
- Momentum conservation: $\sum_q \langle x\rangle_q + \langle x\rangle_g = \langle x\rangle^{CI}_{u+d} + \langle x\rangle^{DI}_{u+d+s} + \langle x\rangle_g = 0.929(64)$

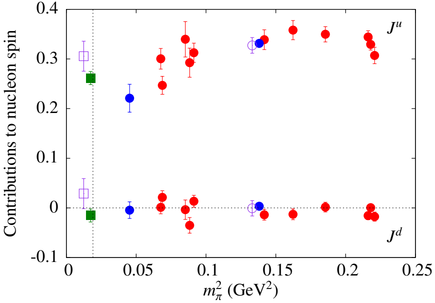
# Nucleon spin?

Spin sum: $\frac{1}{2} = \sum_q \underbrace{\left(\frac{1}{2}\Delta\Sigma^q + L^q\right)}_{J^q} + J^G$



$J^q = A^q_{20}(0) + B^q_{20}(0)$ and $\Delta\Sigma^q = g^q_A$

Disconnected contribution using $\mathcal{O}(150,000)$ statistics for $m_\pi = 373$ MeV and for $m_\pi = 133$ MeV

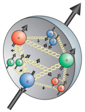



$\Longrightarrow$ Total spin for u-quarks $J^u \lesssim 0.25$ and for d-quark $J^d \sim 0$

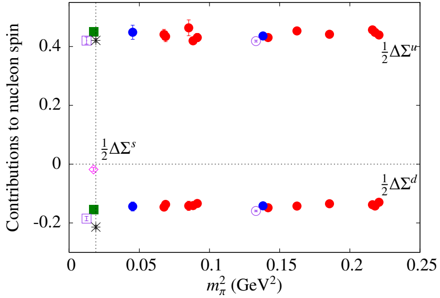
# Nucleon spin?

Spin sum: $\frac{1}{2} = \sum_q \underbrace{\left(\frac{1}{2}\Delta\Sigma^q + L^q\right)}_{J^q} + J^G$

$J^q = A_{20}^q(0) + B_{20}^q(0)$ and $\Delta\Sigma^q = g_A^q$



Disconnected contribution using $\mathcal{O}(150,000)$ statistics for $m_\pi = 373$ MeV and for $m_\pi = 133$ MeV




- $\Delta\Sigma^{u,d}$ consistent with experimental values

# Direct evaluation of parton distribution functions - an exploratory study

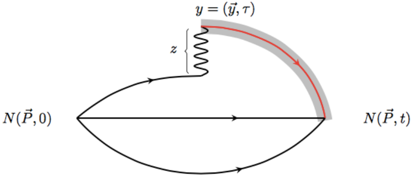
$$\tilde{a}_n(x, \Lambda, P_3) = \int_{-\infty}^{+\infty} dx\, x^{n-1}\, \tilde{q}(x, \Lambda, P_3)\rangle,$$

$$\tilde{q}(x, \Lambda, P_3) = \int_{-\infty}^{+\infty} \frac{dz}{4\pi} e^{-izxP_3} \underbrace{\langle P|\bar{\psi}(z,0)\rangle\gamma_3\, W(z)\psi(0,0)|P\rangle}_{h(P_3,z) \to canbecomputedinLQCD}$$

is the quasi-distribution defined by X. Ji Phys.Rev.Lett. 110 (2013) 262002, arXiv:1305.1539
Exploratory calculations:

- Huey-Wen Lin *et al.* Phys. Rev. D91 (2015) 054510, Clover on $N_f = 2 + 1 + 1$ HISQ, $m_\pi = 310 MeV$ and Jiunn-Wei Chen *et al.*, arXiv:1603.06664
- C.A., K. Cichy, E. G. Ramos, V. Drach, K. Hadjiyiannakou, K. Jansen, F. Steffens, C. Wiese, Phys.Rev. D92 (2015) 014502

- We used the $N_f = 2 + 1 + 1$ TMF ensemble with $m_\pi = 373$ MeV
- Perform HYP-smearing on the gauge links
- Use the stochastic all-to-all propagator in the three-point function
- Extract quasi-distribution for $\frac{2\pi}{L}$, $\frac{4\pi}{L}$, $\frac{6\pi}{L}$



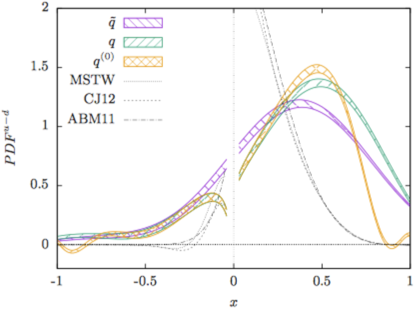
# Comments

Our starting point is

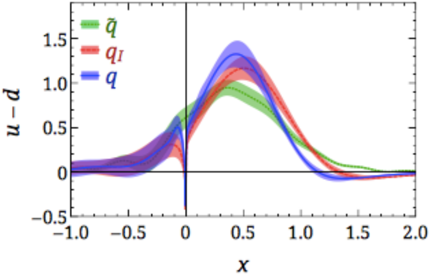
$$q(x,\mu) = \tilde{q}(x,\Lambda,P_3) - \frac{\alpha_s}{2\pi}\tilde{q}(x,\Lambda,P_3)\delta Z_F^{(1)}\left(\frac{\mu}{P_3},\frac{\Lambda}{P_3}\right) - \frac{\alpha_s}{2\pi}\int_{-1}^{1}\frac{dy}{y}Z^{(1)}\left(\frac{x}{y},\frac{\mu}{P_3},\frac{\Lambda}{P_3}\right)\tilde{q}(y,\Lambda,P_3) + \mathcal{O}(\alpha_s^2)$$

- The calculation of the leading UV divergences in $\tilde{q}$ in PT are done keeping $P_3$ fixed while taking $\Lambda \to \infty$ (in contrast to first taking $P_3 \to \infty$ for the renormalization of $q$)
- We still do not have a renormalization procedure
  $\to$ identify the UV regulator as $\mu$ for $q$ and as $\Lambda$ for the case of the quasi-distribution.
- The dependence on the UV regulator $\Lambda$ will be translated, in the end, into a renormalization scale $\mu$ after proper renormalization
- Single pole terms cancel when combining the vertex and wave function corrections, and double poles are reduced to a single pole that are taken care via the principal value prescription

# Preliminary results




Results for 5-HYP steps, $P_3 = 4\pi/L$

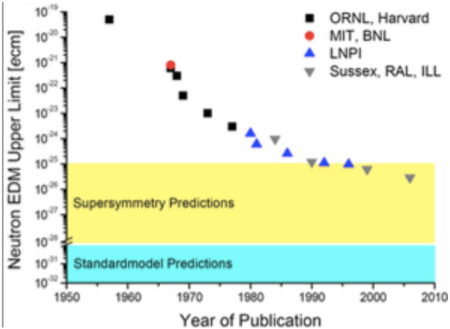


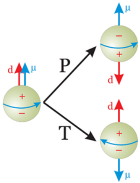

$P_3 = 6\pi/L$, Jiunn-Wei Chen *et al.*, arXiv:1603.06664

Work in progress for the renormalization

# Neutron Electric Dipole Moment (nEDM)

Probe for beyond the standard model physics




Current best upper limit : $|d_n| < 2.9 \times 10^{-26}$e cm (90% C.L. from ILL Grenoble)

# Lattice determination of Neutron Electric Dipole Moment (nEDM)

Add $\theta$-term to the Langragian $\rightarrow$ complex action

- Measure neutron energy in an external electric field
- Simulate with imaginary $\theta$, see e.g. QCDSF, Guo *et al.* 2015
- Assume $\theta$ is small and expand to first order: Compute the CP-violating form factor $F_3(0) \rightarrow$ $|d_n| = \lim_{q^2 \rightarrow 0} \frac{F_3(q^2)}{2m_N}$

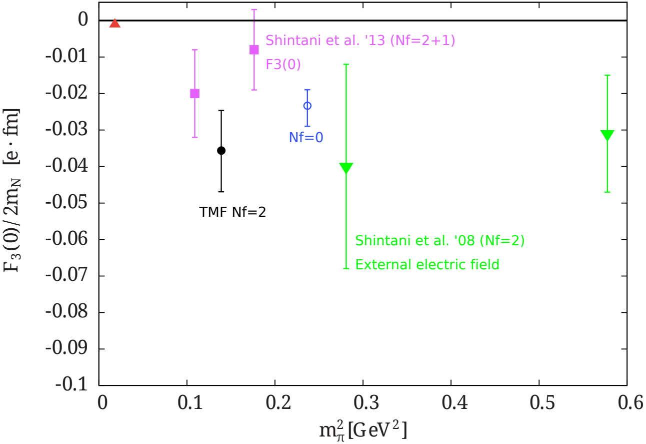
But $F_3(0)$ cannot be determined directly $\rightarrow$ use:

- Fit the $q^2$-dependence
- Use space methods to extract it

# Results on nEDM

- $N_f = 2 + 1 + 1$ twisted mass, $a$ = 0.082 fm, $m_\pi$ = 373 MeV

Use gradient flow to define the topological charge
Comparison of results




ETMC, C. Alexandrou *et al.*x, arXiv:1510.05823

Computation at the physical point under study

# Conclusions

### Future Perspectives

- Confirm $g_A$, $\langle x \rangle_{u-d}$, etc, at the physical point using $N_f = 2$ at a larger volume and with $N_f = 2+1+1$
- Provide predictions for $g_s$, $g_T$, tensor moment, sigma-terms, etc.
- Compute hadron GPDs using new techniques
- Increase statistics on the proton radius using position methods
- Compute gluonic observables
- Nucleon excited states and resonance properties
- ...

## European Twisted Mass Collaboration

European Twisted Mass Collaboration (ETMC)

Cyprus (Univ. of Cyprus, Cyprus Inst.), France (Orsay, Grenoble), Germany (Berlin/Zeuthen, Bonn, Frankfurt, Hamburg, Münster), Italy (Rome I, II, III, Trento), Netherlands (Groningen), Poland (Poznan), Spain (Valencia), Switzerland (Bern), UK (Liverpool)

Collaborators:
A. Abdel-Rehim, S. Bacchio, K. Cichy, M. Constantinou, V. Drach, E. Garcia Ramos, J. Finkenrath, K. Hadjiyiannakou, K.Jansen, Ch. Kallidonis, G. Koutsou, K. Ottnad, M. Petschlies, F. Steffens, A. Vaquero, C. Wiese