Lattice QCD: Hadron Spectrum and Thermodynamics

Z. Fodor

University of Wuppertal, Eotvos University Budapest,
John von Neumann Institute for Computing,
DESY/Zeuthen & Forschungszentrum Juelich

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what is the source of the mass of ordinary matter?
how and when was it generated?

SFB
TR55

Outline

The origin of mass of the visible Universe

source of the mass for ordinary matter (not a dark matter talk)

basic goal of LHC (Large Hadron Collider, Geneva Switzerland):

"to clarify the origin of mass"

e.g. by finding the Higgs particle, or by alternative mechanisms
order of magnitudes: 27 km tunnel and O(10) billion dollars

The vast majority of the mass of ordinary matter

ultimate (Higgs or alternative) mechanism: responsible for the mass of the leptons and for the mass of the quarks

interestingly enough: just a tiny fraction of the visible mass (such as stars, the earth, the audience, atoms)
electron: almost massless, $\approx$1/2000 of the mass of a proton
quarks (in ordinary matter): also almost massless particles

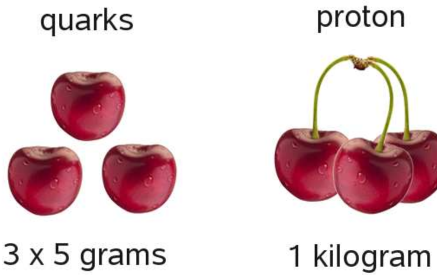
the vast majority (about 95%) comes through another mechanism
$\Longrightarrow$ this mechanism and this 95% will be the main topic of this talk

quantum chromodynamics (QCD, strong interaction) on the lattice

The mass is not the sum of the constituents' mass

usually the mass of "some ordinary thing" is just
the sum of the mass of its constituents (upto tiny corrections)

origin of the mass of the visible universe: dramatically different
proton is made up of massless gluons and almost massless quarks



mass of a quark is ≈5 MeV, that of a proton (hadron) is ≈1000 MeV

Degrees of freedom

Lagrangian contains massless gluons & almost massless quarks
we detect none of them, they are confined
we detect instead composite particles: protons, pions

proton is several hundred times heavier than the quarks
how and when was the mass generated

qualitative picture (contains many essential features):
in the early universe/heavy ion experiment: very high temperatures (motion)
it is diluted by the expansion (of the universe/experimental setup)
small fraction remained with us confined in protons
$\Rightarrow$ the kinetic energy inside the proton gives the mass ($E = mc^2$)

Hadron spectroscopy in lattice QCD

Determine the transition amplitude between:
having a "particle" at time 0 and the same "particle" at time t
$\Rightarrow$ Euclidean correlation function of a composite operator $\mathcal{O}$:

$$C(t) = \langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle$$

insert a complete set of eigenvectors $|i\rangle$

$$= \sum_i \langle 0|e^{Ht}\ \mathcal{O}(0)\ e^{-Ht}|i\rangle\langle i|\mathcal{O}^\dagger(0)|0\rangle = \sum_i |\langle 0|\mathcal{O}^\dagger(0)|i\rangle|^2\ e^{-(E_i - E_0)t},$$

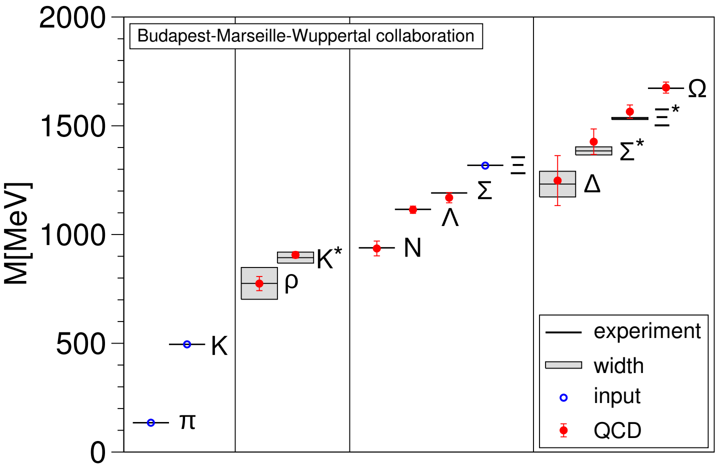
where $|i\rangle$: eigenvectors of the Hamiltonian with eigenvalue E_i.

and $$\mathcal{O}(t) = e^{Ht}\ \mathcal{O}(0)\ e^{-Ht}.$$

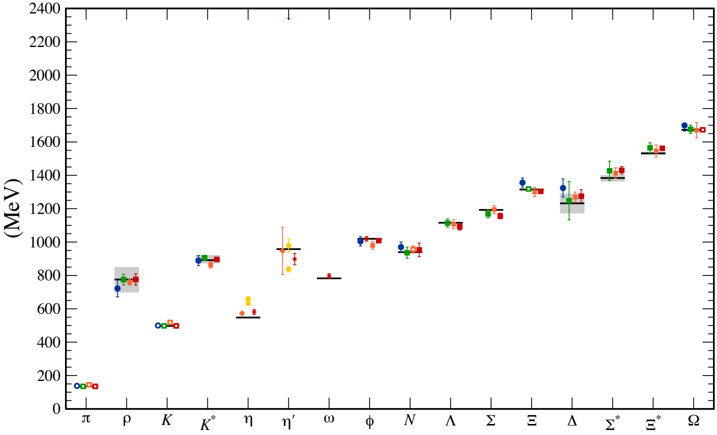
t large $\Rightarrow$ lightest states (created by $\mathcal{O}$) dominate: $C(t) \propto e^{-M\cdot t}$
$\Rightarrow$ exponential fits or mass plateaus M_t=log[C(t)/C(t+1)]

Final result for the hadron spectrum

S. Durr et al., Science 322 1224 2008



Light hadron spectrum summary: A. Kronfeld 1209.3468



results with various actions and fermion formulations(!) are the same

Introduction to isospin symmetry

Isospin symmetry: 2+1 or 2+1+1 flavor frameworks

if 'up' and 'down' quarks had identical properties (mass,charge)

$M_n = M_p, \quad M_{\Sigma^+} = M_{\Sigma^0} = M_{\Sigma^-}, \quad$ etc.

The symmetry is explicitly broken by

- up, down quark mass difference ($m_d/m_u \approx 2$)
- up, down quark electric charge difference (up: 2/3·e down:-1/3·e)

$\Rightarrow$ proton: uud=2/3+2/3-1/3=1 whereas neutron: udd=2/3-1/3-1/3=0

The breaking is large on the quark's level ($m_d/m_u \approx 2$ or charges) but small (typically sub-percent) compared to hadronic scales.

These two competing effects provide the tiny M_n-M_p mass difference $\approx$ 0.14% is required to explain the universe as we observe it

The challenge of computing $M_n - M_p$ (on the 5σ level)

Unprecedented precision is required

$\Delta M_N/M_N = 0.14\% \rightarrow$ sub-permil precision is needed to get a high significance on ΔM_N

$m_u \neq m_d$ $\rightarrow$ 1+1+1+1 flavor lattice calculations are needed $\rightarrow$ algorithmic challenge

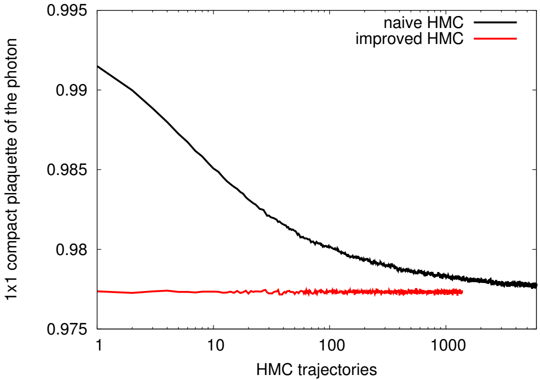
(Previous QCD calculations were typically 2+1 or 2+1+1 flavors)

Inclusion of QED: no mass gap

$\rightarrow$ power-like finite volume corrections expected

$\rightarrow$ long range photon field may cause large autocorrelations

Autocorrelation of the photon field

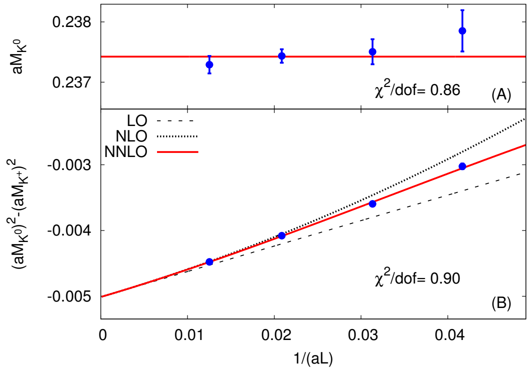


Standard HMC has $\mathcal{O}(1000)$ autocorrelation

Improved HMC has none (for the pure photon theory)

Small coupling to quarks introduces a small autocorrelation

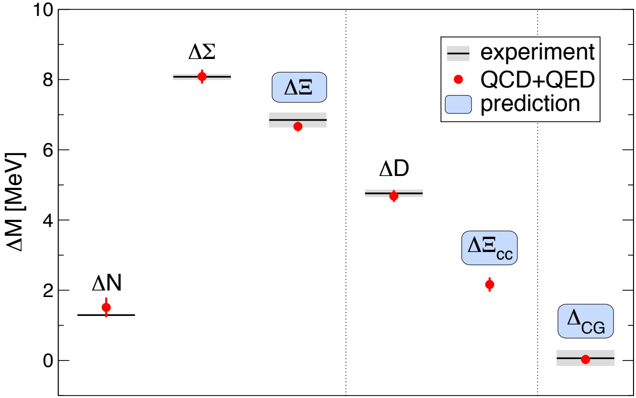
Finite V dependence of the kaon mass



Neutral kaon shows no volume dependence
Volume dependence of the K splitting is perfectly described
$1/L^3$ order is significant

Isospin splittings

splittings in channels that are stable under QCD and QED:



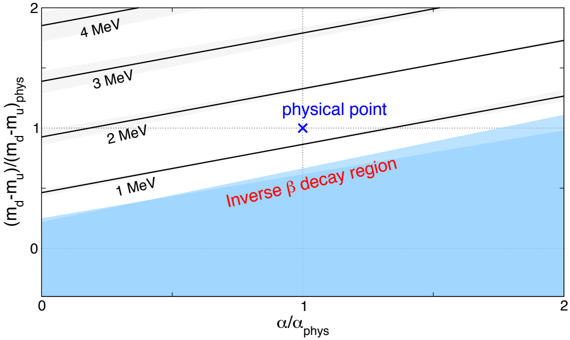
ΔM_N, ΔM_Σ and ΔM_D splittings: post-dictions

ΔM_Ξ, $\Delta M_{\Xi_{cc}}$ splittings and Δ_{CG}: predicitions

Quantitative anthropics

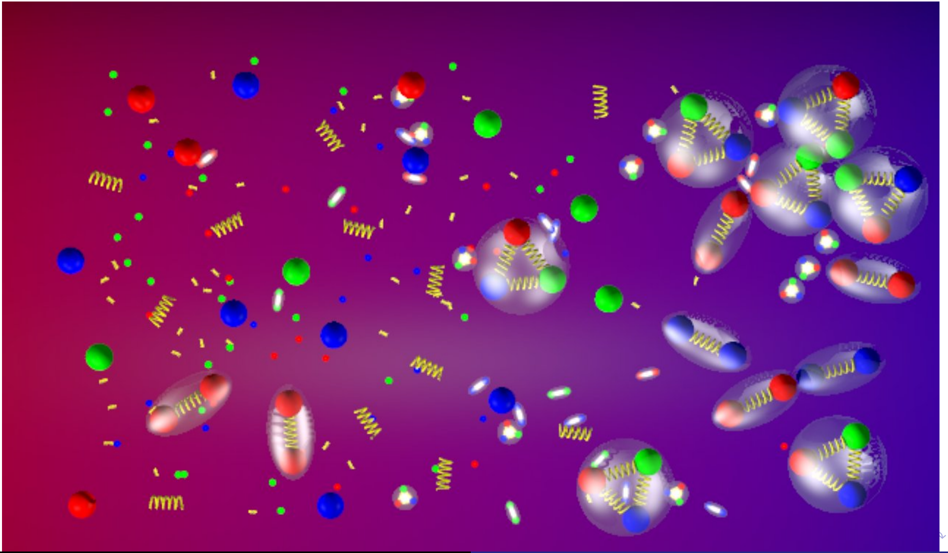
Precise scientific version of the great question:
Could things have been different (string landscape)?

eg. big bang nucleosynthesis & today's stars need $\Delta M_N \approx$ 1.3 MeV



(lattice message: too large or small α would shift the mass)

Reality: smooth analytic transition (cross-over)



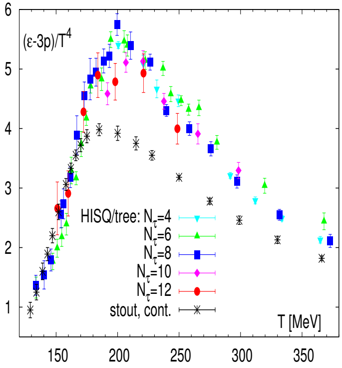
T_c and the equation of state continuum results

long standing discrepancy in T_c is resolved: 157(5) or 154(9) MeV

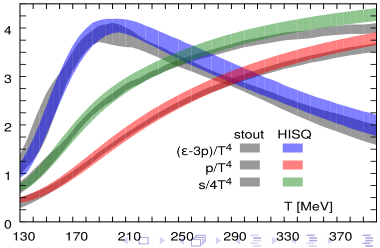
equation of state was also disputed for a long time

is the peak 4? or is it much higher?

before 2014

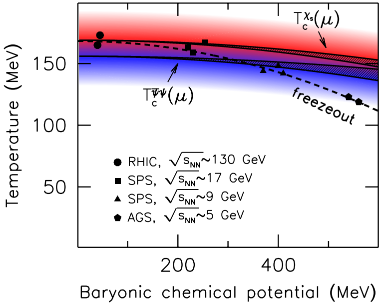


after 2014



Continuum prediction for the curvature: full result

G. Endrodi, Z. Fodor, S.D. Katz, K.K. Szabo, JHEP 1104 2011 001



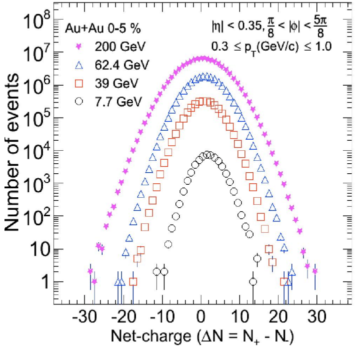
dashed line: freeze-out curve from experiments

lower solid line: T_c from the chiral condensate
upper solid line: T_c from the strange susceptibility

bands (red and blue) indicate the widths of the transition lines
the widths remain in this order approximately constant in μ

Fluctuations in experiments

what fluctuates in a heavy-ion collision?
we have a fixed number of conserved charges (Z=82, A=207)?



imposing kinematical constraints:
consider particles coming only from
a small part of the whole system

charges from subvolumes
will fluctuate
from one event to the other

small enough subvolumes to be a grand canonical ensemble
yet large enough to behave like an ensemble

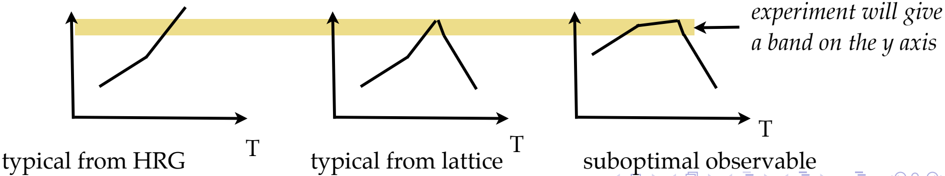
Thermometer/baryometer

older idea, new formulations Gupta et al. Science 332 (2011) 1525, Karsch CEJP 10 (2012) 1234

before freeze-out the system is described with a time-dependent temperature and baryo, charge and strange chemical potentials

assume/test: after freeze-out net bayron, charge and strangeness reflects a system in equilibrium at the freeze-out tempearture

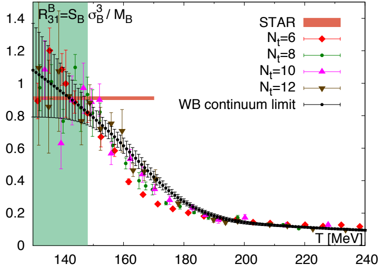
since the fluctuations T and μ dependent, one can compare experimental measurements and lattice predictions to get T and μ
use ratios to eliminate the volume dependence (V is unknown)



Baryon fluctuations: thermometer

Borsanyi et al. Wuppertal-Budapest Coll. Phys.Rev.Lett. 111, 062005 (2013); Phys.Rev.Lett. 113, 052301 (2014)

skewness (third moment) and variance (second moment) ratios for B

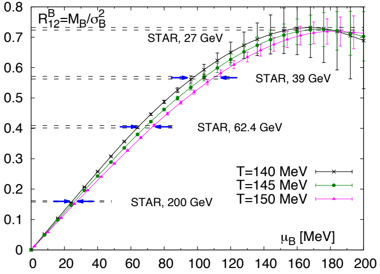


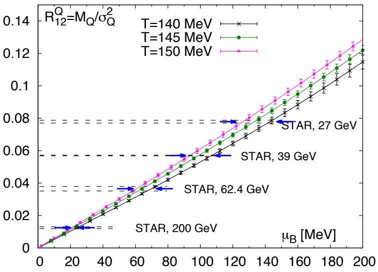
comparison between lattice and STAR for $S_p\sigma_p^3/M_p$ (Q: large errors)
average of the 27, 39, 62, 200 GeV runs (T_c weakly depends on μ)
one can directly read off the temperature: not very precise
lattice: limiting factor $\Rightarrow$ T$<$148 MeV (c_s gave 145(5) MeV)

M/σ^2: good baryometer

Borsanyi et al. Wuppertal-Budapest Coll. Phys.Rev.Lett. 111, 062005 (2013); Phys.Rev.Lett. 113, 052301 (2014)

use M/σ^2 both in the baryon and in the charge sector





compare lattice and the STAR results in a T range 140-150 MeV
directly read off the chemical potential (ordering is different)
consistency between the two values of the μ values

1. how is the mass of ordinary matter generated (what is its source)
   - more than 99.9% of the mass of the visible universe is made up from protons and neutrons (ordinary matter)
     95% of the mass of a proton comes from the kinetic energy within the proton: very different from any other mass
   - the standard model of particle physics (most particularly the theory of strong interaction, QCD) can explain this phenomena
   - full ab-initio calculation of the masses (controlling all systematic uncertainties) $\Rightarrow$ M_N, isospin splittings
2. how was the mass of ordinary matter generated (early universe)
   - transition between the low temperature phase (dominated by color-neutrtal hadrons) and the high temperature phase (dominated by colored objects) $\Rightarrow$ heavy ion collisions
   - though these two phases are fundamentally different there is no singularity, just an analytic cross-over $\Rightarrow$ T_c, EoS, phase diagram