Effective field theories for QCD at non-zero temperature

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1 Motivation and introduction

2 EFTs for heavy quarkonium at finite T

3 Heavy quarkonium and anisotropic QGP

4 Conclusions and Outlook
Motivation and introduction

When do we find a hot QCD medium?

- Transition of nuclear matter into a deconfined phase at high temperature
- Hot medium made of interacting quark and gluons $\rightarrow$ Quark-Gluon Plasma

Cosmology: early Universe likely was an hot and dense medium
How can we hope to reproduce the QGP?

- the high energy heavy-ion colliders, such as the LHC, are the right place

**Time scales for Quark Gluon Plasma**

- Formation time $\tau_0 \sim 1$ fm (in physical units $3.3 \times 10^{-24}$ s)
- Life time of equilibrated deconfined phase $\tau \sim 10$ fm
Motivation and introduction

What comes out from QGP?

Very complicated final state to study
- demanding and challenging experimental analysis
- clean probes are needed...is it possible to have any?

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Motivation and Introduction

Hard Probes for QGP

How can we get information about a so short-lived state?

- A possible way is by exploiting hard probes,
  - jet quenching X. N. Wang and M. Gyulassy (1994)
  - quarkonia suppression T. Matsui and H Satz (1986)

Heavy $Q\bar{Q}$ in medium

Medium effect can dissociate the $Q\bar{Q}$

$$V(r) = -C_F \frac{\alpha_s}{r} \rightarrow -C_F \alpha_s \frac{e^{-m_D(T)r}}{r}$$

- At some $T_d$ the bound state ceases to exist: $m_D > 1/r$

$\Rightarrow$ Suppressed yield of dilepton decay channel $R_{AA}(Q\bar{Q})$
Suppression pattern for the $\Upsilon(nS)$ family at CMS

The more bounded states are less suppressed

How can we better understand this evidence?
EFTs for heavy quarkonium at finite $T$

Energy scales for heavy quarkonium

Many energy scales are there: (perturbative regime and weak coupling)

1) Non-relativistic scales (bound state):

\[ m \gg mv \left( \frac{1}{r} \right) \gg mv^2 (E) \gg \Lambda_{QCD} \]

2) Thermodynamic scales:

\[ \pi T \gg m_D \]

- A weakly coupled quarkonium could be the $\Upsilon(1S)$

\[ m_b \approx 5 \text{ GeV} > m_b \alpha_s \approx 1.5 \text{ GeV} > \pi T \approx 1 \text{ GeV} > m_b \alpha_s^2 \approx 0.5 \text{ GeV} > m_D \approx \Lambda_{QCD} \]

- $\Upsilon(1S)$ may still survive in QGP and be perturbative

- Study the thermal effects on the $\Upsilon(1S)$ spectrum
Towers of EFTs suitable to describe the quarkonium system at a given energy scale
EFT for QCD

How to disentangle the different scales from $L_{QCD}$

\[ L_{QCD} = -\frac{1}{4} F_{\mu\nu}^{a} F_{\mu\nu}^{a} + \bar{Q} \left( i \partial - M \right) Q + L_{Light} \]

- A useful way: Effective Field Theory
  1. Select the right degrees of freedom
  2. Build the effective Lagrangian
  3. Perform calculations with a simplified version of $L_{QCD}$

- We are interested in the spectrum of $Q\bar{Q} \Rightarrow$ binding energy $(Mv^2)$

- The EFT is pNRQCD: $E \sim Mv^2$, N. Brambilla, A. Pineda, J. Soto and A. Vairo (1999)
- The Lagrangian acquires a Schrödinger equation-like form
  \[ V(r) \text{ obtained rigourosly form QCD} \]
EFTs for heavy quarkonium at finite T

pNRQCD IN VACUUM

pNRQCD LAGRANGIAN

- Assuming the hierarchy $m \gg \frac{1}{r} \gg E$

\[
\mathcal{L}_{pNRQCD} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \sum_i \bar{q}_i i D q_i + \int d^3 r Tr \left\{ S^\dagger (i \partial_0 - h_s) S + O^\dagger (i D_0 - h_o) O \\
+ V_A (O^\dagger r \cdot g E S + h.c.) + \frac{V_B}{2} O^\dagger \{ r \cdot g E, O \} + \cdots \right\}
\]

- where we have defined

  - Singlet field $S$, Octet field $O$
  - $h_{s,o} = \frac{p^2}{m} + V_{s,o}^{(0)} + \frac{V_{s,o}^{(1)}}{M} + \cdots$
  - $V_s^{(0)} = -C_F \frac{\alpha_s}{r}$ and $V_o^{(0)} = \frac{1}{2N_c} \frac{\alpha_s}{r}$

All the scales bigger than $Mv^2$ contribute to the potential $V^{(0)}$
We take the scales $\pi T$ and $m_D$ bigger than the binding energy

$$1/r \gg \pi T \gg m_D \gg mv^2,$$


\[ k \sim \pi T \quad k \sim m_D \]

We do a matching from $pNRQCD \rightarrow pNRQCD_{HTL}$, where

\[ V_s(r, T, m_D) = -C_F \frac{\alpha_s}{r} + \delta V_R(r, T, m_D) + i\delta V_I(r, T, m_D) \]

$\delta V_R$: mass of $Q\bar{Q}$ state, $\delta V_I$: related to the width

\[ \frac{i}{k^0 - E + i\frac{\Gamma}{2}} \Rightarrow \begin{cases} E = \langle \text{Re}(V) \rangle \\ \Gamma = -2\langle \text{Im}(V) \rangle \end{cases} \]
The interactions with the medium can break the $Q\bar{Q}$ bound state

$$
\delta V_I(r, T, m_D) = -\frac{N_c^2 C_F}{6} \alpha_s^3 T \\
+ \frac{C_F}{6} \alpha_s r^2 T m_D^2 \left( 2\gamma_E - \log \frac{T^2}{m_D^2} - 1 - 4 \log 2 - 2 \frac{\zeta'(2)}{\zeta(2)} \right) + \frac{4\pi}{9} \log 2N_c C_F \alpha_s^2 r^2 T^3
$$

**Singlet to octet thermal break-up:** dominant if $E \gg m_D$

- Singlet absorbs a gluon from the medium

**Landau damping phenomenon:** dominant if $m_D \gg E$

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Anisotropy in QGP

QGP is a rather complicated system...

- Longitudinal (beam axis) expansion is bigger than the radial expansion
  
  1) Different temperatures
  2) Anisotropic parton momenta

Local momentum anisotropy: $\xi$

The anisotropy effects on the $Q\bar{Q}$ spectrum studied for $\pi T \gg 1/r \sim m_D$


We can address within EFTs the case $1/r \gg \pi T \gg E \gg m_D$

Modelling the anisotropy

$$f(k) = f_{iso} \left( \sqrt{k^2 + \xi (k \cdot n)^2} \right) = \left( e^{\frac{\sqrt{k^2 + \xi (k \cdot n)^2}}{T}} - 1 \right)^{-1}$$
We start with pNRQCD: \( 1/r \gg \pi T \gg E \gg m_D \)

\[
\mathcal{L}_{\text{pNRQCD}} = -\frac{1}{4} F_{\mu \nu}^a F^{a \mu \nu} + \sum_i \bar{q}_i i \slashed{D} q_i + \int d^3 r Tr \left\{ S^\dagger (i \partial_0 - h_s) S + O^\dagger (iD_0 - h_o) O \right\} \\
+ V_A \left( O^\dagger \mathbf{r} \cdot g \mathbf{E} S + h.c. \right) + \frac{V_B}{2} O^\dagger \{ \mathbf{r} \cdot g \mathbf{E}, O \} + \cdots 
\]

- Match pNRQCD onto pNRQCD\text{HTL}
- \( T \) encoded in a redefined potential

\[
\delta \Sigma(E) = -ig^2 C_F r^i \mu^{4-D} \int \frac{d^D k}{(2\pi)^D} \frac{i}{E-h_0-k_0+i\eta} k_i^2 D_{ii} (k_0, k) r^i 
\]

- Momentum region \( k_0 \sim \pi T \) and \( k \sim \pi T \). Since \( \pi T \gg (E - h_0) \)

\[
\frac{i}{E-h_0-k_0+i\eta} = \frac{i}{-k_0+i\eta} - i \frac{E-k_0}{(-k_0+i\eta)^2} + \cdots 
\]

- At leading order in \( \alpha_s \) we obtain

\[
\delta V_s(r, T, \xi) = \frac{\pi \alpha_s C_F T^2}{3} \left( \frac{2}{m} + \frac{N_c \alpha_s r}{4} + \frac{N_c \alpha_s (\mathbf{r} \cdot \mathbf{n})^2}{4r} \right) \frac{\arctan \xi}{\xi} \\
+ \frac{\pi N_c C_F \alpha_s^2 T^2}{12 \xi r} \left( 1 - \frac{\arctan \sqrt{\xi}}{\sqrt{\xi}} \right) (r^2 - 3(\mathbf{r} \cdot \mathbf{n})^2)
\]
**Strategy of the Calculation:** \(1/r \gg \pi T \gg E \gg m_D\)

- Effect of the scale \(E\) within pNRQCD_{HTL}
- Octet unexpanded,
- \(f(k) \sim \frac{T}{k \sqrt{1 + \xi \cos^2 \theta}} + \ldots\)

- Thermal width from the scale \(E\): \(\Gamma = -2 \langle n, l | \text{Im} \delta \Sigma(E) | n, l \rangle\)

\[
\Gamma(T, \xi) = \left(\frac{1}{3} N_c^2 C_F \alpha_s^3 T + \frac{4}{3} \frac{C_F^2 \alpha_s^3}{n^2} T (C_F + N_c)\right) \frac{\sinh^{-1}(\sqrt{\xi})}{\sqrt{\xi}} \sinh^{-1}(\sqrt{\xi}) - \sqrt{\xi} (1 + \xi) \frac{1}{\sqrt{\xi^3}} \langle 2 \ell 0 0 | \ell 0 \rangle \langle 2 \ell 0 m | \ell m \rangle
\]

- Check with \(\xi \to 0\), we recover the known result

  \[N.\;Brambilla,\;M.\;A.\;Escobedo,\;J.\;Ghiglieri,\;J.\;Soto\;and\;A.\;Vairo\; (2010)\]
**Check with known limits**

### Real part of the potential (for $\Upsilon(1S)$)

$$V_s(r, T, \xi) \rightarrow -c_F \frac{\alpha_s}{r} + \frac{\pi}{9} N_c C_F \alpha_s^2 T^2 r + \frac{2\pi}{3m_b} C_F \alpha_s^2 T^2 + O(\xi)$$

### Thermal width

$$\Gamma(T, \xi) \rightarrow \frac{1}{3} N_c^2 C_F \alpha_s^3 T + \frac{4}{3} C_F^2 \alpha_s^3 T (C_F + N_c) + O(\xi)$$

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**Graphs:**
- **Left graph:** Plot of $\text{Re}[V]$ vs. $T$ for different values of $\xi$.
- **Right graph:** Plot of $\Gamma$ vs. $T$ for different values of $\xi$. The graphs show how the real part of the potential and the thermal width vary with temperature and the parameter $\xi$.
**LATTICE AND HOT QCD**

**TRANSITION TEMPERATURE: NON-PERTURBATIVE PROCESS**

- $\epsilon(T, \mu_B = 0)$ against the temperature
- $150 \text{ MeV} < T < 350 \text{ MeV}$ energy density increases
- Change of the degrees of freedom (hadrons $\rightarrow$ QGP)

*A. Bazavov et al (2014)*

- More precise lattice calculation for the $T_c$
- Very important to shape the crossover in heavy-ion collisions

*A. Bazavov et al (2012)*
Interplay between pNRQCD and Lattice at finite temperature

A. Bazavov, M. Berwein, N. Brambilla, P. Petreczky, A. Vairo and J. Weber

- $V_0$ static quark-antiquark potential at $T = 0$
- $F_1$ free energy of the quark-antiquark system

At short distances thermal effects should vanish: $F_1 \rightarrow V_0$

At short distances perturbative calculation should describe the lattice data
Conclusions and Outlook

- Study the QCD phase diagram at finite temperature and density
- Hot QCD medium is established in heavy-ion collisions

- Heavy quarkonia is a useful probe to address the QGP properties
- $Q\bar{Q}$ in QGP is a multi-scale system: effective field theories

- Clear identification of relevant degrees of freedom and physics at different scales
- Temperature and anisotropy of the system taken into account

- No weak coupling regime $\rightarrow$ Non-perturbative techniques: Lattice
- Lattice helps in shaping the QCD phase diagram
- Interplay between EFT and Lattice Gauge Theory