Effective field theories for QCD at non-zero temperature

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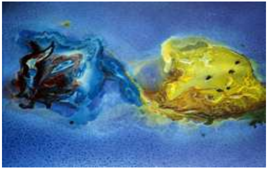
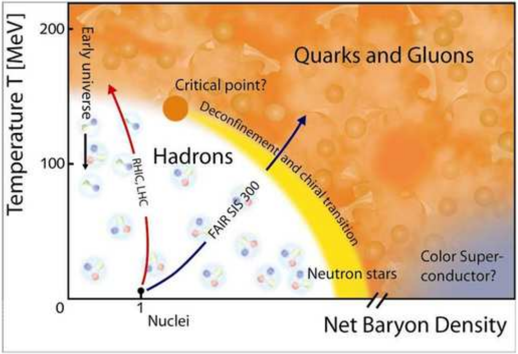


Outline

1. Motivation and introduction
2. EFTs for heavy quarkonium at finite T
3. Heavy quarkonium and anisotropic QGP
4. Conclusions and Outlook

When do we find a hot QCD medium?

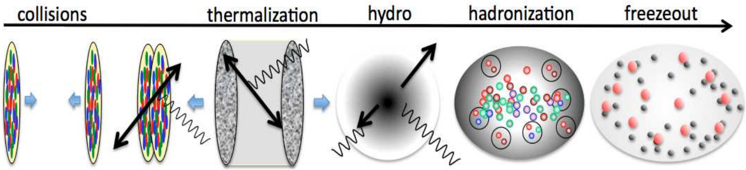
- Transition of nuclear matter into a deconfined phase at high temperature
- Hot medium made of interacting quark and gluons → Quark-Gluon Plasma



- Cosmology: early Universe likely was an hot and dense medium

How can we hope to reproduce the QGP?

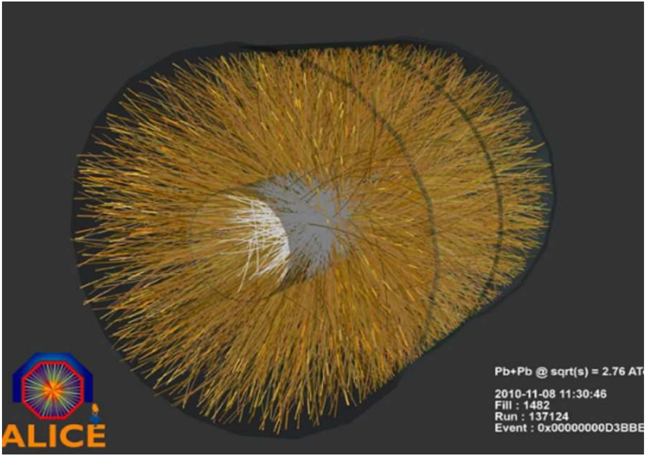
- the high energy heavy-ion colliders, such as the LHC, are the right place



Time scales for Quark Gluon Plasma

- Formation time $\tau_0 \sim 1$ fm (in physical units 3.3×10^{-24} s)
- Life time of equilibrated deconfined phase $\tau \sim 10$ fm

What comes out from QGP?



Very complicated final state to study

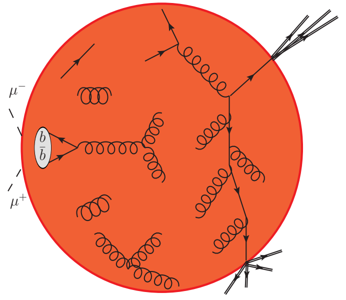
- demanding and challenging experimental analysis
- clean probes are needed...is it possible to have any?

Hard probes for QGP

How can we get information about a so short-lived state?

- A possible way is by exploiting hard probes,
  1. jet quenching *X. N. Wang and M. Gyulassy (1994)*
  2. quarkonia suppression *T. Matsui and H Satz (1986)*

Heavy $Q\bar{Q}$ in medium



- Medium effect can dissociate the $Q\bar{Q}$

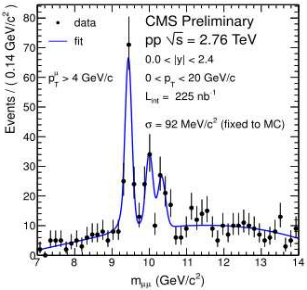
$$V(r) = -C_F \frac{\alpha_s}{r} \to -C_F \alpha_s \frac{e^{-m_D(T)r}}{r}$$

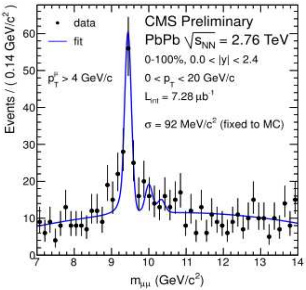
- At some T_d the bound state ceases to exist: $m_D > 1/r$

$\Rightarrow$ Suppressed yield of dilepton decay channel $R_{AA}(Q\bar{Q})$

Quarkonium dissociation at LHC

- Suppression pattern for the $\Upsilon(nS)$ family at CMS
- The more bounded states are less suppressed





- How can we better understand this evidence?

Energy scales for heavy quarkonium

Many energy scales are there: (*perturbative regime and weak coupling*)

1) Non-relativistic scales (bound state):

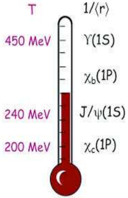
$$m \gg mv \; (1/r) \gg mv^2 \; (E) \gg \Lambda_{\rm QCD}$$

2) Thermodynamic scales:

$$\pi T \gg m_D$$

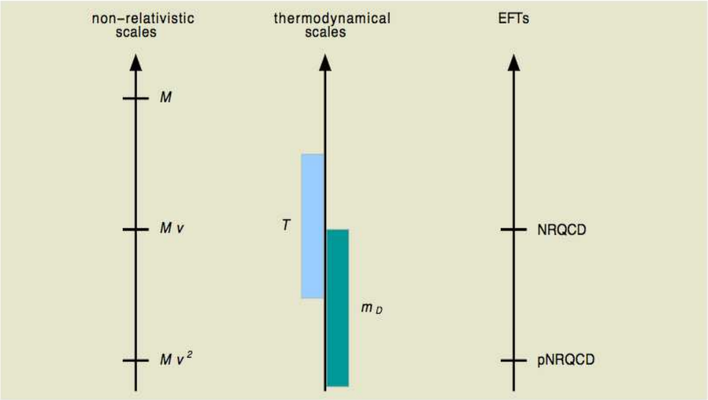
- A weakly coupled quarkonium could be the $\Upsilon(1S)$

$$m_b \approx 5\,\text{GeV} > m_b\alpha_s \approx 1.5\,\text{GeV} > \pi T \approx 1\,\text{GeV} \; > m_b\alpha_s^2 \approx 0.5\,\text{GeV} > m_D \approx \Lambda_{\rm QCD}$$



- $\Upsilon(1S)$ may still survive in QGP and be perturbative
- Study the thermal effects on the $\Upsilon(1S)$ spectrum

EFTs for QCD at $T = 0$ and $T \neq 0$



- Towers of EFTs suitable to describe the quarkonium system at a given energy scale

EFT for QCD

How to disentangle the different scales from

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{\mu\nu,a} F^a_{\mu\nu} + \bar{Q}\left(i\not{D} - M\right) Q + \mathcal{L}_{Light}$$

- A useful way: Effective Field Theory
  1. Select the right degrees of freedom
  2. Build the effective Lagrangian
  3. Perform calculations with a simplified version of $\mathcal{L}_{QCD}$

- We are interested in the spectrum of $Q\bar{Q} \Rightarrow$ binding energy (Mv^2)

- The EFT is pNRQCD: $E \sim Mv^2$, *N. Brambilla, A. Pineda, J. Soto and A. Vairo (1999)*
- The Lagrangian acquires a Schrodinger equation-like form

$$\boxed{V(r) \text{ obtained rigourosly form QCD}}$$

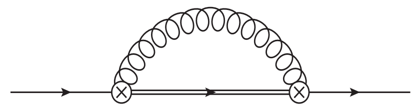
pNRQCD in vacuum

pNRQCD Lagrangian

- Assuming the hierarchy $m \gg \frac{1}{r} \gg E$

$$\mathcal{L}_{\rm pNRQCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \sum_i \bar{q}_i i \not{D} q_i + \int d^3\mathbf{r}\, Tr\left\{ S^\dagger \left(i\partial_0 - h_s\right) S + O^\dagger \left(iD_0 - h_o\right) O \right.$$
$$\left. + V_A \left(O^\dagger \mathbf{r}\cdot g\mathbf{E} S \ + h.c.\right) + \frac{V_B}{2} O^\dagger \left\{\mathbf{r}\cdot g\mathbf{E}, O\right\} + \cdots \right\}$$

- where we have defined
  1. Singlet field S, Octet field O
  2. $h_{s,o} = \frac{\mathbf{p}^2}{m} + V^{(0)}_{s,o} + \frac{V^{(1)}_{s,o}}{M} + \ldots$
  3. $V^{(0)}_s = -C_F \frac{\alpha_s}{r}$ and $V^{(0)}_o = \frac{1}{2N_c}\frac{\alpha_s}{r}$

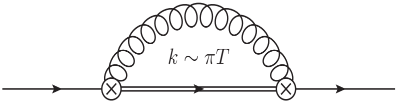


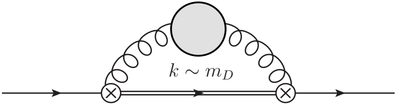
All the scales bigger than Mv^2 contribute to the potential $V^{(0)}$

And if the temperature enters...

- We take the scales πT and m_D bigger than the binding energy

$$1/r \gg \pi T \gg m_D \gg mv^2,$$ *N. Brambilla, J. Ghiglieri, P. Petreczky and A. Vairo (2008)*





We do a matching from

- pNRQCD $\rightarrow$ pNRQCD$_{\mathrm{HTL}}$, where

$$V_s(r,T,m_D) = -C_F\frac{\alpha_s}{r} + \delta V_R(r,T,m_D) + i\delta V_I(r,T,m_D)$$

- δV_R: mass of $Q\bar{Q}$ state, δV_I: related to the width

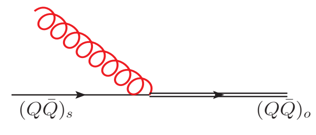
$$\frac{i}{k^0 - E + i\frac{\Gamma}{2}} \Rightarrow \begin{cases} E = \langle \mathrm{Re}(V) \rangle \\ \Gamma = -2\langle \mathrm{Im}(V) \rangle \end{cases}$$

What is a thermal width?

- The interactions with the medium can break the $Q\bar{Q}$ bound state

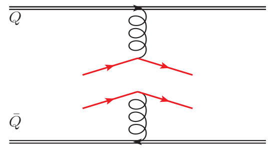
$$\delta V_I(r,T,m_D) = -\frac{N_c^2 C_F}{6}\alpha_s^3 T$$
$$+\frac{C_F}{6}\alpha_s r^2 T m_D^2\left(2\gamma_E - \log\frac{T^2}{m_D^2} - 1 - 4\log 2 - 2\frac{\zeta'(2)}{\zeta(2)}\right) + \frac{4\pi}{9}\log 2 N_c C_F \alpha_s^2 r^2 T^3$$

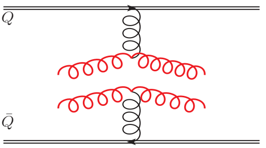
Singlet to octet thermal break-up: *dominant if* $E \gg m_D$



- Singlet absorbs a gluon from the medium

Landau damping phenomenon: *dominant if* $m_D \gg E$





Anisotropy in QGP

QGP is a rather complicated system...

- Longitudinal (beam axis) expansion is bigger than the radial expansion

1) Different temperatures
2) Anisotropic parton momenta

Local momentum anisotropy : ξ

- The anisotropy effects on the $Q\bar{Q}$ spectrum studied for $\pi T \gg 1/r \sim m_D$

Y. Burnier, M.Laine and M. Vepsalainen (2009), A. Dimitru, Y. Gou, and M. Strickland (2009)

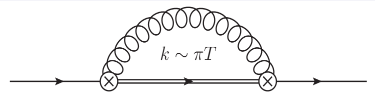
- We can address within EFTs the case $1/r \gg \pi T \gg E \gg m_D$

Modelling the anisotropy

$$f(\mathbf{k}) = f_{iso}\left(\sqrt{\mathbf{k}^2 + \xi(\mathbf{k}\cdot\mathbf{n})^2}\right) = \left(e^{\frac{\sqrt{\mathbf{k}^2+\xi(\mathbf{k}\cdot\mathbf{n})^2}}{T}} - 1\right)^{-1}$$

We start with pNRQCD: $1/r \gg \pi T \gg E \gg m_D$

$$\mathcal{L}_{\rm pNRQCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \sum_i \bar{q}_i i\not{D} q_i + \int d^3\mathbf{r}\, Tr\left\{S^\dagger\left(i\partial_0 - h_s\right)S + O^\dagger\left(iD_0 - h_o\right)O\right.$$
$$\left. + V_A\left(O^\dagger \mathbf{r}\cdot g\mathbf{E}S + h.c.\right) + \frac{V_B}{2}O^\dagger\left\{\mathbf{r}\cdot g\mathbf{E}, O\right\} + \cdots\right\}$$



- Match pNRQCD onto $\mathrm{pNRQCD_{HTL}}$
- T encoded in a redefined potential

$$\delta\Sigma(E) = -ig^2 C_F \frac{r^i}{D-1}\mu^{4-D}\int \frac{d^D k}{(2\pi)^D}\frac{i}{E-h_o-k_0+i\eta}\, k_0^2 D_{ii}(k_0,k) r^i$$

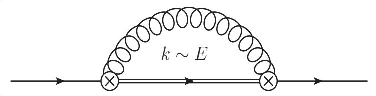
- Momentum region $k_0 \sim \pi T$ and $k \sim \pi T$. Since $\pi T \gg (E - h_0)$

$$\frac{i}{E-h_o-k_0+i\eta} = \frac{i}{-k_0+i\eta} - i\frac{E-k_0}{(-k_0+i\eta)^2} + \cdots$$

- At leading order in α_s we obtain

$$\delta V_s(r,T,\xi) = \frac{\pi\alpha_s C_F T^2}{3}\left(\frac{2}{m} + \frac{N_c\alpha_s r}{4} + \frac{N_c\alpha_s(\mathbf{r}\cdot\mathbf{n})^2}{4r}\right)\frac{\arctan\xi}{\xi}$$
$$+\frac{\pi N_c C_F\alpha_s^2 T^2}{12\xi r}\left(1 - \frac{\arctan\sqrt{\xi}}{\sqrt{\xi}}\right)\left(r^2 - 3(\mathbf{r}\cdot\mathbf{n})^2\right)$$

Strategy of the calculation: $1/r \gg \pi T \gg E \gg m_D$



- Effect of the scale E within pNRQCD$_{\mathrm{HTL}}$
- Octet unexpanded,
- $f(\mathbf{k}) \simeq \frac{T}{k\sqrt{1+\xi\cos^2\theta}} + \ldots$

- Thermal width from the scale E: $\Gamma = -2\,\langle n,l|\mathrm{Im}\delta\Sigma(E)|n,l\rangle$

$$\Gamma(T,\xi) = \left(\frac{1}{3}N_c^2 C_F \alpha_s^3 T + \frac{4}{3}\frac{C_F^2\alpha_s^3}{n^2}T(C_F+N_c)\right)\frac{\sinh^{-1}(\sqrt{\xi})}{\sqrt{\xi}}$$

$$+\left(\frac{1}{4}N_c^2 C_F \alpha_s^3 T + \frac{C_F^2\alpha_s^3}{n^2}T\left(C_F-\frac{N_c}{2}\right)\right)\frac{\left(1+\frac{2}{3}\xi\right)\sinh^{-1}(\sqrt{\xi})-\sqrt{\xi(1+\xi)}}{\sqrt{\xi^3}}\langle 2\,\ell\,0\,0|\ell\,0\rangle\langle 2\,\ell\,0\,m|\ell\,m\rangle$$

- Check with $\xi \to 0$, we recover the known result

  N. Brambilla, M. A. Escobedo, J. Ghiglieri, J. Soto and A. Vairo (2010)

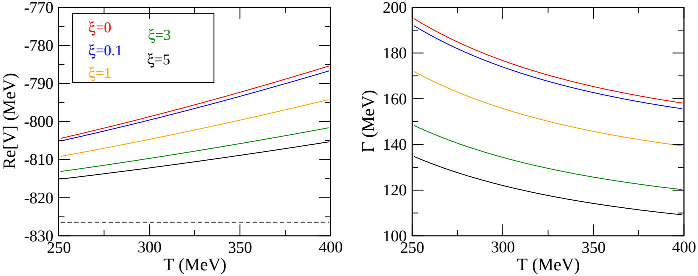
Check with known limits

Real part of the potential (for $\Upsilon(1S)$)

$$V_s(r,T,\xi) \to -c_F\frac{\alpha_s}{r} + \frac{\pi}{9}N_c C_F \alpha_s^2 T^2 r + \frac{2\pi}{3m_b} C_F \alpha_s T^2 + \mathcal{O}(\xi)$$

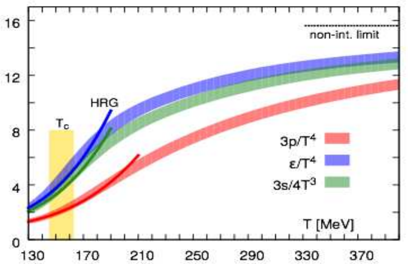
Thermal width

$$\Gamma(T,\xi) \to \frac{1}{3}N_c^2 C_F \alpha_s^3 T + \frac{4}{3}C_F^2\alpha_s^3 T(C_F + N_c) + \mathcal{O}(\xi)$$



Lattice and Hot QCD

Transition temperature: non-perturbative process

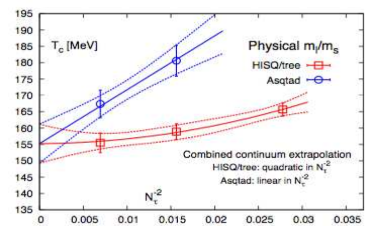


- $\epsilon(T, \mu_B = 0)$ against the temperature
- 150 MeV $< T <$ 350 MeV energy density increases
- Change of the degrees of freedom (hadrons $\rightarrow$ QGP)

A. Bazavov et al (2014)

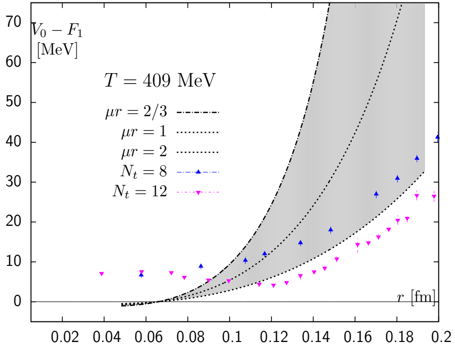
- More precise lattice calculation for the T_c
- Very important to shape the crossover in heavy-ion collisions

A. Bazavov et al (2012)



Work in progress...

- Interplay between pNRQCD and Lattice at finite temperature



A. Bazavov, M. Berwein, N. Brambilla, P. Petreczky, A. Vairo and J. Weber

- V_0 static quark-antiquark potential at $T = 0$
- F_1 free energy of the quark-antiquark system

- At short distances thermal effects should vanish: $F_1 \to V_0$
- At short distances perturbative calculation should describe the lattice data

Conclusions and Outlook

- Study the QCD phase diagram at finite temperature and density
- Hot QCD medium is established in heavy-ion collisions

- Heavy quarkonia is a useful probe to address the QGP properties
- $Q\bar{Q}$ in QGP is a multi-scale system: effective field theories

- Clear identification of relevant degrees of freedom and physics at different scales
- Temperature and anisotropy of the system taken into account

- No weak coupling regime $\rightarrow$ Non-perturbative techniques: Lattice
- Lattice helps in shaping the QCD phase diagram
- Interplay between EFT and Lattice Gauge Theory